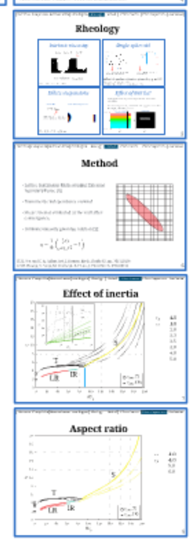
Thank you!



Conclusions

- Taking account to inertia in a dilute suspension of prolate spheroids:
  - The suspension can be shear thinning below Re_c
  - The suspension is shear thickening above Re_c
  - With heavier particles, the suspension becomes more shear thickening everywhere.
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Experiments?

Some challenges:

- High Re_p
- Confinement
- Gravity
- Accuracy

KTH Mechanics

G.W. Woodruff School of Mechanical Engineering

Rheology of dilute suspensions containing prolate spheroids: The effect of inertia

Tomas Rosén, Fredrik Lundell, Minh Do-Quang and Cyrus K. Aidun

Dispersed particle flows

Food

Ref: GorillaAttack/Shutterstock

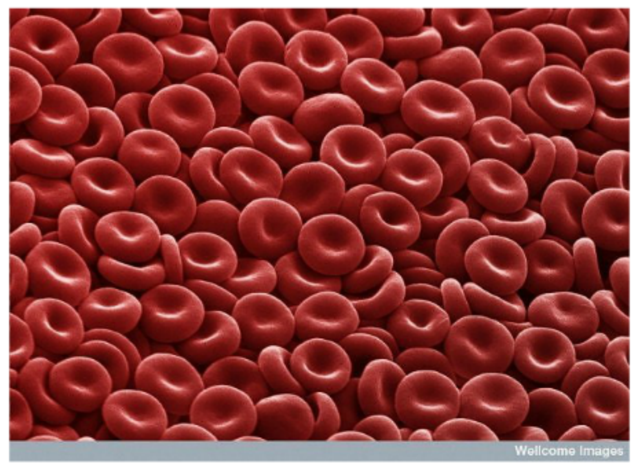
Material processes

Ref: Motif

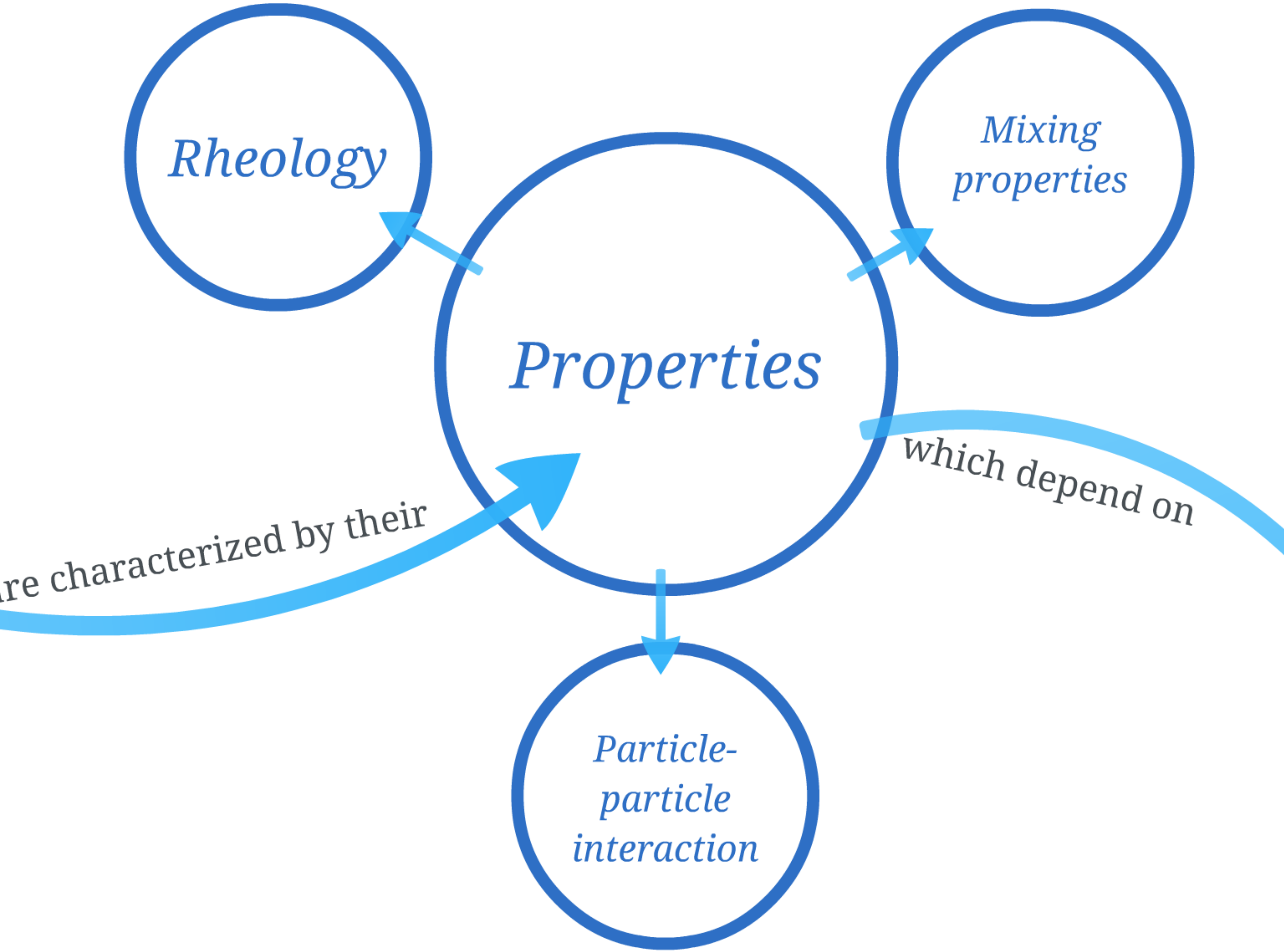
Geophysical

Ref: Snowcrystals.com

Biomedical

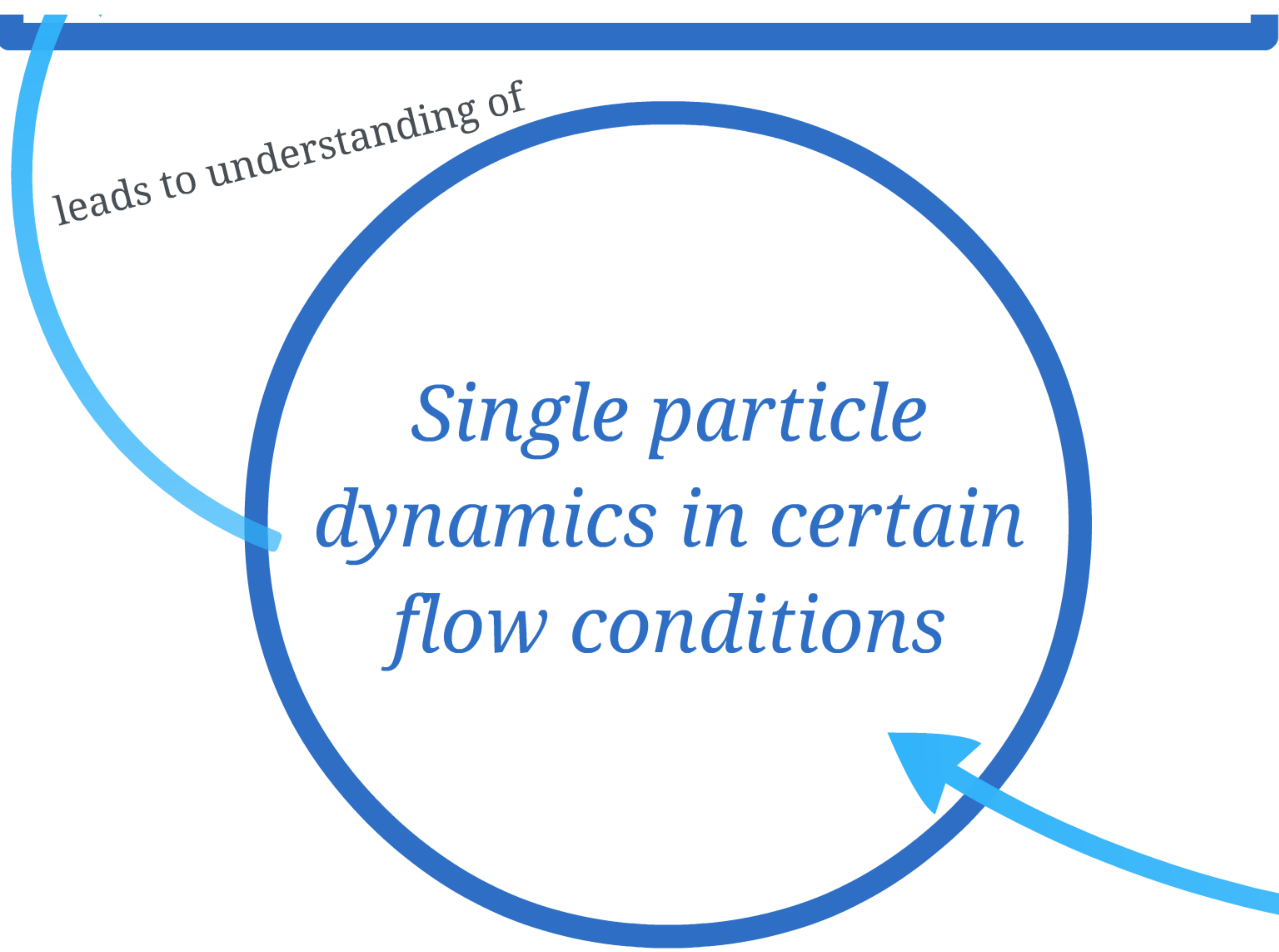


Ref: Wellcome Images


Rheology
Mixing properties
Properties
are characterized by their
which depend on
Particle-particle interaction

Particle distribution in physical and orientation space

which is determined by


leads to understanding of
Single particle dynamics in certain flow conditions

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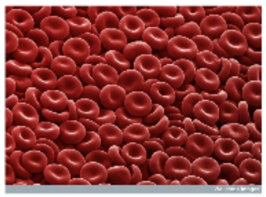

