RESEARCH ARTICLE

Turbulence modulation across the interface of a large deformable drop

Luca Scarbolo* and Alfredo Soldati

Department of Electrical, Industrial and Mechanical Engineering, University of Udine, Italy

In this work momentum transfer across the interface of a single, large deformable droplet in a turbulent channel flow is investigated by direct numerical simulations of turbulence coupled with the diffuse interface approach for interface tracking. A wide range of Weber numbers (ratio between inertia and surface tension) is explored, focusing the analysis to cases of large non-breaking droplets where the two fluids have same density and viscosity and the shear Reynolds number is $Re_\tau = 100$. Results show the presence of wall-blocking effects where the interface-normal velocity is deflected to the interface-tangential direction. These effects depend on surface tension: more deformable is the droplet, the smaller is velocity deflection. In the proximity of the droplet, at distances comparable to the droplet diameter, the interface-normal velocity shows a decay rate that is similar to that observed in the proximity of flat interfaces. As a result, convective effects at the droplet interface are damped, momentum transport across the interface is limited and turbulence intensity inside of the droplet is dramatically reduced.

Keywords: Droplet; Turbulence; Diffuse Interface;

1. Introduction

Turbulent transport phenomena in multiphase dispersed flows play a central role in many industrial and environmental applications. For example prediction of mass and energy transfer between different phases strongly affects the design of chemical reactors, steam reformers and heat exchangers [1–6]. In such systems the overall mass and energy transfer between the fluid phases depends on the local transport phenomena taking place at the interface of each droplet or bubble belonging to the system. Turbulence structures at the liquid-liquid or gas-liquid interfaces, that are responsible for the intensity of the local convective effects, are also responsible for the interface deformations that are governed by the balance between surface tension and local turbulence kinetic energy. In addition the droplet surface acts on the surrounding fluid via a stress tensor which is proportional to the surface tension and the interface local curvature. Those stresses are responsible for the modification of the local turbulent features resulting in a non-trivial coupling between turbulence and surface tension in the proximity of the interface. Thus the accurate numerical description of the transport phenomena involved in those systems requires the detailed resolution of all the turbulent scales at the fluid-fluid interface coupled with the tracking of the deformable interfaces and the surface tension forcing. As a result, computational resources required are beyond those currently available [7]

*Corresponding author. Email: luca.scarbolo@uniud.it

ISSN: 1468-5248 (online only)
© 2011 Taylor & Francis
DOI: 10.1080/14685248.YYYYxxxxxx
http://www.tandfonline.com
and the simulation of the transport phenomena in large scale multiphase dispersed flows is currently done through modeling of both turbulent effects and fluid-fluid interactions [8, 9]. The development of accurate and reliable models requires detailed numerical simulations or experimental observations of simplified multiphase systems, where the leading physical mechanisms can be highlighted with respect to the governing parameters. In spite of its importance, detailed analysis of the flow field across the interface of large deformable droplets in wall-bounded turbulence apparently lack in the literature. Different authors [10, 11] analyzed the effects of large deformable bubbles on the wall shear stress, whereas other authors addressed the problem of droplets and bubbles breakup in turbulent flows [12, 13]. In this work we study a dispersed multiphase system in wall bounded turbulence focusing on the turbulence and surface tension interactions; in particular the behavior of a single, large and deformable droplet released in a fully developed turbulent channel flow is analyzed in detail by direct numerical simulations. The study focuses on the interactions between surface tension and turbulent structures, and in an effort to make the analysis dependent on the minimum number of parameters, the effect of different densities and different viscosities are purposely neglected. The problem considered represents an archetypal model for large scale systems, where swarms of droplets are released in inhomogeneous turbulence. To the best of our knowledge this work represents one of the first attempts to the analysis and modeling of the interaction between turbulence and surface tension focusing on the turbulent field features at the droplet surface.

2. Governing equations

In this work the numerical analysis of a large deformable droplet in turbulence has been studied. The flow field evolution has been described with Direct Numerical Simulations (DNS) of a modified incompressible Navier-Stokes equations coupled with the Diffuse Interface (DI) approach for the interface tracking.

2.1. The Diffuse Interface approach

The DI model is based on the idea that the interface between two fluids is a layer of finite thickness rather than a sharp discontinuity. Across the interfacial layer the physical properties of the fluid components vary in a smooth and continuous way from one fluid to the other. This approach is based on the pioneering work of Van Der Waals [14] who first determined the interface thickness of a critical liquid-vapor mixture. The application of the DI approach is based on the so-called Phase Field Model (PFM) where the state of the system is described, at any time, by a scalar order parameter $\phi$, which is a function of the position vector $\mathbf{x}$. The order parameter directly represents one of the physical properties of the fluid, such as its density, its molar concentration, etc.; all the remaining properties are in turn modeled as proportional to the scalar order parameter $\phi(\mathbf{x})$ [15, 16]. According to the DI approach, the order parameter is mathematically continuous over the entire domain and it shows smooth variations across the interface between single fluid regions, where it assumes mostly uniform values. Coupling the continuous and diffuse representation of the two fluid field with a transport equation of the order parameter, the system evolution can be resolved in time. One of the best-known DI models is the Cahn-Hilliard equation, where the evolution of the order parameter is driven by the minimization of a suitable chemical potential. Cahn and Hilliard [17, 18] generalized the work of Van Der Waals [14] to a time-dependent
system by approximating interfacial diffusive fluxes as being proportional to the chemical potential gradients. In this way the conservation of the phase field is ensured and the diffusion of the interfacial layer, that is the major drawback of the most common interface-tracking methods [19–21], is overcome. Compared to other approaches, the DI modelling grants more accuracy in the computation of the interfacial forces, even if some issues on the local mass conservation may still arise [16]. The convective Cahn-Hilliard equation is written as follows:

