Upscale energy transfer and flow topology in free-surface turbulence

Salvatore Lovecchio,[1] Francesco Zonta,[1] and Alfredo Soldati[1,2]
[1]*Department of Electrical, Management and Mechanical Engineering, University of Udine, 33100, Udine, Italy*
[2]*Department of Fluid Mechanics, CISM, 33100, Udine, Italy*
(Received 20 November 2014; revised manuscript received 23 February 2015; published 19 March 2015)

Free-surface turbulence, albeit constrained onto a two-dimensional space, exhibits features that barely resemble predictions of simplified two-dimensional modeling. We demonstrate that, in a three-dimensional open channel flow, surface turbulence is characterized by upscale energy transfer, which controls the long-term evolution of the larger scales. We are able to associate downscale and upscale energy transfer at the surface with the two-dimensional divergence of velocity. We finally demonstrate that surface compressibility confirms the strongly three-dimensional nature of surface turbulence.

DOI: 10.1103/PhysRevE.91.033010 PACS number(s): 47.27.ek, 47.55.dr

I. INTRODUCTION

The interaction of turbulence with a free surface is important in a number of environmental and geophysical situations: the flow structure at the free surface is dominated by the fluid turbulence underneath and influences transfer rates of mass, momentum, and heat. This flow structure is also proven responsible for the highly nonuniform distribution patterns of biological microorganisms [1,2], which have an additional role onto the chemical and biological species cycles in open water bodies. Understanding the paramount role of turbulence at a free surface may therefore have strong environmental implications at short and long time horizons (global warming, future climate changes). The flow structure of the surface is reminiscent of the inhomogeneous three-dimensional turbulence developing in the flow bulk and follows a dynamics, which does not fully resemble that given by simplified methods based on simulations of two-dimensional turbulence or simulations of three-dimensional homogeneous isotropic turbulence. A specific feature of free-surface turbulence is the long life of large-scale structures [3,4]. This long life is possibly associated with upscale energy transfer mechanisms as predicted by Kraichnan [5], in the context of the turbulent double-cascade scenario: enstrophy goes from large to small scales and energy goes from small to large scale. The turbulence double-cascade scenario has been confirmed by simulations of two-dimensional turbulence (see Ref. [6] and references therein) and also by simulations of three-dimensional homogeneous isotropic turbulence [7], even with rotation and/or stratification [8]. However, no evidence of the energy double cascades has been given for inhomogeneous free-surface turbulent flows.

The aim of this work is to examine inhomogeneous free-surface open channel flows, considering in particular surface topology, energy flux, and local flow compressibility. We demonstrate that the net effect of an upscale energy transfer mechanisms can be observed in three-dimensional free surface flows only at sufficiently high Reynolds number. We also demonstrate that the compressibility factor can be larger than previous theoretical prediction, suggesting that two-dimensional and three-dimensional isotropic models cannot capture completely the dynamics of this type of turbulence [9,10].

II. METHODOLOGY

We consider the dimensionless incompressible continuity and Navier-Stokes equations

$$\frac{\partial u_i}{\partial x_i} = 0, \tag{1}$$

$$\frac{\partial u_i}{\partial t} = -u_j \frac{\partial u_i}{\partial x_j} + \frac{1}{\mathrm{Re}_\tau} \frac{\partial^2 u_i}{\partial x_j \partial x_j} - \frac{\partial p}{\partial x_i} + \delta_{1,i}, \tag{2}$$

where u_i is the ith component of the fluid velocity, p the fluctuating kinematic pressure, $\delta_{1,i}$ the mean pressure gradient driving the flow while $\mathrm{Re}_\tau = hu_\tau/\nu$ is the shear Reynolds number based on channel depth h and shear velocity $u_\tau = \sqrt{h|\,\delta_{1,i}|\,/\rho}$. Equations (1)–(2) are solved by a Fourier-Chebyshev pseudospectral method, assuming free-slip conditions at the free (and flat) surface and no-slip conditions at the bottom (z direction). Periodicity is applied along the streamwise (x) and spanwise (y) directions. We consider two different shear Reynolds numbers: $\mathrm{Re}_\tau^L = 171$ and $\mathrm{Re}_\tau^H = 509$. The size of the computational domain is $L_x \times L_y \times L_z = 2\pi h \times \pi h \times h$, discretized with $128 \times 128 \times 129$ (for Re_τ^L) and $256 \times 256 \times 257$ (for Re_τ^H) grid points. Further details on the numerical method can be found in Refs. [1,11,12].