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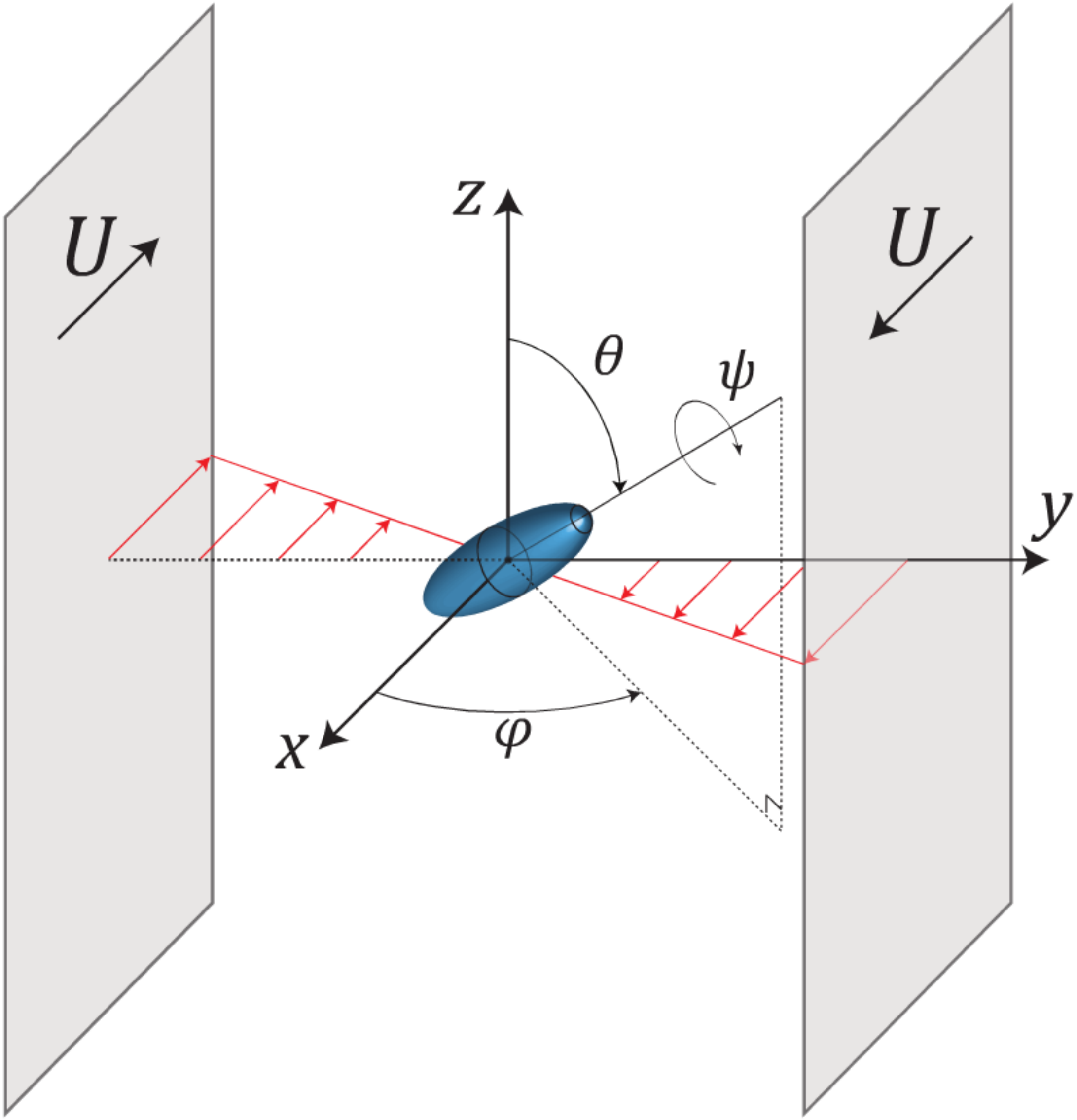
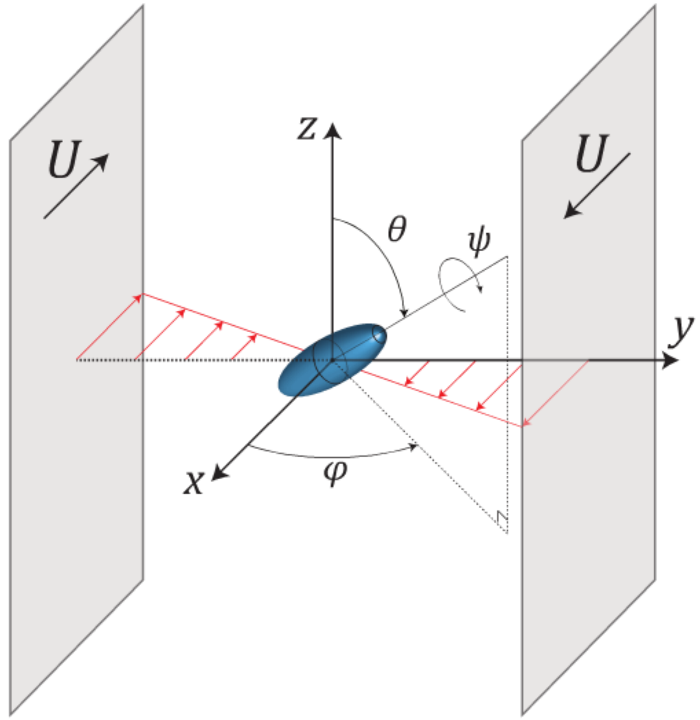


Shear rate:

$$G = \frac{\partial u_x}{\partial y}$$

Flow problem



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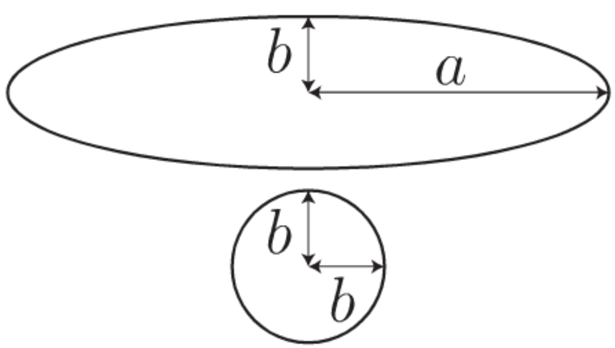
Prolate spheroid



$$\alpha = \frac{\rho_{particle}}{\rho_{fluid}}$$

$$r_p = \frac{a}{b}$$

Flow:

$$\nabla \cdot \vec{u} = 0$$

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u} = -\frac{\nabla p}{\rho} + \nu \nabla^2 \vec{u}$$

$$\vec{u}_{surf} = \vec{\omega} \times \vec{r}$$

Particle:

$$\mathbf{I} \cdot \frac{d\vec{\omega}(t)}{dt} + \vec{\omega}(t) \times [\mathbf{I} \cdot \vec{\omega}(t)] = \vec{T}(t)$$