\[
\frac{\partial \phi}{\partial t} = -u \cdot \nabla \phi + \nabla (M \nabla \mu),
\]

(1)

where \(u\) is the velocity field, \(M = M(\phi)\) is the mobility (or Onsager coefficient) that controls the interface relaxation time and \(\mu\) is a chemical potential that controls the interfacial layer behavior. Eq. (1) models the evolution in time of a diffuse interface in particular it is able to either represent the conservative advection of a diffuse interface [22–24] or the creation and dissolution of interfaces in either breakup or critical phenomena [25, 26]. The chemical potential \(\mu\) is defined in terms of the free energy functional \(f[\phi]\) as follows:

\[
\mu = \frac{\delta f[\phi(x)]}{\delta \phi},
\]

(2)

where \(f[\phi]\) assumes suitable definitions according to the problem under analysis and it is a conservative, thermodynamically consistent functional. The DI representation of an immiscible binary mixture of isothermal fluids is given by the following free energy functional:

\[
f[\phi(x)] = f_{id} + \frac{1}{2} \kappa |\nabla \phi|^2
\]

(3)

and, in this case, the scalar order parameter \(\phi\) represents the relative concentration of the two fluid components. The first term on the right-hand-side of eq. (3) is the ideal part of the free energy and accounts for the tendency of the system to separate into pure fluids clusters. For two immiscible fluids, the phobic behavior is described by a double-well formulation which shows two minima corresponding to the two stable fluid phases:

\[
f_{id} = \frac{\alpha}{4} \left( \phi - \sqrt{\frac{\beta}{\alpha}} \right)^2 \left( \phi + \sqrt{\frac{\beta}{\alpha}} \right)^2,
\]

(4)

where \(\alpha\) and \(\beta\) are two positive constants defining the properties of the interface. Since the two fluids are allowed to mix into the interfacial layer, they store a mixing energy which is kept in account by the non-local term \(1/2\kappa|\nabla \phi|^2\) of eq. (3). The mixing energy stored into the interfacial layer is controlled by the positive parameter \(\kappa\) and it is the origin of the surface tension in to the DI approach here adopted. The relative concentration equilibrium profile across the interface is given by the competition of the two terms appearing in the free energy formulation and can be obtained by minimizing the free energy functional with respect to the variations of the order parameter:

\[
\mu = \frac{\delta f[\phi]}{\delta \phi} = 0 \Rightarrow \alpha \phi^3 - \beta \phi - \kappa \nabla^2 \phi = 0.
\]

(5)
Integration of eq. (5) for a one-dimensional planar interface, where $\phi(z \to \pm \infty) = \phi_{\pm}$, yields two stable solutions $\phi_{\pm} = \pm \sqrt{\beta/\alpha}$ and the following non-uniform solution:

$$\phi(z) = \phi_+ \tanh \left( \frac{z}{\sqrt{2} \xi} \right).$$

(6)

The capillary width $\xi = \sqrt{\kappa/\beta}$ is the interface length scale; in particular $0.9 \phi_\mp \leq \phi \leq 0.9 \phi_+$ in a layer of 4.164$\xi$ that contains the 98.5% of the interface surface tension $[15, 23]$. At the equilibrium, the surface tension $\sigma$ is defined as the specific energy stored into the interfacial layer; applying the definition to eq. (6), the following value of the surface tension is obtained:

$$\sigma = \kappa \int_{-\infty}^{+\infty} \left( \frac{d\phi}{dz} \right) dz = \frac{\sqrt{8} \kappa^2 \beta^2}{3 \alpha}. \quad (7)$$

Eq. (7) allows to define the free energy parameters $\alpha$, $\beta$ and $\kappa$ to achieve the desired surface tension value, given the interface thickness $\xi$ and the equilibrium solutions $\phi_{\pm}$. The derivation described above has been adopted and reviewed by several authors $[27, 28]$ moreove the convergence of eq. (1) to the “sharp interface limit” has been recently proven by Khatavkar $[29]$, Yue $[30]$ and Magaletti $[31]$ among the others. In particular, although the fictitious widening of the interface necessary for its numerical resolution$^1$, the DI approach can describe the desired value of $\sigma$ by a correct tuning of the free energy functional coefficients $[23]$. It has been shown that the surface tension $\sigma$ can be correctly recovered by the model in out-of-equilibrium conditions (i.e. non-steady interfaces) when a correct scaling between the capillary width $\xi$ and the interface mobility $M$ is adopted.