III. RESULTS

A. Spectral fluxes

With the aim of quantifying downscale and upscale energy transfer contributions across spatial flow scales, we adopt a filtering approach [13,14] by applying a low-pass Gaussian filter $G_l(\mathbf{k}) = \exp(-|\mathbf{k}|^2\Delta^2/24)$ of size Δ to the field variables written in the wave number space ($\mathbf{k}$). The scales of the velocity field are thus divided into scales larger than Δ (unfiltered) and smaller than Δ (filtered). The turbulence kinetic energy transport equation for the unfiltered scales ($\overline{q} = \overline{u_i}^{(\Delta)}\overline{u_i}^{(\Delta)}$) is thus [13,14]:

$$\frac{\partial \overline{q}}{\partial t} + \frac{\partial \overline{q}\overline{u_i}^{(\Delta)}}{\partial x_j} = \frac{\partial}{\partial u_j}\left(-2\overline{p}^{(\Delta)}\overline{u_j}^{(\Delta)} - 2\overline{u_i}^{(\Delta)}\tau_{ij} + \frac{1}{\mathrm{Re}_\tau}\frac{\partial \overline{q}}{\partial x_j}\right)$$
$$-\frac{2}{\mathrm{Re}_\tau}\frac{\partial \overline{u_i}^{(\Delta)}}{\partial x_j}\frac{\partial \overline{u_i}^{(\Delta)}}{\partial x_j} + 2\tau_{ij}\overline{S_{ij}}^{(\Delta)}, \tag{3}$$

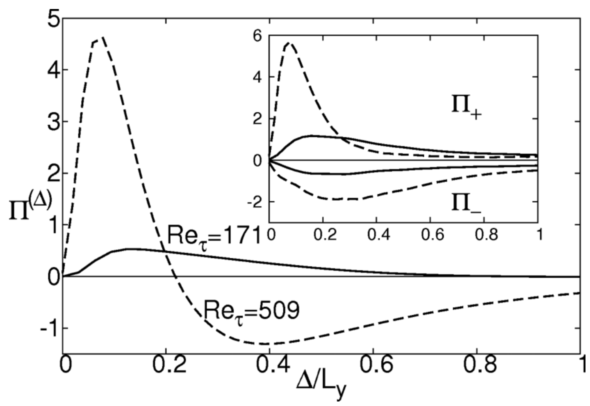

FIG. 1. Time- and surface-averaged energy flux $\Pi^{(\Delta)}$ as a function of the Gaussian filter size (Δ) at the channel surface ($z_0 = 0$) for Re_τ^L (solid line) and Re_τ^H (dashed line). The contribution of the positive energy flux (downscale energy transfer) and of the negative energy flux (upscale energy transfer) are also shown in the inset.

where $\tau_{ij} = \overline{u_i u_j}^{(\Delta)} - \overline{u_i}^{(\Delta)}\overline{u_j}^{(\Delta)}$ is the filtered stress and $\overline{S_{ij}}^{(\Delta)} = 1/2(\partial\overline{u_i}^{(\Delta)}/\partial x_j)$ is the unfiltered rate-of-strain tensor. The following dissipation rate,

$$\Pi^{(\Delta)} = -\tau_{ij}\overline{S_{ij}}^{(\Delta)} = -[\overline{u_i u_j}^{(\Delta)} - \overline{u_i}^{(\Delta)}\overline{u_j}^{(\Delta)}]\frac{\partial\overline{u_i}^{(\Delta)}}{\partial x_j},$$

is the energy flux transferred between small and large scales across the scale of the filter size. The contribution of the downscale and upscale energy flux can be computed as $\Pi_+^{(\Delta)} = \frac{1}{2}(\Pi^{(\Delta)} + |\Pi^{(\Delta)}|)$ and $\Pi_-^{(\Delta)} = \frac{1}{2}(\Pi^{(\Delta)} - |\Pi^{(\Delta)}|)$, respectively.

The time and surface averaged energy fluxes for both Reynolds numbers are shown in Fig. 1 as a function of the dimensionless filter size, Δ/L_y, and are normalized by the plane averaged value of the viscous dissipation $\epsilon_0 = (2/\mathrm{Re}_\tau)S_{ij}S_{ij}$ at the free surface [13]. For the low Reynolds number Re_τ^L, we observe that the energy flux peaks at $\Delta/L_y \simeq 0.1$, and then monotonically decreases to zero, indicating the predominance of the downscale energy transfer mechanism across the entire scale range. For the high Reynolds number Re_τ^H, the maximum is reached approximately at the same location $\Delta/L_y \simeq 0.1$, but the energy flux becomes negative for $\Delta/L_y > 0.2$, indicating a strong and persistent upscale energy transfer at larger scales. In the inset of Fig. 1, positive and negative energy flux contributions are shown for Re_τ^L and Re_τ^H, with evidence of both cascades coexisting in the entire range of scales. However, for Re_τ^L the contribution of the upscale energy transfer is always smaller than the downscale energy transfer. For Re_τ^H the upscale energy transfer is proportionally more important, with a peak occurring for scales larger than those corresponding to the peak of the downscale energy transfer.