Fluid inertia:

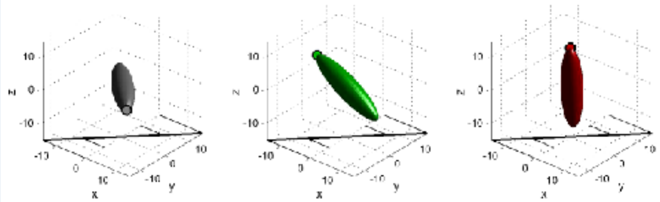
$$Re_p = \frac{4Ga^2}{\nu}$$

Particle inertia:

$$St = \alpha \cdot Re_p$$

Rotational states

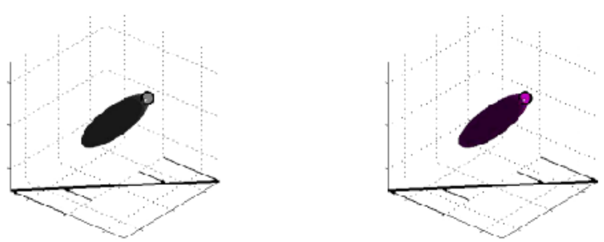
Without inertia



Jeffery orbits

G. B. Jeffery, Proc. R. Soc. Lond. A 102, pp. 161 (1922)

States associated with Particle inertia

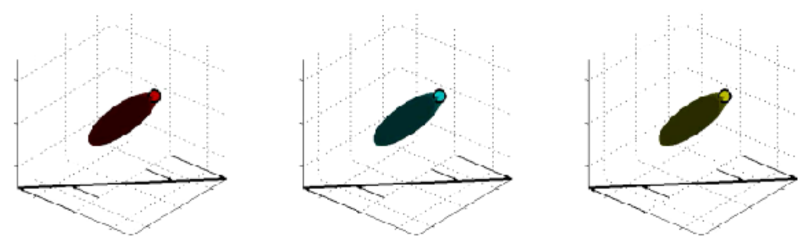


Tumbling (T)

Rotating w. const. ang. vel. (R)

F. Lundell, A. Carlsson, Phys. Rev. E 81, 016323 (2010)

States associated with Fluid inertia



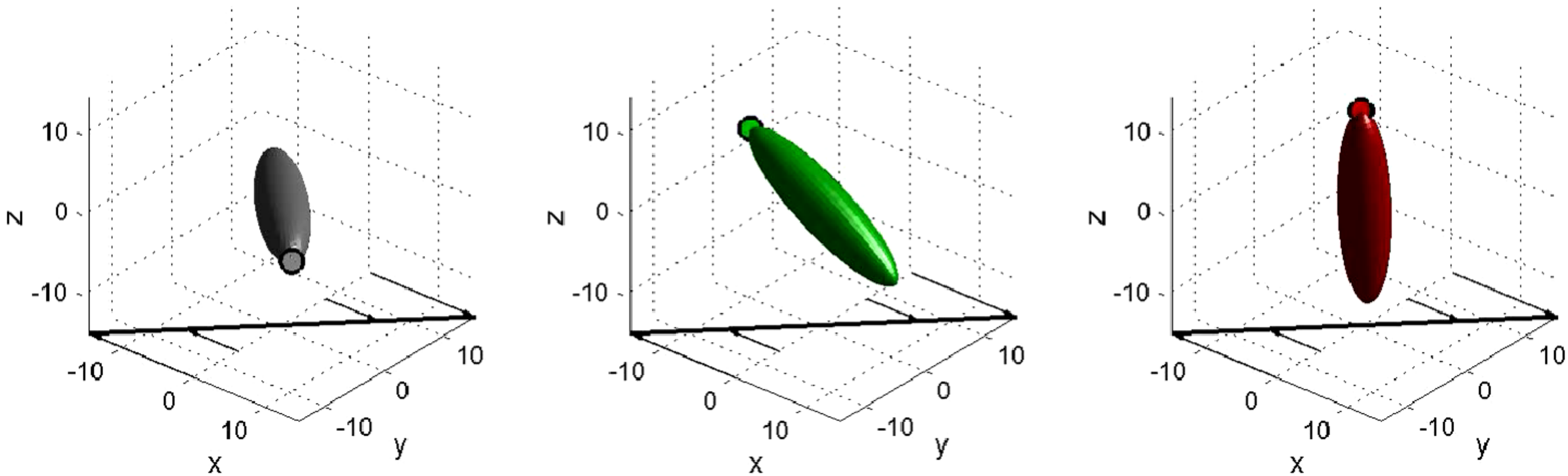
Log-rolling (LR)

Inclined rolling (IR)

Steady state (S)

