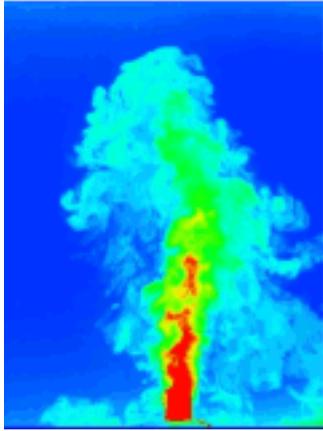


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Particle tracking in LES flow fields: conditional Lagrangian statistics of filtering error

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Particle tracking in LES flow fields: conditional Lagrangian statistics of filtering error

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In this paper, we examine the conditional Lagrangian statistics of the pure filtering error, which affects particle tracking in large-eddy simulations of wall-bounded turbulence. A-priori tests are performed for the reference case of turbulent channel flow, and statistics are computed along the trajectory of many particles with different inertia, initially released in near-wall regions where either a sweep event or an ejection event is taking place. It is shown that the Lagrangian probability density function (PDF) of the filtering error is, in general, different from the Eulerian one, computed at fixed grid points. Lagrangian and Eulerian PDFs become similar only in the long-time limit, when the filtering error distribution is strongly non-Gaussian and intermittent. Results also show that the distribution of the short-time error in the homogeneous directions can be approximated by a Gaussian function. Due to flow anisotropy effects, which are particularly significant for small-inertia particles, such approximation does not hold in the wall-normal direction.

Keywords: large-eddy simulation; Lagrangian sub-grid modeling; turbulent dispersed flows

1. Introduction

The use of large-eddy simulation (LES) to investigate particle-laden turbulent flows has become customary in academia [1–5] and is also emerging in industry [6,7] due to the capability of LES to tackle flow configurations closer to practical applications than those affordable by direct numerical simulation (DNS). However, applicability of LES to dispersed flows is limited by the fact that only the filtered flow velocity field is available: Such flow field reproduces the dynamics of the large scales of motion, and prevents particles from interacting with the small (unresolved) sub-grid scales (SGS) of turbulence, which may strongly influence clustering of inertial particles [8], and leads to significant underestimation of particle preferential concentration and deposition rates. This effect is particularly important in wall-bounded turbulence [9–12], where it results in poor predictions of near-wall accumulation at steady state (when particle concentration gradients become constant in time) [13]. In view of this issue, there is currently a general consensus about the need to model the effect of SGS turbulence on particle dynamics [14]. More specifically, SGS

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models should be able to replicate velocity field fluctuations that drive particle deposition and re-entrainment fluxes when wall-normal concentration gradients are still developing, a condition commonly found in most engineering and environmental applications. Wrong estimation of the instantaneous balance between deposition and re-entrainment fluxes sums up over time and gives the observed accumulated deviations from DNS [10] or experimental results [15]. In this context, a first necessary step is the precise quantification of filtering effects on the near-wall coherent structures that drive wall-normal particle transport (the so-called sweep and ejection events) [16], and in turn on their interaction with the dispersed phase at the particle scale. To this aim, present Eulerian statistics are not sufficient because they provide time-independent description of the error behaviour that can be used only for modelling particle transport at statistically steady state, when particle motion is unaffected by changes in wall-normal concentration gradients. These changes must be accounted for in deposition/re-entrainment models, and time-dependent Lagrangian statistics are thus required to provide the necessary information at times much shorter than those required to reach the steady state. To the best of our knowledge, this kind of statistics is currently unavailable and previous efforts to characterise filtering effects on particle transport focused on the Eulerian moments of $\delta \mathbf{u}$ and on its probability density function (PDF), computed at fixed grid points.

A possible way to address this issue is to carry out a-priori tests in which the flow and particle parameters are in the correct range to explore the specific conditions for which LES are proven to be inadequate [17,18]. In a previous work [19], we focused on the error that is introduced in Lagrangian particle tracking (LPT) in LES flow fields due to filtering of the fluid velocity seen by particles along their trajectory, $\mathbf{x}_p(t)$. In particular, we quantified a lower bound for this error, indicated as $\delta \mathbf{u} \equiv \delta \mathbf{u}(\mathbf{x}_p(t), t)$ hereinafter (see Section 2 for a more formal definition) for the reference case of particle-laden turbulent channel flow, considering an ideal situation in which the exact dynamics of the filtered velocity field is available and time accumulation of the error along particle trajectory (which would otherwise lead to progressive divergence between DNS and LES trajectories) is eliminated a priori. In such situation $\delta \mathbf{u}$ is equal to the difference between the particle-to-fluid relative velocity measured in DNS and that measured in LES, and therefore depends on the time evolution of particle concentration (particle number density) n_p along the wall-normal direction. A similar investigation was carried out by Geurts and Kuerten [13] for the same flow configuration at higher Reynolds numbers. Both studies [13,19] indicate that the filtering error is strongly intermittent (at the subgrid scale) and affected by flow anisotropy. Guidelines were also suggested to develop stochastic closure models for particle motion, based on the observation that the error behaviour is independent of the Reynolds number, of the filter type, and also of particle inertia when proper normalisation is applied. Filtering errors can be corrected using either an Eulerian [20,21] or a Lagrangian approach [22–26]. The Eulerian approach is aimed at retrieving the unresolved SGS velocity fluctuations at each grid point within the computational domain. The Lagrangian approach is based upon the idea of reproducing the velocity of the fluid seen by particles along their trajectory, $\mathbf{u}(\mathbf{x}_p(t), t)$, indicated simply as \mathbf{u}_s hereinafter. The findings of Bianco et al. [19] support the idea of developing a purely Lagrangian stochastic model, based on a modified Langevin equation written for \mathbf{u}_s . The whole stochastic process would be a function of a proper state vector, in our case $\mathbf{Z} = (\mathbf{x}_p, \mathbf{u}_p, \mathbf{u}_s)$ with \mathbf{u}_p the particle velocity, as follows:

$$d\mathbf{u}_s(t) = \mathbf{A}(t; \mathbf{Z}, \overline{F(\mathbf{Z})})dt + \mathbf{B}(t; \mathbf{Z}, \overline{G(\mathbf{Z})})d\mathbf{W}(t) + \gamma(t), \quad (1)$$

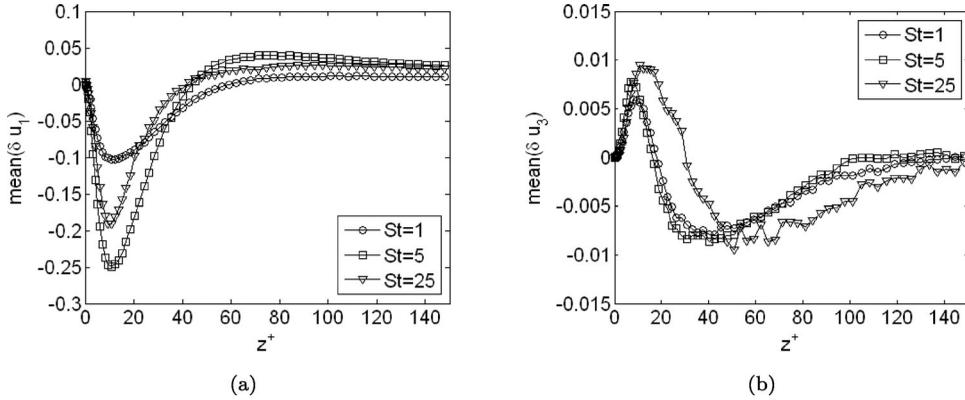


Figure 1. Mean values of the pure filtering error $\delta \mathbf{u}$ as a function of wall distance, z^+ , at varying particle inertia: (a) streamwise component, $\langle \delta u_x \rangle$; (b) wall-normal component, $\langle \delta u_y \rangle$. Profiles refer to the a-priori tests performed with a cut-off filter of width $CF=4$ in the Fourier space. From Bianco et al. [19]

where \mathbf{A} and \mathbf{B} are the usual drift and diffusion terms, whereas \bar{F} and \bar{G} denote any filtered fluid field variable taken at the particle location or any filtered field obtained from the stochastic process employed in the model. The additional term γ in Equation (1) represents a coloured process and/or a Poisson process that takes into account all those properties of the flow field that are relevant for particle transport. However, this term cannot be approximated by a simple diffusive process. The results discussed by Bianco et al. [19] further suggest that a one-point PDF model [27], namely a Langevin model that does not incorporate spatial information about sweeps and ejections, cannot reproduce phenomena related to two-point statistics like preferential concentration and wall accumulation. Spatial information could be added through a real two-point model, a quite complicated approach in non-homogeneous flows, or through a modified Langevin model in which the residual effect of coherent structures is introduced via a counting process that depends on the mean flow properties evaluated at the particle locations. This idea has been recently investigated by Guingo and Minier [28], who developed a model in which particle interaction with sweep and ejection events is simulated explicitly by including the main geometrical features of these events in a statistical description of $\delta \mathbf{u}$. Since the goal is to develop a physically sound Lagrangian stochastic model, Lagrangian analysis appears clearly inevitable [29]. Thus, to pursue the path indicated by Guingo and Minier [28], for LES, the Eulerian characterisation of $\delta \mathbf{u}$ provided by Bianco et al. [19] and by Kuerten and Geurts [13,19] is not sufficient because it does not single out filtering effects on deposition from those on re-entrainment, nor it allows to inspire or benchmark time evolution models of these effects. To elaborate, in Figure 1 (taken from Bianco et al. [19]), we show the wall-normal behaviour of the mean error $\langle \delta \mathbf{u} \rangle$ (streamwise and wall-normal components, with brackets denoting variables averaged in time and space over the homogeneous flow directions). This quantity is given by two contributions, which cannot be isolated from Figure 1, and yet they must be characterised statistically to be included in Equation (1): the density-weighted filtering error for particles entrained in sweeps, $\langle \delta \mathbf{u} \cdot n_p \rangle_{sw}$, and the density-weighted filtering error for particles entrained in ejections, $\langle \delta \mathbf{u} \cdot n_p \rangle_{ej}$. When deposition and re-entrainment fluxes do not balance [30,31], both error components are non-zero and, similarly to the particle

drift velocity [30,32], depend on the instantaneous particle concentration in the coherent areas near the wall ($n_{p,sw}$ in sweeps, $n_{p,ej}$ in ejections).