B. Flow topology and energy cascade

The physical mechanisms leading to downscale or upscale energy transfer are still to be fully investigated, with new promising theories (for two-dimensional turbulence) supporting the importance of large-scale strain and vortex thinning in the dynamics of the upscale energy transfer [14]. At the surface of our three-dimensional numerical experiment, turbulence is continuously regenerated by upwellings advecting energy at the surface, where turbulence dynamics appears somehow two dimensionalized and characterized by strongly different features [3,15].

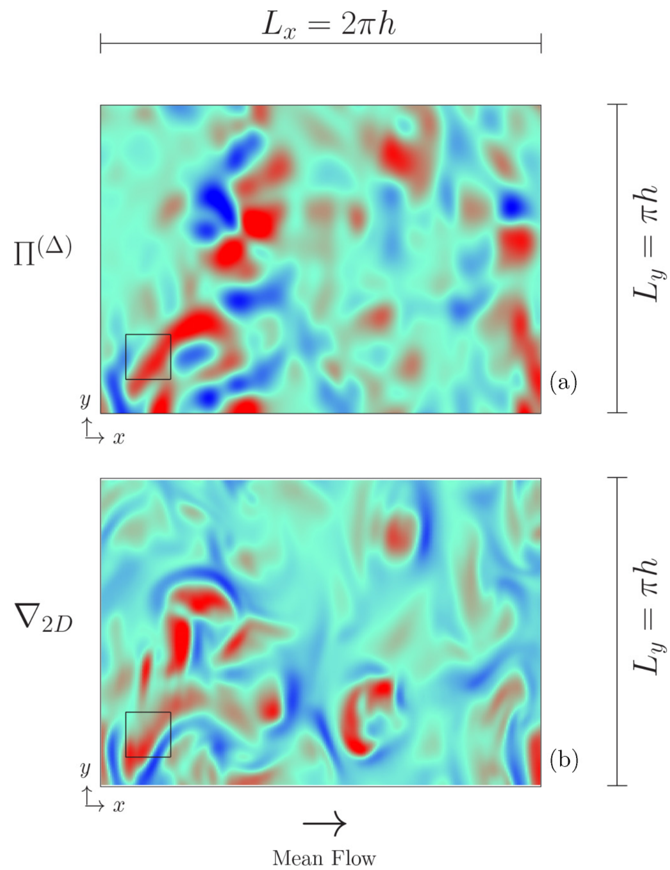

FIG. 2. (Color online) Contour maps of the energy flux $\Pi^{(\Delta)}$ (a) and of the two-dimensional surface divergence ∇_{2D} (b) computed at the free surface for $\mathrm{Re}_\tau = 171$.

To analyze the flow topology, we use the two-dimensional surface divergence of velocity $\nabla_{2D} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$. The two-dimensional divergence is associated with the exchange of mass and momentum between surface and bulk of the flow. Regions characterized by $\nabla_{2D} > 0$ are regions of local flow expansion, generated by bulk flow upwellings. Regions characterized by $\nabla_{2D} < 0$ are regions of local flow compression leading to downwellings. Contour maps of the instantaneous energy flux $\Pi^{(\Delta)}$ (computed for $\Delta/L_y = 0.2$, a scale at which upscale and downscale energy transfer are both significant for the two Re_τ examined) and of the instantaneous two-dimensional surface divergence ∇_{2D} are shown *vis a vis* for low and high Reynolds numbers in Fig. 2 and Fig. 3, respectively. Local energy fluxes and flow sources/sinks have a distinctly inhomogeneous distribution but nonetheless exhibit a remarkable correspondence, which can give a possible explanation of surface turbulence mechanisms. Regions of downscale energy transfer ($\Pi^{(\Delta)} > 0$) seem to correspond to regions of flow expansion (upwellings, $\nabla_{2D} > 0$), whereas regions of upscale energy transfer ($\Pi^{(\Delta)} < 0$) seem to correspond to regions of flow compression (downwellings, $\nabla_{2D} < 0$). We measured this correspondence via the following correlations computed