T. Rosén, F. Lundell, C. K. Aidun, J. Fluid Mech., submitted (2013)

Without inertia



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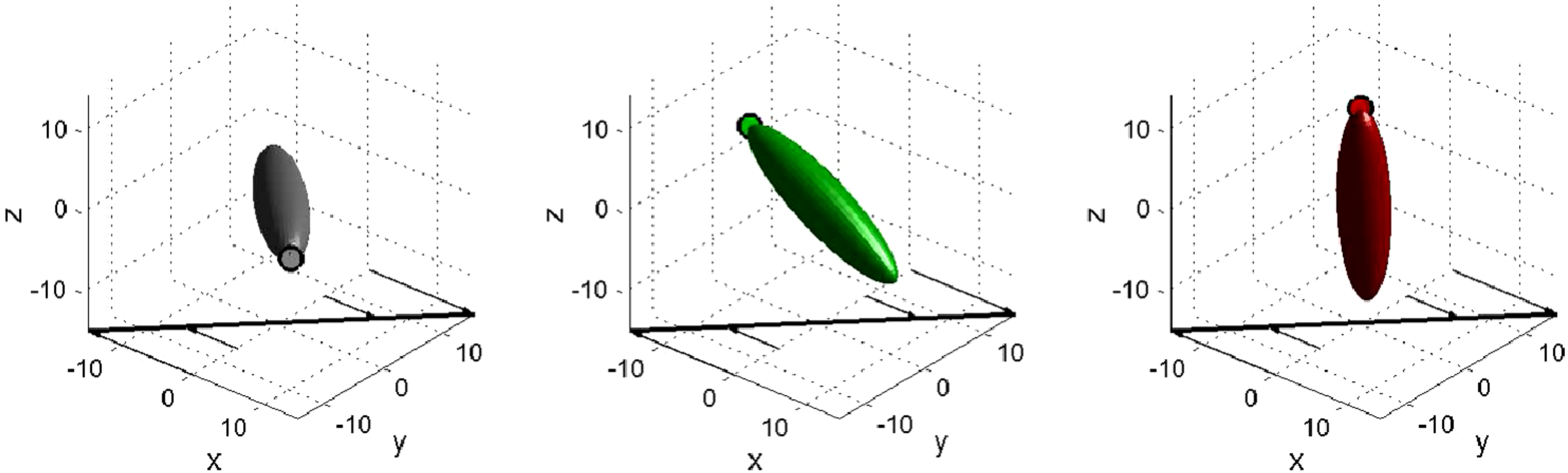

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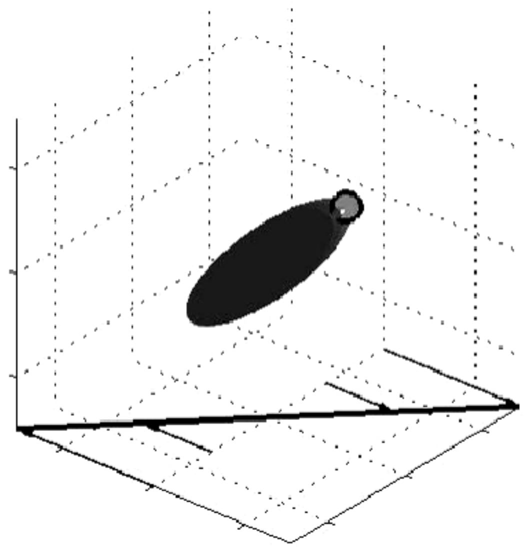

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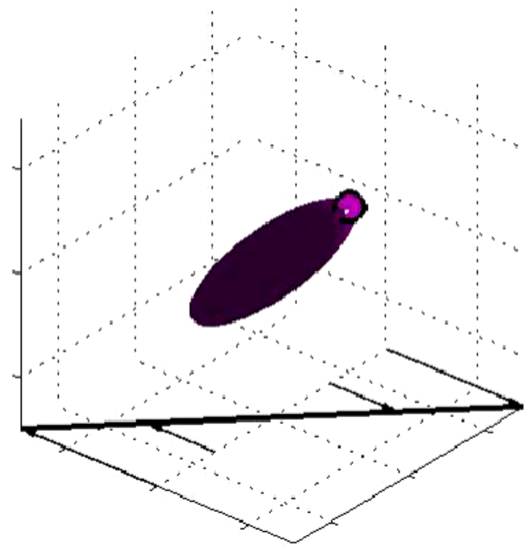

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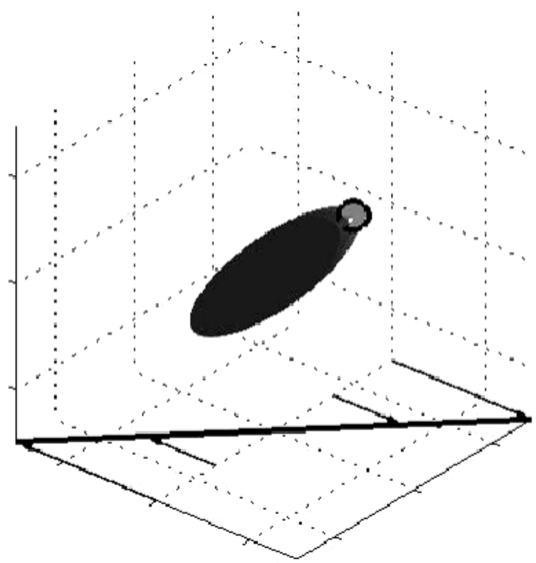
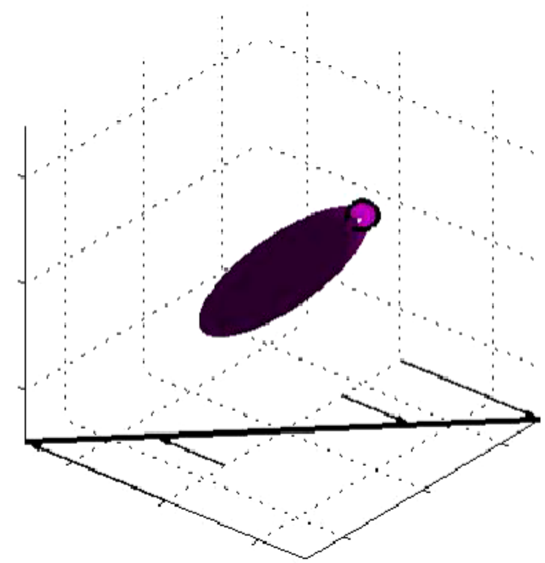


Tumbling (T)