Existing studies must therefore be integrated with quantitative analyses of the statistical properties (PDFs in particular) of $\delta \mathbf{u}$, conditioned to specific initial conditions for particle interaction with the near-wall structures. This is precisely the aim of the present work, which is based on a-priori tests similar to those performed by Bianco et al. [19], but scrutinises the statistical properties of the error discriminating between particles initially trapped in a fluid sweep and particles initially trapped in a fluid ejection. The time evolution of these properties is then examined as particle concentration gradients develop in the wall-normal direction.

2. Problem formulation and numerical methodology

We used pseudo-spectral DNS [31] to compute the turbulent Poiseuille flow of air (incompressible and Newtonian) in a three-dimensional channel at $Re_\tau = u_\tau h/\nu = 150$, where u_τ is the friction velocity, ν is fluid viscosity and h is the channel half-height. This value of Re_τ was chosen to match the simulation parameters used by Bianco et al. [19] and allow direct comparison of statistics. Based on the findings of Geurts and Kuerten [13], we expect present results to be relevant up to much higher Reynolds numbers (e.g. $Re_\tau \simeq 900$) provided that appropriate re-scaling is applied.

The reference geometry consists of two flat parallel walls with periodic boundary conditions in the streamwise (x) and spanwise (y) directions and no-slip conditions at the walls (located at $z = 0$ and $z = 2h$). The size of the computational domain is $L_x \times L_y \times L_z = 4\pi h \times 2\pi h \times 2h$, corresponding to $1885 \times 942 \times 300$ wall units (obtained using ν and u_τ for adimensionalisation, and identified with the superscript ‘+’ hereinafter), discretised with $128 \times 128 \times 129$ grid nodes. A two-level, explicit Adams–Bashforth scheme for the non-linear terms, and an implicit Crank–Nicolson method for the viscous terms in the Navier–Stokes equations are employed for time advancement.

We tracked three sets of $-O(10^5)$ –particles, characterised by values of the Stokes number, St , equal to 1, 5 and 25. The Stokes number is defined here as $St = \tau_p/\tau_f$, where $\tau_p = \rho_p d_p^2/18\mu$ is the particle response time (based on particle density ρ_p , particle diameter d_p and fluid dynamic viscosity μ) and $\tau_f = \nu/u_\tau^2$ is the fluid characteristic timescale. Values of the Stokes number were selected based on previous studies of particle preferential concentration in turbulent channel flow at the same Reynolds number [16,31] here, in which it was shown that maximum segregation and preferential concentration occurs for particles with $St = 25$. The two smaller values were thus chosen to highlight trends in the behaviour of the filtering error as preferential concentration effects become weaker. Particles are modelled as pointwise rigid spheres that are much heavier than the fluid ($\rho_p/\rho \simeq 769$). Particles are injected into the flow at concentration low enough to consider dilute flow conditions (no inter-particle collisions) and one-way coupling between the two phases (no turbulence modulation by particles). Particle-wall interaction is modelled considering perfectly elastic rebound, while periodic boundary conditions are applied along x and y . The Lagrangian equation of particle motion in vector form reads as [32,33]

$$\frac{d^2 \mathbf{x}_p}{dt^2} = \frac{d\mathbf{v}_p}{dt} = \frac{\mathbf{u}_s - \mathbf{v}_p}{\tau_p} (1 + 0.15 Re_p^{0.687}), \quad (2)$$

where $Re_p = d_p |\mathbf{v}_p - \mathbf{u}_s|/\nu$ is the particle Reynolds number. Equation (2) is solved using a fourth-order Runge–Kutta scheme for time advancement and an interpolation scheme based on sixth-order Lagrangian polynomials for computation of \mathbf{u}_s interpolation at particle location.