2.2. Coupling with the flow field

The evolution of the velocity field $u$ is described by the incompressible Navier-Stokes equations provided by a phase field-dependent surface force $[32]$:

$$\nabla \cdot u = 0, \quad (8)$$

$$\frac{\partial u}{\partial t} = -u \cdot \nabla u - \nabla p + \nu \nabla^2 u + \mu \nabla \phi, \quad (9)$$

where $p$ is the pressure term and $\nu$ is the kinematic viscosity. The coupled Chan-Hilliard/Navier-Stokes (CHNS) equations (1), (8) and (9) is the so-called “Model-H” $[33]$, where the surface tension forcing $\mu \nabla \phi$ is derived from the Korteweg stress $[34]$.

$^1$At least three mesh-points are necessary to fully resolve the interface with the current methodology employed. Larger number of mesh-points can be required according to the accuracy of the numerical scheme adopted.
3. Numerical simulations

3.1. Geometry and numerical scheme

With reference to the schematic of Fig. 1 the evolution of a single fluid droplet of diameter $d$ released in a fully developed turbulent channel flow is analyzed. The two fluids are considered immiscible, incompressible, Newtonian, density-matched and viscosity-matched. The interface between them is physically maintained by the surface tension $\sigma$. The mobility $M = M(\phi)$ is kept constant [23, 31], leading to a single relaxation time for the interface. With this assumptions the system is set to its simplest configuration, allowing to isolate the surface tension effects and the turbulence interface interactions. The reference coordinate system is located at the center of the channel and $x$-, $y$- and $z$-axes point in the stream-wise, span-wise and wall-normal directions, respectively. The size of the channel is $4\pi h \times 2\pi h \times 2h$ in $x$, $y$, and $z$ directions, respectively and $h$ is the channel half-height. The droplet is initialized by superposing the phase field $\phi$ over a fully developed turbulent flow obtained from previous single phase DNSs in a statistically steady state. The CHNS equations (1), (8) and (9) have been rewritten in a non-dimensional form, where the superscript “−” indicates non-dimensional quantities. The scaling variables here adopted are $U_\tau$, $h$, and $\phi_+$, where $U_\tau = \sqrt{\tau_w/\rho}$ is the shear velocity based on the wall shear stress $\tau_w$ and $\phi_+ = \sqrt{\beta/\alpha}$ is one of the two stable solutions given by the chemical potential (2).

\[
\frac{\partial \phi^-}{\partial t^-} = -\mathbf{u}^- \cdot \nabla \phi^- + \frac{1}{Pe} \nabla^2 \mu^-,
\]

\[
\nabla \cdot \mathbf{u}^- = 0,
\]

\[
\frac{\partial \mathbf{u}^-}{\partial t^-} = -\mathbf{u}^- \cdot \nabla \mathbf{u}^- - \nabla p^- + \frac{1}{Re_\tau} \nabla^2 \mathbf{u}^- + \frac{1}{\sqrt{8} We \cdot Ch} \frac{1}{\mu^-} \nabla \phi^-,
\]

where the dimensionless form of the chemical potential (2) yields to the following:

\[
\mu = \phi^3 - \phi - Ch^2 \nabla^2 \phi^-.
\]

Through non-dimensionalization procedure, the following dimensionless groups appear:

\[
Re_\tau = \frac{U_\tau h}{\nu}, \quad Pe = \frac{U_\tau h}{M \beta}, \quad We = \frac{\rho U_\tau^2 h}{\sigma}, \quad Ch = \frac{\xi}{h}.
\]

$Re_\tau$ is the shear Reynolds number, that is the ratio between inertial forces and viscous forces and $Pe$ is the Peclet number that represents the interface relaxation time. $We$ is the Weber number, that is the ratio between inertial forces and the surface tension and $Ch$ is the Cahn number that represents the dimensionless capillary width. In our approach, $Re_\tau$, $We$, $Ch$ and $Pe$ are macroscopic input parameters defined considering the physical fluid properties, the flow regime, the simulated surface tension and the diffuse interface modeling. Once fixed the shear Reynolds number, the value of the surface tension is chosen by changing the Weber number. When considering immiscible fluids, the interface thickness depends on the numerical algorithm only, thus the Cahn number can be fixed to the smallest
possible value. To obtain results independent from $Ch$, the Peclet number should be properly chosen: for this reason the scaling proposed by Khatakar [29] has been adopted. Equations (10)-(13) have been solved using a pseudo-spectral approach based on the transformation of the variables into the wave-number space through Fourier transforms along two periodic directions $x$ and $y$ and Chebyshev transforms along the wall-normal direction $z$. Periodicity conditions have been applied along the homogeneous directions $x$ and $y$ for both velocity field and order parameter, whereas at the walls no-slip velocity has been imposed. Following [23] and [15], no flux boundary condition $\partial \phi / \partial z = 0$ has been imposed for $\phi$, yielding to a normal contact angle between the interface and the channel walls. The detailed numerical procedure for the discretization of the Cahn-Hilliard equation can be found in [23], whereas the extension to the span-wise direction has been obtained following [16, 35]. The description of the numerical approach for the solution of the Navier-Stokes equations can be found in [35]. In this work the shear Reynolds number based on the half channel height is $Re_\tau = 100$, leading to a fully developed turbulent flow and, following [29], the Peclet number is $Pe = 2.56 \cdot 10^5$. We considered four different Weber numbers, $We = 0.0053, 0.0106, 0.0212, 0.0424$ and a droplet diameter $d = 0.8$. This diameter is much larger than the Kolmogorov length scale $\eta \approx \kappa$ at all the positions in the domain: the ratio between the Kolmogorov length scale and the droplet diameter is $0.06 \leq \eta / d \leq 0.13$. The simulations were run on a $256 \times 128 \times 129$ fixed cartesian grid which is fine enough to resolve the smallest length scale of the turbulent flow ($\Delta z \sim \Delta y \sim \eta$), while the time step $\Delta t = 10^{-4}$ has been chosen to resolve correctly the smallest temporal scales and respond to the numerical stability requirements associated with the grid resolution.