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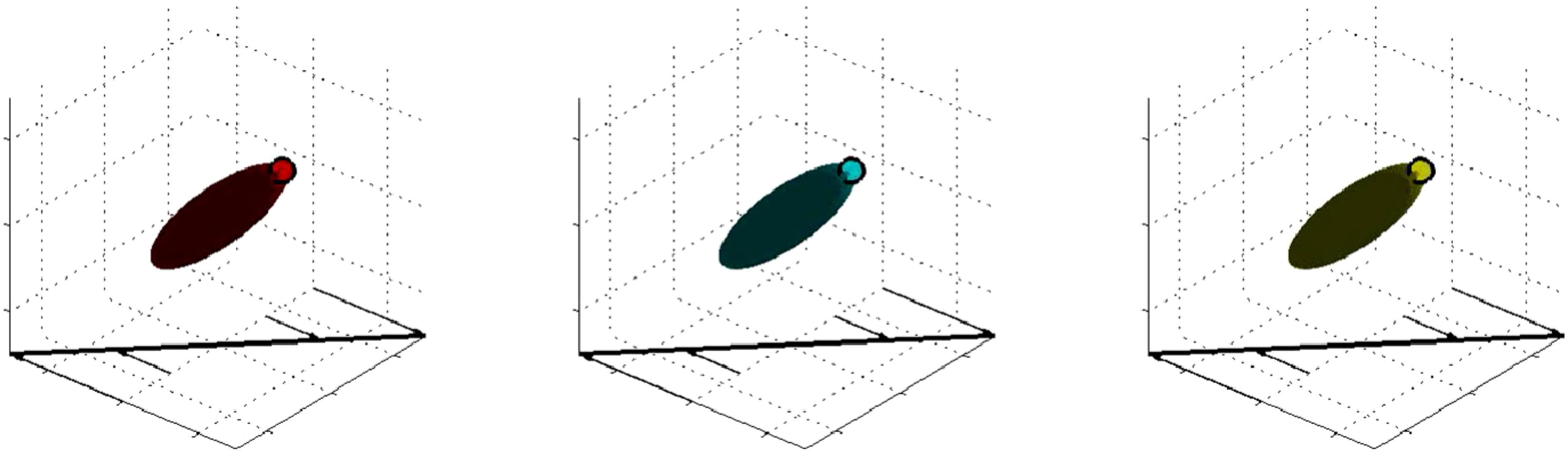

Log-rolling (LR) Inclined rolling (IR) Steady state (S)

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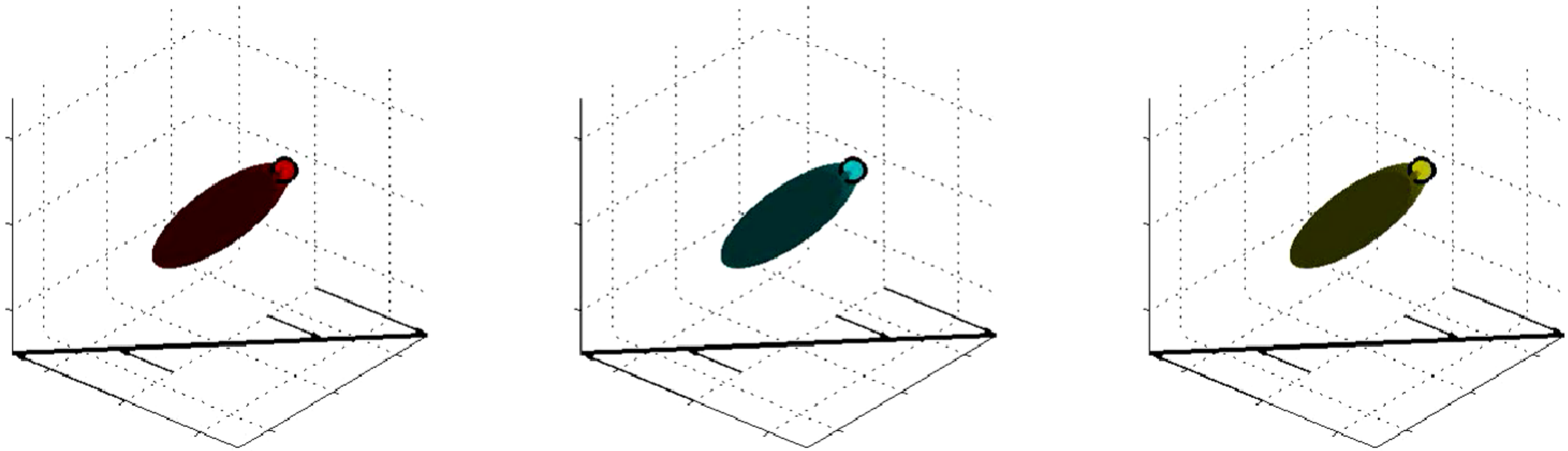