In the a-priori tests, LPT is carried out replacing \mathbf{u}_s in Equation (2) with the filtered fluid velocity field, $\bar{\mathbf{u}}(\mathbf{x}_p, t)$. This field is obtained through explicit filtering of the DNS velocity in the wave number by a cut-off filter applied in the homogeneous directions. Several filter widths have been examined (see also Bianco et al. [19]): results shown in this paper refer to a filter width $CF = 4$ with respect to DNS (corresponding to 32×32 Fourier modes). Data are not filtered in the wall-normal direction z since the wall-normal resolution in LES is often DNS like. Since we want to examine the error due to filtering in isolation, we consider an ideal situation in which all further sources of error affecting particle tracking in LES¹ can be disregarded, thus making the exact dynamics of the resolved velocity field available. Moreover, time accumulation of the pure filtering error on the particle dynamics is prevented by computing first the trajectory of each particle in the DNS flow fields and then forcing the particle to evolve in the filtered DNS fields following its DNS trajectory. This means that, at each time step n and for each particle k , we impose

$$\mathbf{x}_{p,k}^{f\text{-DNS}}(t^n) = \mathbf{x}_{p,k}^{\text{DNS}}(t^n) \equiv \mathbf{x}_{p,k}(t^n) ; \quad \mathbf{v}_{p,k}^{f\text{-DNS}}(t^n) = \mathbf{v}_{p,k}^{\text{DNS}}(t^n) \equiv \mathbf{v}_{p,k}(t^n), \quad (3)$$

where superscripts f – DNS and DNS identify particle position and velocity in filtered DNS (i.e. a-priori LES) and in DNS, respectively. The pure filtering error affecting the fluid velocity seen by the k th particle at time t^n is computed as

$$\delta \mathbf{u} \equiv \delta \mathbf{u}(\mathbf{x}_{p,k}(t^n), t^n) = \mathbf{u}(\mathbf{x}_{p,k}(t^n), t^n) - \bar{\mathbf{u}}(\mathbf{x}_{p,k}(t^n), t^n) \equiv \mathbf{u}_s - \bar{\mathbf{u}}_s. \quad (4)$$

This error is made non-dimensional using the shear velocity: $\delta \mathbf{u}^+ = \delta \mathbf{u}/u_\tau$.

To evaluate the probability distribution of $\delta \mathbf{u}$, a-priori tests were carried out selecting two possible initial conditions for the location of particle injection into the flow: for each set tracked, particles were released either in regions where a sweep event was taking place or in regions where an ejection event was detected. To identify these regions, the sign of the streamwise fluid velocity fluctuation u' and of the fluid Reynolds stress component $|u'|w'$ was evaluated on five wall-parallel monitor planes in the near-wall region (located at $z^+ = 4, 6, 8, 10, 12$ in the present flow configuration) [16]. An event is recorded at a given location when on at least four of the five planes u' and $|u'|w'$ have the same sign: $u' > 0$ and $|u'|w' < 0$ for a sweep; $u' < 0$ and $|u'|w' > 0$ for an ejection. Results discussed in this paper were obtained considering, for each value of the Stokes number, a sample of 10,000 particles released randomly within each so-identified sweep region on the reference monitor plane at $z^+ = 8$, and a sample of 10,000 particles released randomly within each ejection region on the same plane. For comparison purposes, also particles with initial uniform distribution in the $z^+ = 8$ plane were tracked. In all cases, the initial particle velocity was set equal to that of the fluid at the particle location.

3. Results and discussion

In this paper, we focus on the Lagrangian PDFs of the filtering error, which were computed at different times along particle trajectories conditioned by the particle initial position within a sweep (referred to as sweep-conditioned PDFs hereinafter) or within an ejection (referred to as ejection-conditioned PDFs hereinafter). This initial condition forces particle

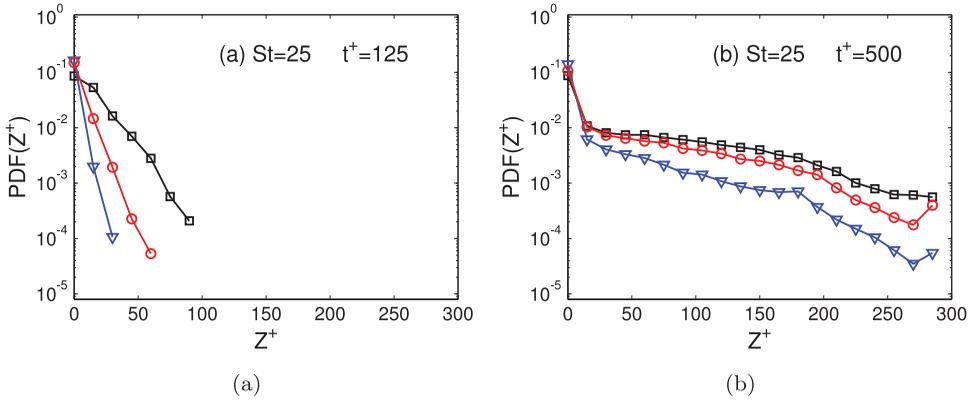


Figure 2. (a) Conditional PDF of particle-to-wall distance at $t^+ = 125$ and $t^+ = 500$. (b) For $St = 25$ particles released in sweeps (∇) or in ejections (\square) at $z^+ = 8$. For comparison, the PDF of particles uniformly released at $z^+ = 8$ (\circ) is also shown.