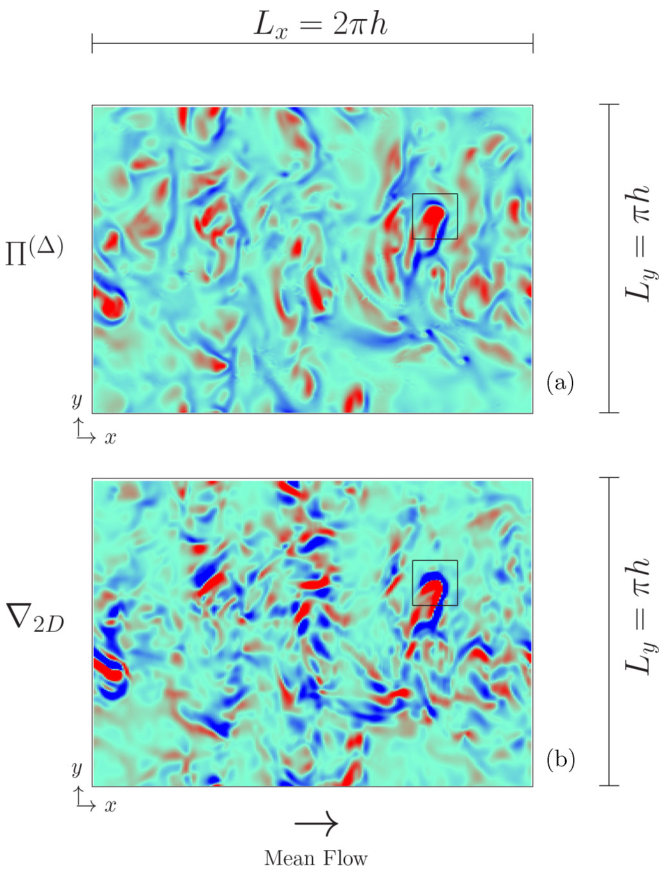

FIG. 3. (Color online) Contour maps of the energy flux $\Pi^{(\Delta)}$ (a) and of the two-dimensional surface divergence ∇_{2D} (b) computed at the free surface for $\mathrm{Re}_\tau = 509$.

for $\Delta/L_y = 0.2$:

$$\langle \nabla_{2D}^{\mathrm{sign}} \Pi_{\mathrm{sign}}^{(\Delta)} \rangle = \frac{\langle \nabla_{2D}^{\mathrm{sign}}(x-x',y,z_0)\Pi_{\mathrm{sign}}^{(\Delta)}(x,y,z_0)\rangle}{\nabla_{2D,\mathrm{rms}}^{\mathrm{sign}}(z_0)\Pi_{\mathrm{sign,rms}}^{(\Delta)}(z_0)},$$

where sign $= +/-$ and z_0 is the vertical coordinate of the free surface. Results are shown in Fig. 4(a) for $\mathrm{Re}_\tau = 171$ and in Fig. 5(a) for $\mathrm{Re}_\tau = 509$. The correlation between $\Pi_+^{(\Delta)}$ and ∇_{2D}^+ and between $\Pi_-^{(\Delta)}$ and ∇_{2D}^-, is strong for both Reynolds numbers within a length roughly corresponding to the filter size, and then drops for larger x/L_y. This strong spatial correlation suggests a causal relation between the downscale/upscale energy transfer and the surface renewal mechanism based on upwellings and downwellings [3,13]. To show this striking correspondence, in Figs. 4(b)–4(c) and in Figs. 5(b)–5(c) we present an enlargement of the small square area depicted in Figs. 2–3. Contour maps of Figs. 4(b)–4(c) and of Figs. 5(b)–5(c) show an almost perfect coincidence of downscale/upscale energy transfer regions with the expansion/compression regions: the overlapping streamlines complete this evident correspondence.

The occurrence of downscale/upscale energy transfer can be directly linked to the behavior of the third-order structure function, $S_3 = \langle(\delta_r u)^3\rangle$, with $\delta_r u = [u(x+r) - u(x)]$ being the longitudinal velocity increments [16]. In particular, the sign of S_3 indicates the direction of the energy flux: for negative

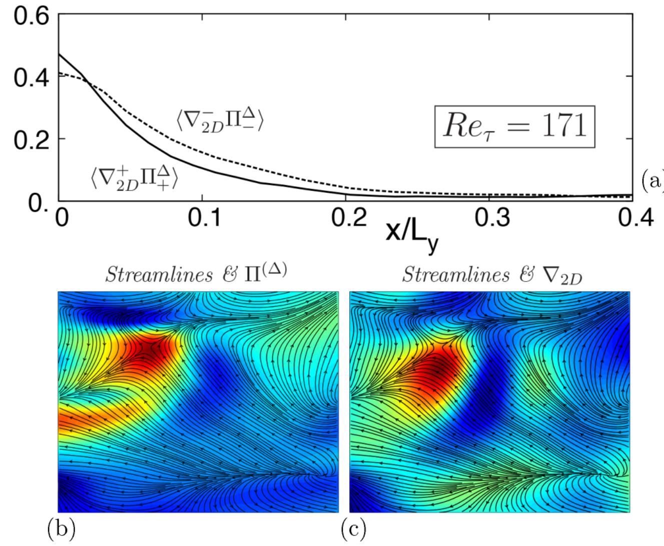

FIG. 4. (Color online) Correlation coefficient $\langle\nabla_{2D}^+\Pi_+^\Delta\rangle$ (solid line) and $\langle\nabla_{2D}^-\Pi_-^\Delta\rangle$ (dashed line) between the positive (negative) energy flux Π_+^Δ (Π_-^Δ) and the positive (negative) two-dimensional surface divergence ∇_{2D}^+ (∇_{2D}^-) computed along the streamwise direction x and averaged in time for $\mathrm{Re}_\tau^L = 171$ (a). We also show the spatial distribution of $\Pi^{(\Delta)}$ (b) and ∇_{2D} (c) on the small square area depicted in Fig. 2. Flow streamlines have been superposed to each contour map to highlight regions of local flow expansion (sources) and of local flow compression (sinks).