3.2. Accuracy issues

The pseudo-spectral scheme adopted can resolve accurately the interfacial layer with a minimum number of three mesh-points [15, 16]. With reference to eq. (6) the interface thickness is $l_\xi = 4.164 \cdot Ch$, yielding to a variation of the order parameter $-0.9 \leq \phi \leq 0.9$. Choosing $Ch = 0.036$, the interfacial layer is described by three mesh-points along $x$ and $y$ directions, where a uniform discretization is adopted ($\Delta y \sim \Delta x = 4\pi h/255$). Along the $z$ direction, a finer non-uniform discretization is adopted: $l_\xi$ is described by a minimum number of seven mesh-points. The DI models cannot completely fulfill local mass conservation [36] and this issue has been observed also in this work. Thanks to the accuracy of the numerical scheme adopted, however, the mass loss is in any case small and the estimated jumps of the interface-normal velocity component across the interface are of $O(10^{-5})$ [16]. In particular, after the entire simulation ($3 \cdot 10^5$ time-steps, corresponding to $\sim 40$ channel length covered by the mean flow), losses of volume $V^-$ (or equivalently of mass $m$) range from $4\%$ to $14\%$ for $We = 0.0053$ to $We = 0.0424$, respectively. This corresponds to a maximum reduction of $6\%$ of the equivalent droplet diameter $d_{eq} = \sqrt{3V^-/4\pi}$ and a relative mass transfer $\Delta (m/m_0)/\Delta t = 5 \cdot 10^{-3}$. A collection of the relevant parameters of each simulation is reported in Tab. 1.

The Chebyshev transform, adopted along the wall-normal direction, produces a non-uniform grid spacing highly refined at the walls where large gradients have to be resolved.
4. Results and Discussion

In this work, the effects of surface tension on the turbulent velocity fluctuations at the interface of a large deformable droplet released in a turbulent channel flow are investigated. To study the features of the turbulent flow in proximity to a deformable interface moving in a non-homogeneous flow, first the velocity field over the droplet interface has been examined (Sect. 4.2), then the behavior of the flow field at both sides of the interface has been analyzed (Sect. 4.3) and, finally, the turbulence inside of the droplet has been compared with that outside the droplet (Sect. 4.4). To examine these different issues of the problem, the analysis has been performed adopting different viewpoints, so that the investigate features have been highlighted. In Fig. 2, the three different fluid domains considered for the analysis are reported: velocity field over the droplet surface (Fig. 2-a), velocity field across the interface of the droplet (Fig. 2-b) and turbulent field inside and outside of the droplet (Fig. 2-c). Time-independent statistical results have been obtained by time averaging over a window $\Delta T_{sim} = 30$, that corresponds to $\sim 30$ eddy turnover times $T_e = h/U$. All the results reported in this section are measured in wall-units that are obtained normalizing by $U, \rho, \nu$ and $\phi$.

4.1. Droplet behavior and turbulent features

After released in the fully developed turbulent channel flow (Fig. 1), the droplet is advected by the flow field, moving with non-zero stream-wise velocity $u$. Due to the turbulent fluctuations, the droplet also moves along the span-wise and wall-normal directions with a meandering trajectory. However, during the entire simulation, the droplet never reaches wall distances closer than $20w.u.$ In addition the shape of the droplet is modified by the local flow field: the time averaged deformation $\langle S/S_0 - 1 \rangle$ is reported in Tab. 1 for each Weber number, where $S = S(t)$ is the instantaneous external area of the droplet and $S_0 = S_0(t)$ is the area of an equivalent sphere computed from the the instantaneous droplet volume (brackets denote time averaging). For the set of Weber numbers considered in this work, the average droplet deformation is small ($0.03\% < \langle S/S_0 - 1 \rangle < 0.7\%$) and no breakup is observed. The presence of the interface is responsible for the modification of the local turbulent structures and in particular of turbulent transport at length scales $l \sim d$. In Fig. 3 the vortical structures near the interface are shown for two different $We$: snapshots of simulation $W1$ (small deformations) and $W4$ (large deformations) at non-dimensional time $t = 1600$ are reported in Fig. 3-a and Fig. 3-b, respectively. Coherent structures are identified using the second invariant $Q$ of the velocity gradient tensor $\nabla u$ [37]:

$$Q = \frac{1}{2}(\Omega_{ij}\Omega_{ij} - S_{ij}S_{ij}),$$

where $\Omega_{ij} = 1/2(u_{ij} - u_{ji})$ and $S_{ij} = 1/2(u_{ij} + u_{ji})$ are the anti-symmetric and symmetric components of $\nabla u$, respectively. The vorticity magnitude $|\omega|$ has been superimposed to the vortices identified with iso-surfaces $Q = Q_t > 0$, where the threshold value $Q = 0.0038$ has been chosen for a non biased representation of the eddies. When the deformability is low ($W1$, Fig. 3-a), the region of fluid near the droplet is populated by a larger number of turbulent structures than in the large deformability case ($W4$, Fig. 3-b). The reduced number of coherent structures observed is probably due to the elastic behavior of the interface that first damps the turbulent forcing, releasing it at different lengths scales, at different times,
and at different positions over the surface. The release of energy acts as a non-synchronous damper in the turbulent chain, producing fluid motions which are non-coherent with those responsible for the energy accumulation. By contrast the higher number of coherent structures observed in the low deformability case (W1) is probably due to the deviation of the flow produced by the interface that yields to structures smaller in size than the typical channel flow vortices. These structures are coupled with peaks of vorticity magnitude, which are likely due to the shear stress induced by the interface, while, when the interface is more deformable, the vorticity levels appear to be comparable to those registered in the buffer layer (see vorticity contours in Fig. 3).

4.2. Velocity fluctuations over the droplet surface

In this section the surface-normal velocity fluctuations $u_n^*$ and the surface-tangential velocity fluctuations $u_t^*$ measured over the surface of the droplet are analyzed. Since the droplet is a finite size body moving in a non-homogeneous flow field, the definition of homogeneous statistical directions is not trivial, therefore Probability Density Functions (PDF) of the normal and tangential velocity fluctuations are presented. The interface has been defined as the position where $\phi = 0$ and the velocity components have been projected on the directions normal and tangential to the surface. Since two tangential vectors are identified at each interface location, only the magnitude of the tangential component has been considered $u_t^* = |u_t|$. To compare the results with those available in the literature in absence of mean flow, the velocity fluctuations $u^*$ have been defined with respect the droplet center of mass velocity $u_{cm}$: $u^* = u - u_{cm}$. Adopting this decomposition, also the convective effects are better highlighted. In Fig. 4, PDFs of $u_n^*$ and $u_t^*$ are shown for the different Weber numbers. Increasing $We$, a reduction of the interface-normal component (Fig. 4-a) coupled with a strengthening of the interface-tangential component (Fig. 4-b) is observed. This effect can be attributed to the wall-blocking effects, as indicated by Perot and Moin [38], who showed how the flow streams impacting solid interfaces are deflected from the interface-normal direction to the interface-tangential directions (intercomponent energy exchanges). In our case, responsible for the wall blocking effect is the local normal stress $\tau_n = \sigma / \bar{r} \cdot n$ that arises at the droplet interface because of the surface tension and the finite curvature radius $\bar{r}$. A first effect is due to the dependency of $\tau_n$ on the surface tension: the wall-blocking effects increase when $We$ is reduced and, in turn, intercomponent exchanges are larger for smaller $We$ (as observed also by Trontin [39]). A second effect is related to the normal stress dependency on the local deformations: large and energetic structures impacting on the interface act to reduce the local curvature, corresponding to a reduction of $\tau_n$. This can explain the dependence of $u_t^*$ on $We$ confined in a range of small velocities ($0.5 \leq |u_n^*| \leq 0$) observed in Fig. 4. The presence of intercomponent exchange effects dependent on $We$ is also consistent with the increment of small coherent structures observed in Fig. 3.