interaction with near-wall coherent structures, as demonstrated in Figure 2, where we show the sweep-conditioned and ejection-conditioned PDFs of the particle-to-wall distance for the $St = 25$ particles at times $t^+ = 125$ (panel a) and $t^+ = 500$ (panel b). Also shown is the PDF obtained when particles are injected with uniform distribution on the $z^+ = 8$ plane (circles). Results for the other particle sets ($St = 1$ and 5) are qualitatively similar and are not shown for brevity. As expected, particles released within sweeps are more likely to remain confined in the near wall region, whereas particles released within ejections can be more easily entrained toward the center of the channel. Particles injected uniformly exhibit an intermediate behaviour, thus supporting the adequacy of the criterion chosen to disentangle particles-turbulence interaction near the wall. Moreover, in Figure 2(b), it is shown that in the long time, sweeps are more efficient than ejections (the curve is very close to initially random conditions). That explains the mechanism underlying net flux of particles towards the wall experienced by inertial particles, confirming previous analysis [16]. The sweep-conditioned and ejection-conditioned Lagrangian PDFs of the filtering error components are reported in Figures 3 and 4, for the same times of Figure 2. Also shown is the Gaussian PDF with the same mean and standard deviation as the $PDF(\delta u_i)$. From a collective examination of these figures, Lagrangian results confirm that the filtering error is typically stochastic [13,19]. However, Lagrangian PDFs at short times (e.g. $t^+ = 125$) appear significantly different with respect to those at later times and to their Eulerian counterpart (shown in Figures 12 and 13 of the paper by Bianco et al. [19]). In particular, it seems that the error distribution in the homogeneous directions x and y can be approximated with reasonable accuracy by a Gaussian function in the short-time limit, with the exception of few rare events characterised by very large values of δu_i (especially in the spanwise direction). This was not observed for the Eulerian PDFs, which always deviate significantly from a Gaussian distribution [19]. In the wall-normal direction, the error distribution for the high-inertia particles ($St = 25$, circles) is still very close to Gaussian. However, effects due to the anisotropy of the flow show up even at short times for the low-inertia particles, as indicated in particular by the broad tails of $PDF(\delta u_z)$ for the $St = 1$ particles in Figures 3(c) and 4(c). These features are due to filtering effects on coherent structures and, in turn, on particles-turbulence interaction mechanisms. As shown by Bianco et al. [19], filtering weakens the intensity of both sweeps and ejections, leading to underestimation

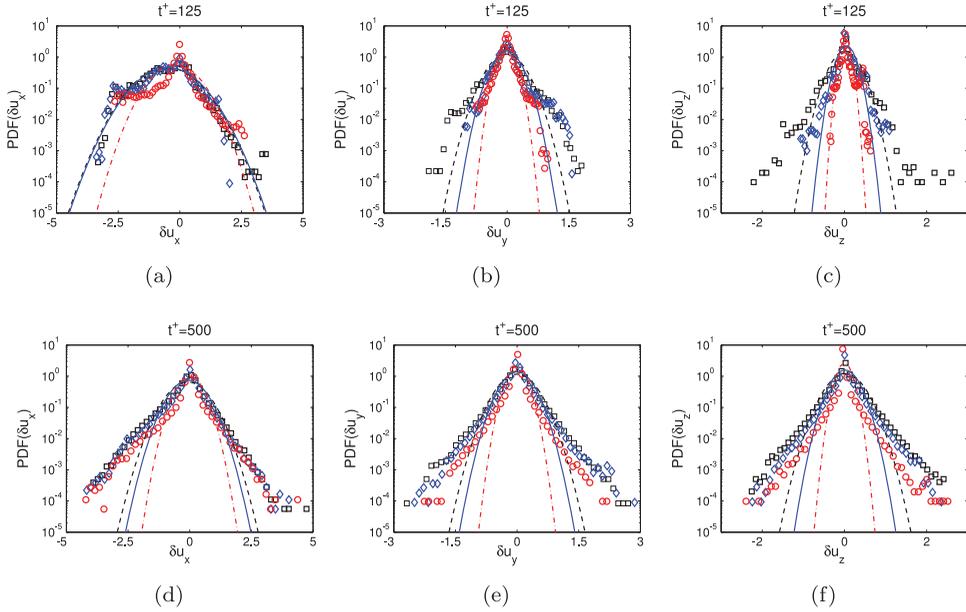


Figure 3. Instantaneous PDF of the filtering error components, δu_i , at varying Stokes numbers (symbols), evaluated for particles initially released in sweeps at $z^+ = 8$ (cut-off filter with $CF=4$). The corresponding Gaussian PDFs (lines) with equal standard deviation are also reported. Panels: (a–c) $t^+ = 125$, (d–f) $t^+ = 500$. Symbols: \square $St = 1$, \diamond $St = 5$, \circ $St = 25$. Lines: — — — $St = 1$, — — — — $St = 5$, - · - · - $St = 25$.