S_3, energy goes from large to small scales (downscale energy transfer), whereas for positive S_3 energy goes from small to large scales (upscale energy transfer). Our results on the behavior of S_3 at the channel center and at the free surface

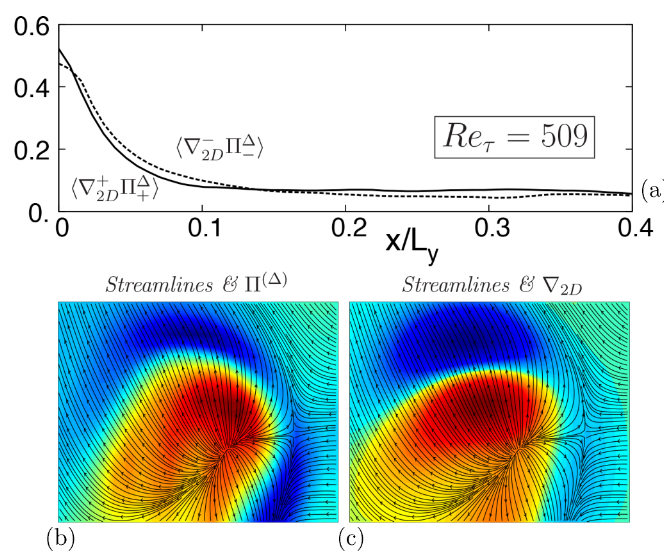

FIG. 5. (Color online) Correlation coefficient $\langle\nabla_{2D}^+\Pi_+^\Delta\rangle$ (solid line) and $\langle\nabla_{2D}^-\Pi_-^\Delta\rangle$ (dashed line) between the positive (negative) energy flux Π_+^Δ (Π_-^Δ) and the positive (negative) two-dimensional surface divergence ∇_{2D}^+ (∇_{2D}^-) computed along the streamwise direction x and averaged in time for $\mathrm{Re}_\tau^H = 509$ (a). We also show the spatial distribution of $\Pi^{(\Delta)}$ (b) and ∇_{2D} (c) on the small square area depicted in Fig. 3. Flow streamlines have been superposed to each contour map to highlight regions of local flow expansion (sources) and of local flow compression (sinks).

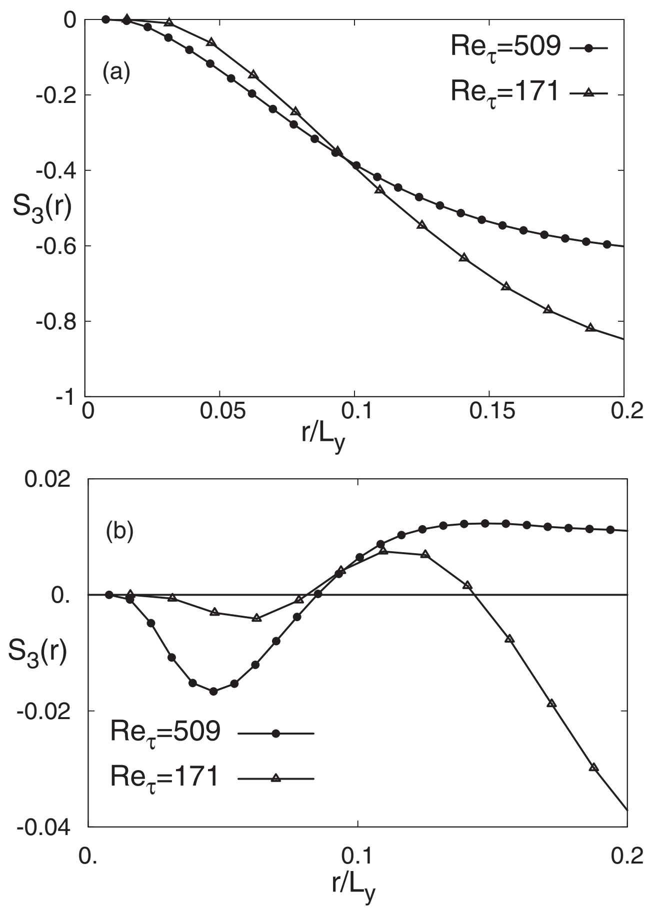

FIG. 6. Third-order structure function $S_3(r)$ computed at the channel center (a) and at the free surface (b) for both $\mathrm{Re}_\tau^L = 171$ and $\mathrm{Re}_\tau^H = 509$. S_3 is shown as a function of r/L_y.