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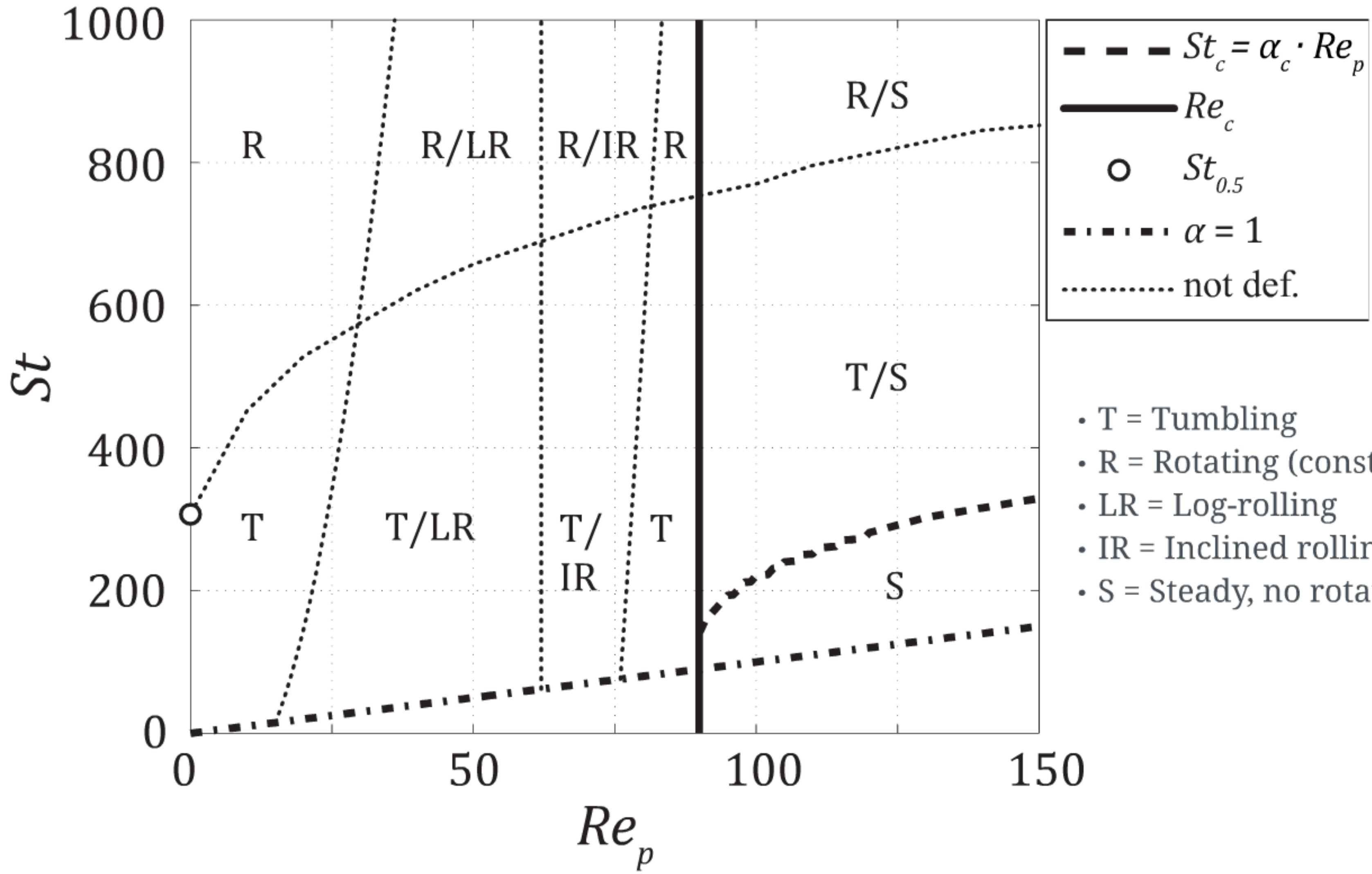
State diagram



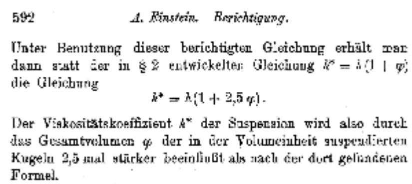
- T = Tumbling
- R = Rotating (constant ang. vel.)
- LR = Log-rolling
- IR = Inclined rolling
- S = Steady, no rotation

T. Rosén, F. Lundell, M. Do-Quang, C. K. Aidun, J. Fluid Mech., submitted (2013)

Rheology

Intrinsic viscosity

$$\nu_{disp} = \nu_{liq} \cdot (1 + \eta \cdot \Phi)$$


592 A. Einstein. Berichtigung.

Unter Benutzung dieser berichtigten Gleichung erhält man dann statt der in § 2 entwickelten Gleichung $k^* = k(1 + \varphi)$ die Gleichung

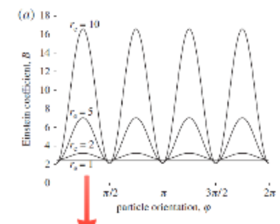
$$k^* = k(1 + 2{,}5\,\varphi).$$

Der Viskositätskoeffizient k^* der Suspension wird also durch das Gesamtvolumen φ der in der Volumeneinheit suspendierten Kugeln 2,5 mal stärker beeinflußt als nach der dort gefundenen Formel.

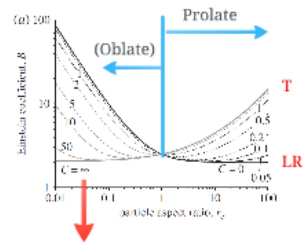
For spheres: $\eta = 2.5$

Einstein, A., Annalen der Physik 339, pp.591 (1911)

Single spheroid



Largest contribution when particle aligned with strain direction



Intrinsic viscosity dependent on rotational orbit

Jeffery's hypothesis (slender spheroids): $\eta = 2.0$

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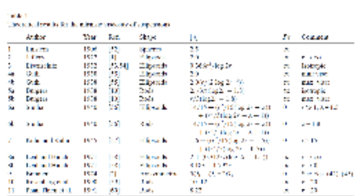
Dilute suspensions

Slender fibres:

$$\eta = 2Aa_{xxyy}$$

Orientational tensor determined from an orientational distribution

Other elongated particles:



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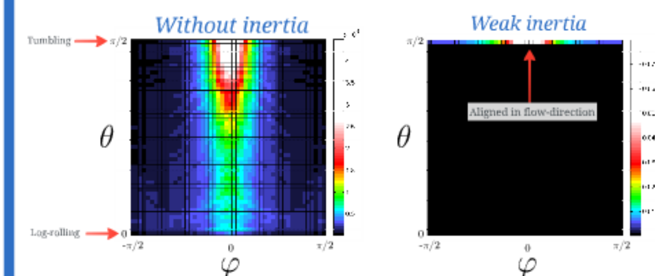