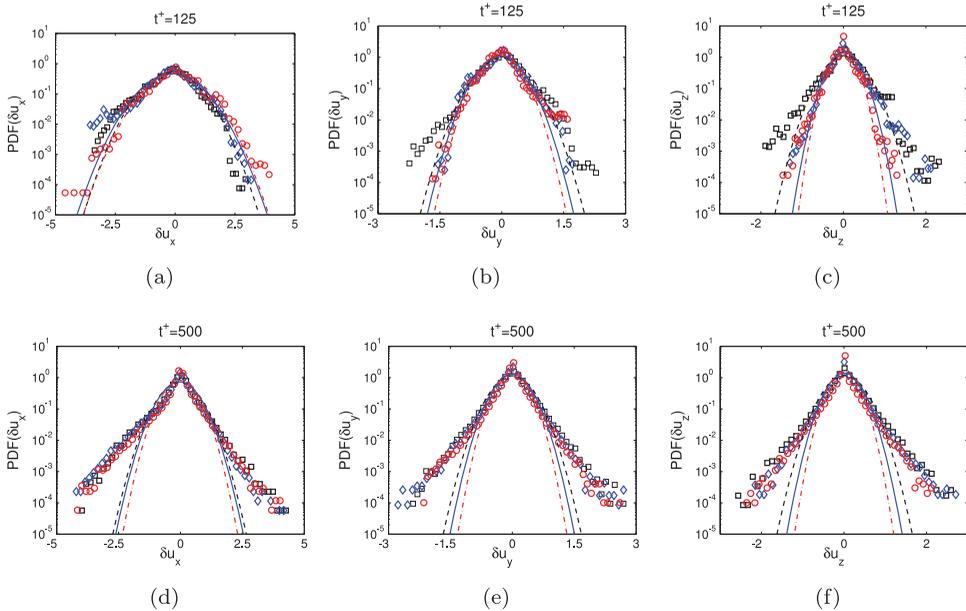


Figure 4. Instantaneous PDF of the filtering error components, δu_i , at varying Stokes numbers (symbols), evaluated for particles initially released in ejections at $z^+ = 8$ (cut-off filter with $CF = 4$). The corresponding Gaussian PDFs (lines) with equal standard deviation are also reported. Panels: (a–c) $t^+ = 125$, (d–f) $t^+ = 500$. Lines and symbols are as in Figure 3.

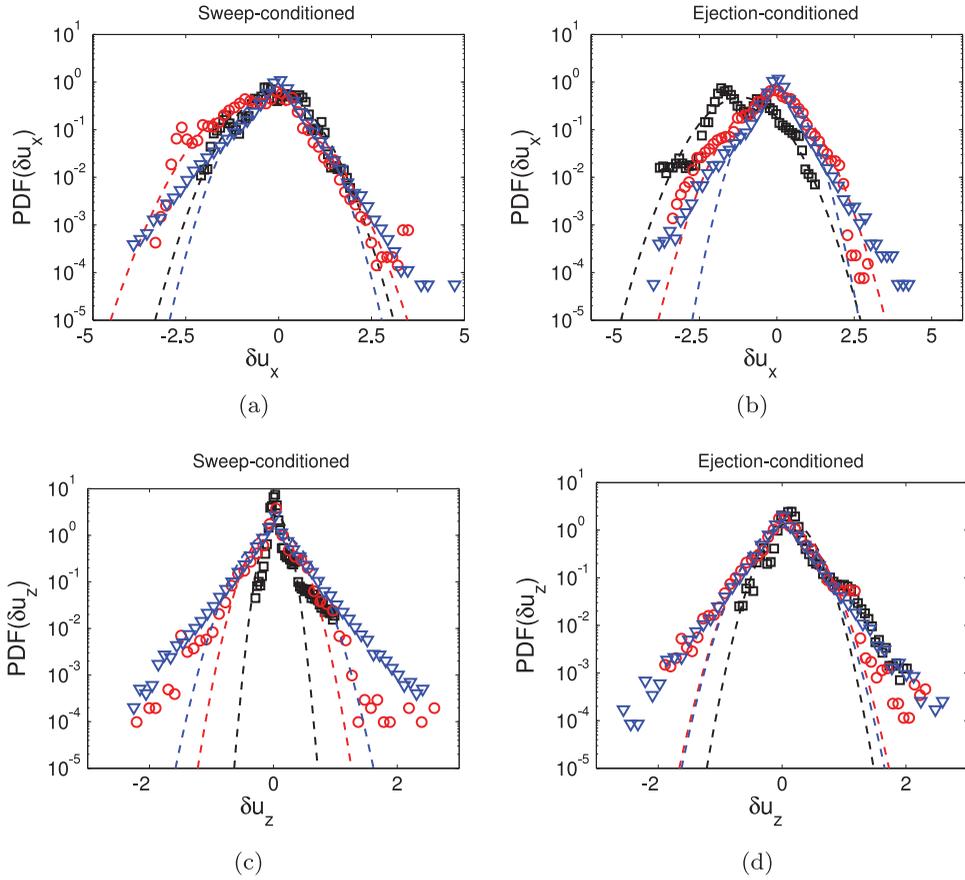


Figure 5. Time evolution of the conditioned Lagrangian PDF of the filtering error components, $\text{PDF}(\delta u_i)$, evaluated for the $St = 1$ particles. The corresponding Gaussian PDFs (lines) with equal standard deviation are also reported. Panels: (a), (c) sweep-conditioned PDF; (b), (d) ejection-conditioned PDF. Symbols: \square $t^+ = 25$, \circ $t^+ = 125$, ∇ $t^+ = 500$. Lines: $---$ $t^+ = 25$, $---$ $t^+ = 125$, $---$ $t^+ = 500$.