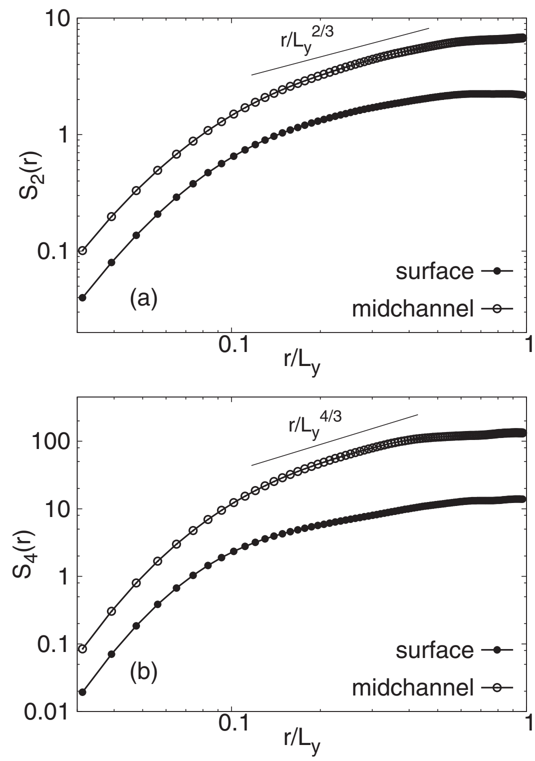

FIG. 7. Second-order [$S_2(r)$, (a)] and fourth-order [$S_4(r)$, (b)] structure functions for $\mathrm{Re}_\tau^L = 171$ expressed as a function of r/L_y and computed at the surface and at the center of the channel. The solid lines indicate the observed scaling in the inertial range. At the channel center (midchannel, $-\circ-$), $S_2 \simeq r^{2/3}$ and $S_4 \simeq r^{4/3}$.

are shown (as a function of r/L_y) in Fig. 6 for both Re_τ^L and Re_τ^H. At the channel center [Fig. 6(a)], S_3 is always negative for both Re_τ, indicating the predominance of the downscale energy transfer: in the bulk, energy flows from large to small scales only. A similar behavior is observed at the free surface for Re_τ^L [$-\blacktriangle-$ in Fig. 6(b)], with only a narrow range of scales (around $r/L_y \simeq 0.1$) were S_3 is positive (i.e., a limited upscale energy transfer). The situation changes for Re_τ^H at the free surface [$-\bullet-$ in Fig. 6(b)]: S_3 is negative at small length scales but turns positive (displaying a plateau) for $r/L_y > 0.1$. This indicates the occurrence of a downscale energy transfer for $r/L_y < 0.1$, which is replaced by a persistent upscale energy transfer for $r/L_y > 0.1$. The length scale at which S_3 changes sign ($r/L_y \simeq 0.1$) can be regarded as the average size of an upwelling. This may be understood from a simple physical interpretation of the third-order structure function. By drawing an imaginary circle around a given flow field, the radial vector of the velocity difference between the center of the circle and its circumference indicates the energy direction: the energy flows out of the circle, if the circle encloses an upwelling, whereas the energy flows in, if the circle encloses a downwelling. The upscale energy transfer has been recently associated [7] also to the behavior of the second-order $S_2 = \langle(\delta_r u)^2\rangle$ and fourth-order $S_4 = \langle(\delta_r u)^4\rangle$ structure functions. In particular, structure functions were found to scale as $S_p \simeq r^{p/3}$ in the inertial range. Here, we try to quantify the behavior of $S_2(r)$ and $S_4(r)$ in free surface flows, where no clear indication of these behaviors is available [18]. Our results are shown in Figs. 7–8 as a function of the dimensionless displacement r/L_y and for both Re_τ^L and Re_τ^H. Specifically, we compare the behavior of the structure functions at the free surface ($-\bullet-$) and at the channel center ($-\circ-$). The range of scales where we observe an algebraic scaling, although not extremely large, is however sufficient to propose a trend behavior [19]. As expected, where turbulence is three dimensional (channel center) $S_p \simeq r^{p/3}$ for both Re_τ^L and Re_τ^H. To quantify carefully the scaling behavior of the structure functions at the free surface, we use the extended self-similarity (ESS) representation [17]. The slope (ξ_p) of the pth-order structure function ($S_p \simeq r^{\xi_p}$) is obtained by plotting $\langle(\delta_r u)^p\rangle$ versus $\langle(\delta_r u)^3\rangle$ on a log-log plot, and by computing $\xi_p = d \log S_p / d \log S_3$. The value of ξ_p measured for $p \leqslant 6$ is shown in Fig. 9 and compared with the Kolmogorov $p/3$ scaling (dotted line). Deviations from the Kolmogorov scaling are seen for $p \geqslant 4$ for both Re_τ^L and Re_τ^H (with larger deviations for Re_τ^H), and are likely due to intermittency phenomena occurring at the free surface.