4.3. Statistics across the interface of the droplet

To clarify the intercomponent exchange discussed in Sect. 4.2, the behavior of the turbulent field across the interface is analyzed. Turbulence kinetic energy $k = u^* \cdot u^*$, vorticity magnitude $\omega/\omega_0 = |\nabla \times u|/\omega_0$ and Root Mean Square (RMS) of the velocity fluctuations have been analyzed across the interface (identified by the surface where $\phi = 0$). The vorticity has been normalized with the average vorticity magnitude $\omega_0$ measured in the buffer layer of a single phase flow, and
the same velocity decomposition presented in Sect. 4.2 has been maintained: \( \mathbf{u}^* = \mathbf{u} - \mathbf{u}_{cm} \). With reference to the interface points \( p_i \) shown in Fig. 2-b, the analysis has been performed along the directions normal to the droplet surface at \( p_i \). Then, considering the droplet center of mass position \( \mathbf{x}_{cm} = (x_{cm}, y_{cm}, z_{cm}) \), the interface points \( p_i \) with wall-normal coordinate \( z_{cm} \) have been considered: \( p_i = p_i(x_{\phi=0}, y_{\phi=0}, z = z_{cm}) \). Along those interface-normal directions, the quantities under analysis have been measured at distances \( \delta \) from the interface ranging from \( -20w.u. \leq \delta \leq 60w.u. \), where \( \delta = 0 \) represents the intersection with the droplet interface and \( \delta \) increases moving outside of the droplet (see Fig. 2-b). With the adoption of this framework, the quality of the statistical samples is improved: since \( z_{cm} \) is in confined in the channel buffer layer for most of the time, measures are made in regions of limited wall-normal non-homogeneity. Furthermore, the velocity fluctuations \( \mathbf{u}^* \) show a nearly flat profile in the center of the channel. As a result, the accuracy of the statistical analysis is increased. The measured quantities have been time and ensemble averaged (denoted by brackets), where ensemble averaging has been made over all the realizations obtained at the same distance \( \delta \) from the interface. The evolution of the turbulence kinetic energy and the evolution of the vorticity magnitude along the interface normal direction are shown in Fig. 5-a and Fig. 5-b, respectively. The turbulent kinetic energy is damped moving from outside of the droplet to the droplet interface for all the Weber numbers considered (Fig. 5-a). The damping rate \( \Delta k/\Delta \delta \) is larger near the interface and it reduces at larger \( \delta \). Inside of the droplet \( (\delta < 0) \) a damping of 80\% of \( k \) is observed with respect to the value measured at \( \delta = 60w.u. \) and, across the interface (half-thickness of 7.4\( w.u. \) measured at \( \phi = \pm 0.9 \) [15]), \( k \) shows a magnitude comparable to that observed inside the droplet. The results indicate that the damping effects take place in a region outside of the droplet and, as expected, the interface effects reduce moving far from the interface. Near the interface a vorticity production \( (\omega/\omega_0 > 1) \) can be observed for all the considered Weber numbers and at both sides of the interface (Fig. 5-b). In the outer side of the interface, production is due to the intercomponent energy transfer (wall-blocking effect [38]) that yields to the generation of the small coherent structures highlighted in Sect. 4.1. Since the wall-blocking effect depends on the surface tension, the vorticity production peak increase when \( We \) reduces. This result is consistent with the results of Trontin [39] and Li [40] and with the wall-blocking mechanism discussed in Sect. 4.2. The peak of production is registered in a region of 7\( w.u. \leq \delta \leq 14w.u. \), then vorticity production reduces increasing the distance from the interface and, at \( \delta = 60w.u. \), \( \omega/\omega_0 \approx 1.1. \) This behavior indicates a weaker flow field deviation with respect to the near interface region, where the blocking effect is larger. By contrast, at \( \delta = 60w.u. \), the droplet effects on the turbulent kinetic energy are still important. The evolution of the RMSs of the interface-normal and the interface-tangential velocity fluctuations along the interface normal direction are shown in Fig. 6-a and Fig. 6-b, respectively. The RMSs of the interface-normal fluctuations decrease moving from large \( \delta \) to the droplet surface (Fig. 6-a), in particular their behavior appear to be almost independent from \( We \) and the profiles collapse on a single line. The interface effects are non-negligible along the whole distance considered \( \delta = 60w.u. \), thus statistics extended\(^\text{1}\) to larger \( \delta \) are needed to define the complete range of spatial interaction. At distances \( \delta \geq d/2 \), the RMSs decay is compared with the \( \delta^{1/3} \) scaling law showing a qualitative consistency with the theory developed by Hunt and Graham [41] for a solid interface in a free-stream flow. 

\(^{1}\)The extension to larger distances can lead to an increasing statistical non-homogeneity of the sample, thus different analysis framework should be adopted.
indicated that the distortion in the velocity field can be seen up \( \delta \propto L \) where \( L \) is the size of the largest vortex of the flow and that the interface-normal RMS decays as \( \delta^{1/3} \) in regions close to distances \( L \). In our case the droplet is expected to interact with eddies of size spanning from the Kolmogorov length scale \( \eta_k (2.5 \text{w.u} - 11 \text{w.u.}) \) to the droplet diameter \( d = 80 \text{w.u.} \), furthermore the droplet interface is deformed by large and energetic structures. As a result discrepancies from theory [41] are observed. The RMSs of the interface-tangential components (Fig. 6-b) show a fast decay in a region close to the droplet interface \( 15 \text{w.u.} \) (corresponding to large vorticity production), while smaller reduction rates are observed at larger distances. The simultaneous damping of \( u_*^t \) and \( u_*^n \) is the origin of the reduction of \( k \) and is probably associated to the vorticity production \( \omega/\omega_0 \) that yields to an incomplete energy transfer between the two components. These effects are large in the near interface regions \( \delta \leq 15 \text{w.u.} \), where the flow field deflection (and the vorticity production) is large, while they reduce moving far from the interface.