of particle fluxes to and off the wall. The much broader tails of $\text{PDF}(\delta u_z)$ for the $St = 1$ particles compared to that of $St = 5$ and $St = 25$ particles at short times, observed in Figure 3(c), can be explained considering that, starting from the initial condition imposed on particle velocity ($v_{p,i} = u_{s,i}$), the error distribution evolves in time depending on the particle Stokes number: low-inertia particles reach local equilibrium with the surrounding fluid faster than high-inertia particles and their dynamics starts to be affected by filtering after a much shorter transient upon injection. In the homogeneous directions, however, the Stokes number dependence of the PDFs is marginal, as was also found for the Eulerian PDFs [19].

To examine the time evolution of the sweep-conditioned and ejection-conditioned error distribution, in Figure 5 we compare directly the Lagrangian PDFs computed at three different times of the simulation ($t^+ = 25$, $t^+ = 125$ and $t^+ = 500$) for the reference case of $St = 1$ particles. The PDF of the spanwise error component (not shown) is very similar to that of the streamwise error. Figure 5 shows that, as time progresses, the error distribution is characterised by broader tails that cover a wider range of values and deviates

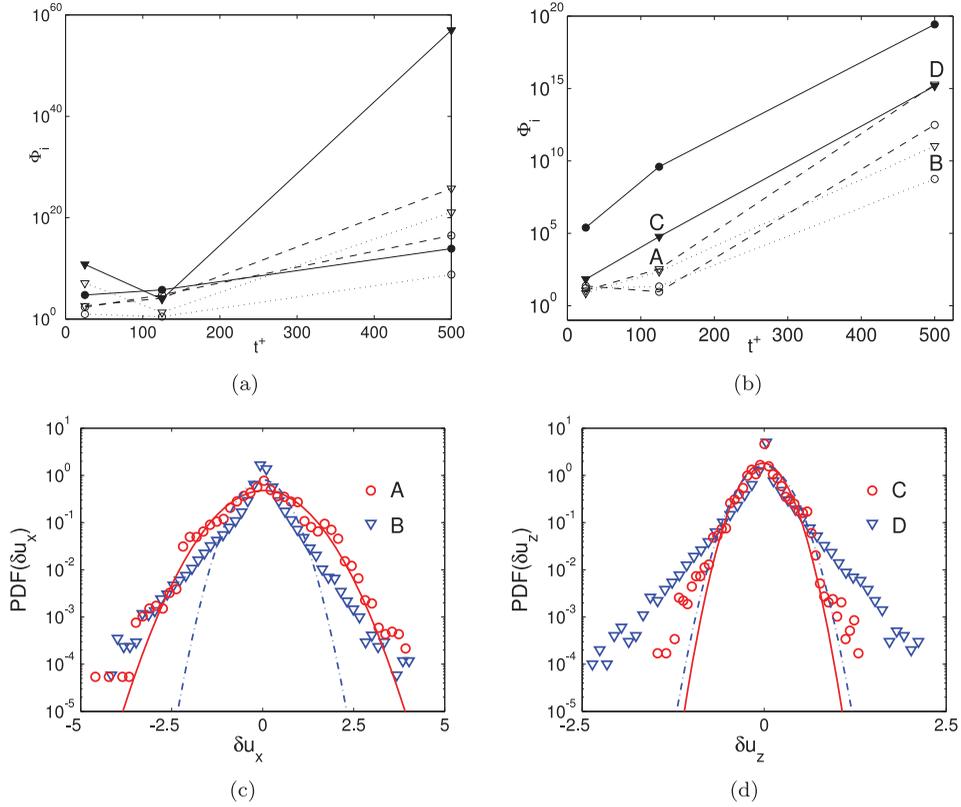


Figure 6. Time evolution of the deviation from Gaussianity, Φ_i , for sweep-conditioned PDFs (a) and for ejection-conditioned PDFs (b). Symbols: \bullet , \circ $St = 5$, \blacktriangledown , \triangledown $St = 25$. Lines: $\cdots\cdots$ Φ_x , $---$ Φ_y , $---$ Φ_z . Symbols associated with letters A to D in panel (b) correspond to the instantaneous ejection-conditioned Lagrangian PDF of the streamwise and wall-normal filtering error for the $St = 25$ particles, shown in panels (c) and (d) respectively. Symbols: \circ $t^+ = 125$, \triangledown $t^+ = 500$. Lines correspond to GPDF(δu_i): $---$ $t^+ = 125$; $---$ $t^+ = 500$.

significantly from Gaussianity: eventually, the Lagrangian conditional PDFs recover the Eulerian behaviour, which is the expected long-time asymptote. We remark here that these findings are common to all error components regardless of the particle Stokes number and of the initial condition imposed on particle-turbulence interaction.