C. Flow compressibility

Examination of the previous results allows us to infer the following scenario for the dynamics of downscale/upscale energy transfer in free-surface turbulence. Downscale energy

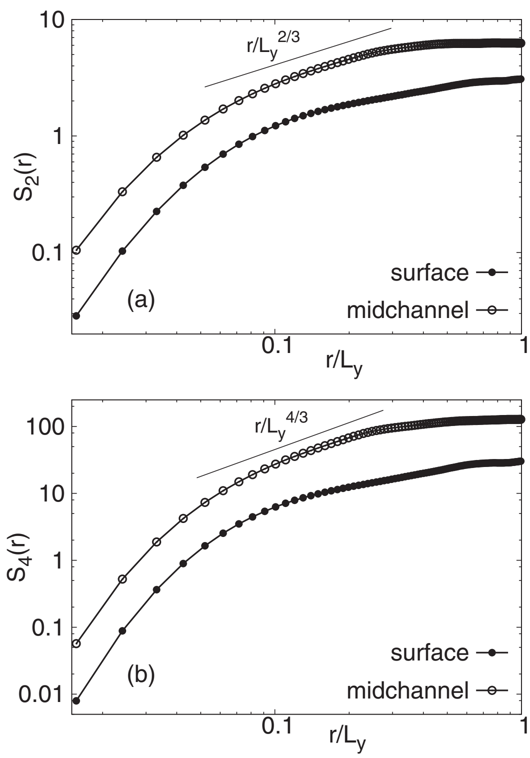

FIG. 8. Second-order [$S_2(r)$, (a)] and fourth-order [$S_4(r)$, (b)] structure functions for $\mathrm{Re}_\tau^H = 509$ expressed as a function of r/L_y and computed at the surface and at the center of the channel. The solid lines indicate the observed scaling in the inertial range. At the channel center (midchannel, $-\circ-$), $S_2 \simeq r^{2/3}$ and $S_4 \simeq r^{4/3}$.

transfer is associated with regions of local flow expansion ($\nabla_{2D} > 0$) caused by plumes and upwellings reaching the free surface from below. Since upwellings are produced by the fully three-dimensional flow field, they follow their three-dimensional turbulence nature, with a dynamics maintained also in the initial stage of their joining the surface. This reminiscence of the three-dimensional turbulence explains the regions of downscale energy transfer observed at the free surface. At later stages, and once attached at the free surface, upwellings gradually forget their three dimensionality and move according to the basic laws of two-dimensional

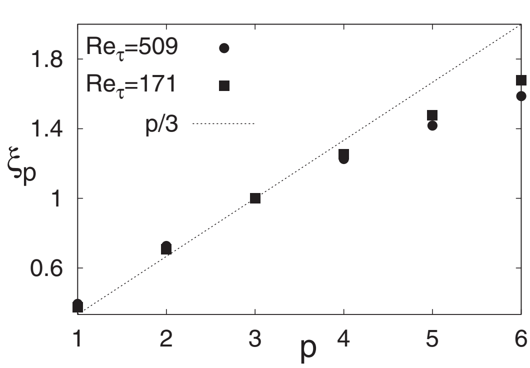

FIG. 9. Structure function scaling exponent ξ_p at the free surface for both Re_τ^L and Re_τ^H. The dotted line indicates the classical Kolmogorov scaling $p/3$.

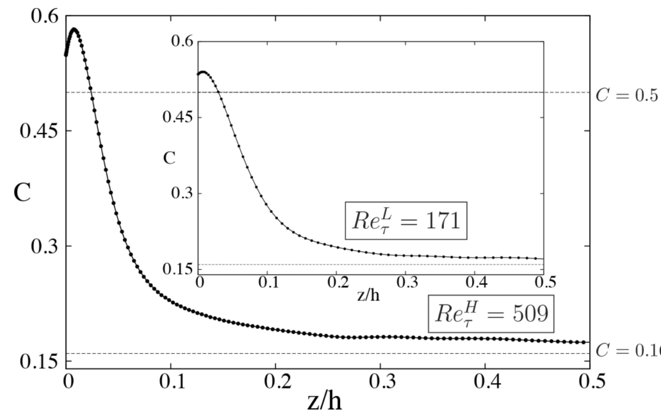

FIG. 10. Behavior of the compressibility factor C along the wall-normal direction z/h for Re_τ^H and for Re_τ^L (inset). Note that $z = 0$ represents the free surface. Values of the compressibility factor (i) for a two-dimensional cut of a three-dimensional homogeneous isotropic turbulent flow ($C \simeq 0.16$) and (ii) for a compressible Kraichnan flow ($C = 0.5$) are also shown [10].