### 4.4. Velocity fluctuations inside the droplet

In Sect. 4.3 the damping of the turbulent kinetic energy across the droplet interface has been shown. To quantify the modulation effects produced by the presence of the interface on the fluid embedded inside of the droplet, the velocity fluctuations inside the droplet have been measured. In this case the usual turbulent channel flow decomposition \( u' = u - \bar{u}(z) \) has been adopted to compare the fluctuating field \( u' \) with the single phase turbulent channel flow statistics. The PDFs\(^1\) of the velocity fluctuations measured inside of the droplet (points \( p_i \) in Fig. 2-c) have been compared with the PDFs of the velocity fluctuations measured outside of the droplet: a channel slab parallel to the walls with the same height of the deformed droplet and centered into the droplet center of mass has been considered (points \( p_j \) in Fig. 2-c). In both cases the statistical ensemble has been restricted to the points of the volume of fluids contained in the channel flow buffer layer (defined as the region \( 30 \text{w.u.} \) far from both walls). Even if the droplet surface sometimes reaches positions \( 20 \text{w.u.} \) close to to the walls, the largest part of its volume lies into the buffer layer, thus the PDFs of the velocity fluctuations can be compared with the single phase turbulent channel flow statistics [42]. A schematic of the fluid domains considered for this analysis is shown in Fig. 2-c, while the PDFs of \( u' \), \( v' \) and \( w' \) are shown in Fig. 7-a, Fig. 7-b and Fig. 7-c, respectively. The presence of the droplet shows a small influence on the outer flow field, except for stream-wise fluctuations \( u' \leq -3 \) where some damping effects can be observed. These effects could be due to two different mechanisms: first, the droplet moves in a region characterized by a negative skewness factor [43], thus the damping effects introduced are expected to affect more the negative velocity components. Second, the meandering motion of the droplet from the channel center to the near-wall regions introduce an higher mean flow component that can damp the negative fluctuations and enhance the positive components. Since the velocity difference is smaller for the positive components, their enhancement is negligible compared to the negative fluctuations damping. All the three velocity components are damped inside the droplet (dots), with respect to the velocity fluctuations measured outside the droplet (lines). Due to the damping of turbulence kinetic energy discussed in Sect. 4.3, the velocity fluctuations near the droplet interface are small and, as a result, the momentum transfer across the interface is limited. Inside of the droplet,

\(^1\)As in Sect. 4.2, the results analysis through Probability Density Functions allow to treat the sample non homogeneity.
the negative skewed stream-wise velocity distribution is shifted toward an almost centered shape (Fig. 7-a). The increased isotropy of $u'$, observed inside the droplet reducing $W_e$, can be explained by the redistribution of the external turbulent forcing over the entire interface produced by the surface tension. This process is more effective when surface tension is high, by contrast a more deformable interface can easily transfer the external field features to the fluid inside of the droplet through local interface deformations.

5. Conclusions

Numerical analysis of the turbulent transport phenomena in large dispersed multiphase systems requires computational resources that are beyond the current available limits, thus modeling is required. The development of suitable models for the study of systems where bubbles and droplets swarms are transported and dispersed can be simplified through the analysis of a single droplet released in a turbulent flow. In this work a dispersed multiphase system has been simplified to its archetypal model: a single deformable droplet released in a turbulent channel flow. In particular only the surface tension effects have been considered, neglecting density and viscosity mismatches. The velocity fluctuations relative to the droplet center of mass, measured at the droplet interface have been analyzed in terms of interface-normal and interface-tangential components. The normal components are reduced increasing the surface tension, whereas the tangential components show an opposite behavior, according to the presence of the wall-blocking effects [38–40]. The presence of the interface produces a deviation of the surrounding velocity field, that results in a vorticity generation dependent on the surface tension and in a damping of the turbulence intensity near the interface. These effects are experienced at larger distances from the interface increasing the surface tension. Furthermore the turbulent features observed at the surface of the droplet show similarities to those observed for solid walls in free-stream flows [41]. As a result, the convective effects at the droplet surface are damped, the momentum transport across the interface is reduced and the turbulence inside the droplet is weaker.

6. Acknowledgments

The authors acknowledge PRACE for awarding us access to resource JUROP A based in Germany at Julich. The EU COST Action MP0806 “Particles in turbulence” is also acknowledged.

References

REFERENCES


[33] D. Korteweg, Sur la forme que prennent les equations du mouvements des fluides si l’on tient compte des forces capillaires causees par des variations de densite considerees mais continues et sur la theorie de la capilarite dans l’hypothese d’une variation continue de la densite, Archives Neerlandaises des Sciences Exactes et Naturelles Series II, 6 (1901), pp. 1–23.


Table 1. Summary of the simulation parameters and average deformation \((S/S_0 - 1)\) for each simulation.