For modelling purposes, it is useful to quantify the observed deviation from Gaussianity as a function of time. To capture the global tendency, we propose to use the following integral parameter:

$$\Phi_i = \sqrt{\int_{\min(\text{PDF}(\delta u_i))}^{\max(\text{PDF}(\delta u_i))} (\text{PDF}(\delta u_i) - \text{GPDF}(\delta u_i)) / \text{GPDF}(\delta u_i)^2 d\delta u_i}, \quad (5)$$

where GPDF denotes the Gaussian PDF having the same standard deviation as the Lagrangian PDF and $i = x, y, z$. The time evolution of Φ_i associated with the sweep/ejection-conditioned errors for the two largest particle sets ($St = 5$ and $St = 25$) is shown in Figure 6(a) and 6(b), respectively. Confirming the trend observed in Figure 5, the deviation from Gaussianity of Lagrangian conditioned PDFs increases significantly in time.

Eventually, the values of Φ_i become very large due to the huge increase of the difference PDF(δu_i) – GPDF(δu_i) in the PDF tails, where the values of GPDF(δu_i) are very low compared to PDF(δu_i) (see Figure 6(b) and 6(c), for instance). For the sweep-conditioned case, shown in Figure 5(a), Φ_i is always larger for the largest $St = 25$ particles, especially in the wall-normal direction. The proportionality between Φ_i and St is lost in the ejection-conditioned case, shown in Figure 5(b). For the filter width considered in the present work, the largest deviation from Gaussianity is obtained for the $St = 25$ particles only in the homogeneous directions; the values of Φ_i in the wall-normal direction being maximum for the intermediate $St = 5$ particles. It is worth emphasising that the integral parameter used here is clearly too rough to allow a refined sensitivity analysis but this finding shows clearly that the combination of filtering, inertia and flow anisotropy effects may change the error distribution in a complex fashion.

4. Conclusions

The PDFs discussed in this paper were obtained from a-priori tests performed to characterise the Lagrangian statistical properties of the pure filtering error for the reference case of particle-laden turbulent channel flow. In addition to the original Lagrangian approach adopted, the novelty of the study also lies in conditioning the statistics with different initial conditions for particle interaction with near-wall sweep and ejection events. This allows us to differentiate the error distribution behaviour between particles trapped in sweeps (relevant for deposition phenomena) and particles entrained in ejections (relevant for entrainment phenomena). By examining the time evolution of the conditioned PDFs, we show that the error distribution at short times can be described by a Gaussian function along the homogeneous flow directions, but not in the wall-normal direction. Eventually, Lagrangian PDFs evolve showing increasing deviations from Gaussianity (which we quantified through a suitable integral parameter) and become similar to the Eulerian PDFs. We remark here that filtering alone is sufficient to change the time behaviour of Lagrangian statistics, whereas this is not observed with Eulerian statistics (e.g. the time correlations of velocity Fourier modes [34]). Our results also show that the long-term error distribution always exhibits a strongly non-Gaussian and intermittent nature, which is determined by the flow anisotropy. The flow anisotropy may also explain why the conditional PDFs examined here are different from previous DNS results exhibiting strong intermittency [17,18]. These findings may provide useful information to develop a Lagrangian sub-grid model that can give accurate predictions for clustering and wall-accumulation of inertial particles in wall-bounded flows. In particular, albeit stochastic, small-scale effects are not far from being Gaussian with some dependency on inertia and anisotropy. This observation does not imply that the filtering error can be approximated by a diffusion process, but supports the idea of developing an anisotropic Lagrangian model along the lines recently proposed by Chibbaro and Minier [27].

Future developments of this work are concerned first with the increase of the flow Reynolds number at which tests are performed to investigate on possible scaling of the filtering error behaviour with Re_τ . Based on the findings of Geurts and Kuerten [13], who observed little or no Re_τ dependence of the filtering error up to Re_τ , such scaling behaviour analysis would require an increase of the Reynolds number by at least one order of magnitude (and simulations with very high costs for nowadays computers). It would be also useful to evaluate the Lagrangian conditioned PDFs in a-posteriori tests (based on *real* LES), in which only an approximation of the dynamics of the filtered flow field is available.

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Note

1. If in an LES the equation of particle motion is solved with the filtered fluid velocity, three sources of error can be distinguished with respect to DNS [9]. The filtering (or *subgrid*) error is introduced because equations are solved with the filtered velocity. Second, a *modelling* error occurs because a real LES does not provide the exact filtered velocity to the particle equations, but only an approximation due to the limitations of the sub-grid model. Third, an *interpolation* error is made, since the LES is performed on a much coarser grid than the DNS. However, this error can be considered negligible when high-order interpolation schemes are used [9].

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