turbulence (hence following the upscale energy transfer). The relative strength of the upscale energy transfer mechanism is small for low Re_τ, but increases for increasing Re_τ. Therefore, it would be natural to hypothesize that, especially for increasingly high Reynolds numbers, the three-dimensional nature of the surface is progressively lost. In fact, it is not so, as we will show by examining the flow compressibility, also in connection with previous results obtained for two-dimensional and three-dimensional homogeneous turbulence [9,10,20]. The degree of compressibility is quantified by the dimensionless compressibility factor

$$C = \frac{\langle (\nabla \cdot \mathbf{v})^2 \rangle}{\langle |\nabla \mathbf{v}|^2 \rangle}. \tag{4}$$

The value of C as a function of the vertical direction z/h is shown in Fig. 10 for both Reynolds numbers. The range of abscissae in Fig. 10 is limited to the top half of the channel from the free surface down to the channel centerline ($z/h = 0.5$). For both Re_τ, compressibility peaks close to the surface and drops down almost asymptotically to $C \simeq 0.16$ (dashed line in Fig. 10) at the channel half height. This limiting value represents the theoretical prediction of C for homogeneous isotropic turbulence [10]. The line $C = 0.5$ in Fig. 10 is instead the theoretical prediction based on the Kraichnan compressible flow as reported in Refs. [10,21]. This value was thought to represent the upper limit of the compressibility factor as computed from three-dimensional simulations of homogeneous isotropic turbulence with suitable boundary conditions [10,20]. Our data show that the compressibility C of a three-dimensional free-surface channel flow exceeds the theoretical threshold value $C = 0.5$. In accordance with Ref. [22], in which an increase of the compressibility factor C was associated with an important upscale energy transfer, we find that compressibility is larger for larger Re_τ, when the upscale energy transfer is proportionally stronger. We also remark that a compressibility factor larger than the critical value $C = 0.5$ is of significant importance: it suggests the occurrence of extreme events (velocity sources and sinks, associated to pointlike structures), which might have strong influence on the dispersion of chemical species and particles in free-surface flows [1] but also in other flow instances [23].

IV. CONCLUSIONS

We used direct numerical simulation of an open channel flow at two different Reynolds numbers ($\mathrm{Re}_\tau^L = 171$ and $\mathrm{Re}_\tau^H = 509$) to analyze the behavior of the energy flux ($\Pi^{(\Delta)}$) and of the local flow compressibility (C) in free-surface turbulence. The value of the energy flux closely relates to the downscale or upscale energy transfer: a positive energy flux indicates a downscale energy transfer, whereas a negative energy flux indicates an upscale energy transfer. For the lower Reynolds number case (Re_τ^L), we observe that the mean energy flux is always positive, indicating the predominance of the downscale energy transfer across the entire scale range. For increasing Re_τ, we observe the formation of an upscale energy transfer, which controls the dynamics of larger scales, with important implications for all transport mechanisms influenced by these scales. We also found that downscale/upscale energy transfer correlates with the behavior of the two-dimensional surface divergence ∇_{2D}: downscale energy transfer is associated with regions of local flow expansion ($\nabla_{2D} > 0$), while upscale energy transfer is associated with regions of local flow compression ($\nabla_{2D} < 0$). There are two possible sources of energy transport at a turbulent free surface. First, energy can be exchanged between the free surface and the bulk simultaneously at all spatial scales. Second, mass and energy transport between the bulk and the surface does not occur simultaneously at all spatial scales. Rather, the fluid enters the surface at flow upwellings (positive flow divergence) and leaves the surface at flow downwellings (negative flow divergence). Both of these mechanisms are at play in free-surface flows: the former dominates at lower Re_τ, while the latter becomes prominent for increasing Re_τ. This scenario is consistent with the behavior of the structure functions of the velocity difference $S_p = \langle(\delta_r u)^p\rangle$ (and in particular of S_3). Despite its two-dimensional geometry, free-surface turbulence has specific features, which cannot but partially be observed in two-dimensional computations. We computed the compressibility factor C and we found that the free-surface compressibility factor can trespass the theoretical threshold $C = 0.5$ (Kraichnan compressible flow), which was considered an upper limit in previous computations [10,20]. These findings are also particularly important because they open new intriguing perspectives to model and parametrize the free-surface dynamics in large eddy simulations (LES), where the adopted subgrid-scale stress models are usually absolutely dissipative, i.e., they only provide for the downscale energy transfer. Based on current results, we suggest that an upscale energy transfer must be taken into account to ensure accurate predictions using LES [24,25].

ACKNOWLEDGMENTS

We acknowledge CINECA supercomputing center (Bologna, Italy) and ISCRA Computing Initiative for generous allowance of computer resources. We also acknowledge Regione Autonoma Friuli Venezia Giulia under Grant No. PAR FSC 2007/2013. We wish to thank the anonymous reviewers for the valuable comments and suggestions made on the previous version of this manuscript.

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