<table>
<thead>
<tr>
<th>Simulation</th>
<th>(We)</th>
<th>(Re_{\gamma})</th>
<th>(Ch)</th>
<th>(Pe)</th>
<th>((S/S_0 - 1))</th>
</tr>
</thead>
<tbody>
<tr>
<td>W1</td>
<td>0.0053</td>
<td>100</td>
<td>0.036</td>
<td>(2.56 \cdot 10^2)</td>
<td>(0.39 \cdot 10^{-3})</td>
</tr>
<tr>
<td>W2</td>
<td>0.0106</td>
<td>100</td>
<td>0.036</td>
<td>(2.56 \cdot 10^2)</td>
<td>(1.28 \cdot 10^{-3})</td>
</tr>
<tr>
<td>W3</td>
<td>0.0212</td>
<td>100</td>
<td>0.036</td>
<td>(2.56 \cdot 10^2)</td>
<td>(2.41 \cdot 10^{-3})</td>
</tr>
<tr>
<td>W4</td>
<td>0.0424</td>
<td>100</td>
<td>0.036</td>
<td>(2.56 \cdot 10^2)</td>
<td>(6.54 \cdot 10^{-3})</td>
</tr>
</tbody>
</table>

Figure 1. Computational domain and problem under analysis: a single deformable droplet is released in a fully developed turbulent channel flow.

Figure 2. Sketch of the fluid domains adopted for data analysis. Statistics on the droplet surface (a) are obtained considering all the points \(p_i\) on the droplet surface, where \(n\), \(t_1\) and \(t_2\) are the interface-normal vector and the interface-tangential vectors, respectively. Statistics across the interface (b) are computed considering first the interface-normal directions at points \(p_i(x, y, z_{cm})\) of the interface, where \(z_{cm}\) is the wall-normal coordinate of the droplet center of mass. Averages are made over the ensemble of points located at distance \(\delta\) from the interface. Statistics inside of the droplet (c) are obtained considering the points \(p_i\) the volume of fluid inside the droplet interface; statistics outside of the droplet are made on the points \(p_j\) of the volume of fluid external to the droplet and limited to the channel flow buffer layer \((30w.u. \leq z \leq 170w.u.)\)
Figure 3. Interactions between vortical structures and droplet for different Weber numbers at time $t = 1600$: (a) simulation $W_1$, (b) simulation $W_4$. Vortices are identified with iso-surfaces of the second invariant of the velocity gradient: $Q = 0.0038$ ($Q$-criterion). Contour plots of the local vorticity magnitude $\omega = |\omega|$ are superposed to the vortices. The droplet interface is located by the iso-surface $\phi = 0$ and is rendered in red. The entire computational domain along the wall-normal direction is shown (portion of the bottom wall is shown for clarity), while only a portion of it is shown along the length-wise and span-wise directions ($\Delta x \times \Delta y = 400 \times 250$). Near the more deformable droplet ($W_4$), the observed turbulent structures density is reduced and their size is larger compared to those near the stiffer droplet ($W_1$). Peaks of vorticity magnitude are observed near the stiffer droplet.
Figure 4. Probability Density Functions (PDF) of the velocity fluctuations $u^*$ (computed with respect to the droplet center of mass velocity) on the surface of the droplet at various Weber numbers $We$: (a) surface-normal velocity fluctuations $u^*_n$; (b) surface-tangential fluctuations $u^*_t$. Interface-normal components increase with the droplet deformability (increasing $We$), whereas tangential components reduce with the interface deformability.
Figure 5. Statistics across the droplet interface at various Weber numbers $We$: (a) averaged turbulent kinetic energy $\langle k \rangle = \langle u^* \cdot u^* \rangle$ (where $u^*$ is computed with respect to the droplet center of mass velocity); (b) average normalized vorticity magnitude $\langle \omega/\omega_0 \rangle = \langle (\nabla \times u)/\omega_0 \rangle$. Turbulent kinetic energy decays moving closer to the droplet, while vorticity increases in the proximity of the interface. Vorticity production peak decreases with the droplet surface tension (decreasing $We$).
Figure 6. Statistics across the droplet interface at various Weber numbers $W_e$: (a) interface-normal velocity fluctuations $\langle RMS(u^*_n) \rangle$; (b) interface-tangential velocity fluctuation $\langle RMS(u^*_t) \rangle$. Velocity fluctuations $u^*$ are computed with respect to the droplet center of mass velocity. Both velocity components are damped approaching the droplet interface. Near the droplet $\langle RMS(u^*_t) \rangle$ shows a faster decay with respect to $\langle RMS(u^*_n) \rangle$. A qualitative comparison with the scaling $\delta^{0.3}$ at distances $\delta \propto d$ [41] is proposed.
Figure 7. Probability Density Functions (PDF) of the velocity fluctuations $u'$ (computed with respect to the channel flow mean velocity), measured inside the droplet (dots), in the buffer layer outside the droplet (lines) and for a single phase flow (SP) at various Weber numbers $We$: (a) stream-wise fluctuations $u'$; (b) span-wise fluctuations $v'$; (c) wall-normal fluctuations $w'$. Velocity fluctuations are damped inside of the droplet, whereas the effects produced on the external velocity components are of small entity.