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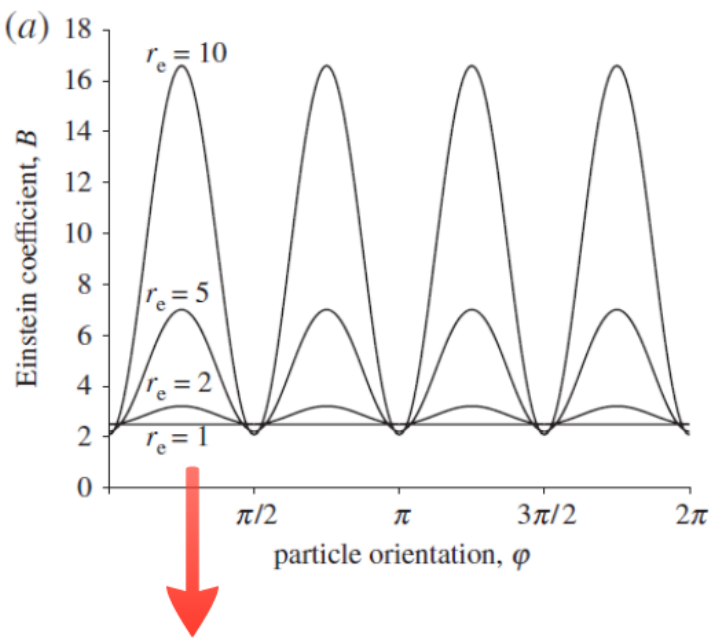

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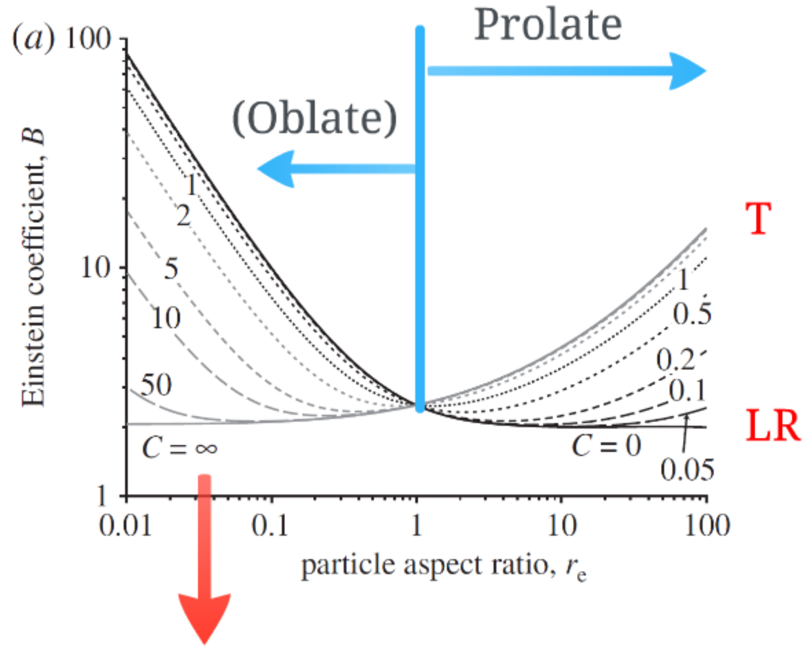

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Other elongated particles:

Table 1
Theoretical results for the intrinsic viscosity of suspensions

	Author	Year	Ref.	Shape	$[\eta]$	$Pé$	Comment
1	Einstein	1906	[52]	Spheres	2.5	∞	
2	Jeffery	1922	[1]	Ellipsods	2.0	∞	$r \to \infty$
3	Eisenschitz	1932	[53,54]	Ellipsoids	$0.366r^2/\log 2r$	∞	Isotropic
4a	Guth	1938	[55]	Ellipsoids	2.0	∞	min: visc:
4b	Guth	1938	[55]	Ellipsoids	$2.0(r/(2\log 2r^{-3}))$	∞	max: visc:
5a	Burgers	1938	[10]	Rods	$2r/3\pi(\log 2r - 1.8)$	∞	isotropic
5b	Burgers	1938	[10]	Rods	$r/3(\log 2r - 1.8)$	∞	max: visc:
6a	Simha	1940	[56]	Ellipsoids	$14/15 + (r^2/15(\log 2r - \lambda)) + (r^2/5(\log 2r - \lambda + 1))$	0	$r \gg 1, \lambda = 1.5$
6b	Simha	1940	[56]	Rods	$14/15 + (r^2/15(\log 2r - \lambda)) + (r^2/5(\log 2r - \lambda + 1))$	0	$\lambda = 1.8$
7	Kuhn and Kuhn	1945	[57]	Ellipsoids	$1.6 + ((r^2/15(\log 2r - 1.5)) + ((r^2/5(\log 2r - 0.5))$	0	$r > 15$
8a	Leal and Hinch	1971	[58]	Ellipsoids	$2 + (0.312r/(\log 2r - 1.5))$	∞	$r \to \infty$
8b	Leal and Hinch	1971	[58]	Ellipsoids	$3.183 - 1.792r$	∞	$r \to 0$
9	Brenner	1974	[2]	Axisymmetric	$5Q_1 - Q_2 + 2Q_3$	0	See Eqs. (42)–(45)
10	Rosenberg et al.	1990	[59]	Rods	10–12	∞	$r = 20$
11	Phan Thien et al.	1991	[60]	Rods	8.22	∞	$r = 20$

J. K. Batchelor, J. Fluid Mech. 41, pp. 545 (1970)
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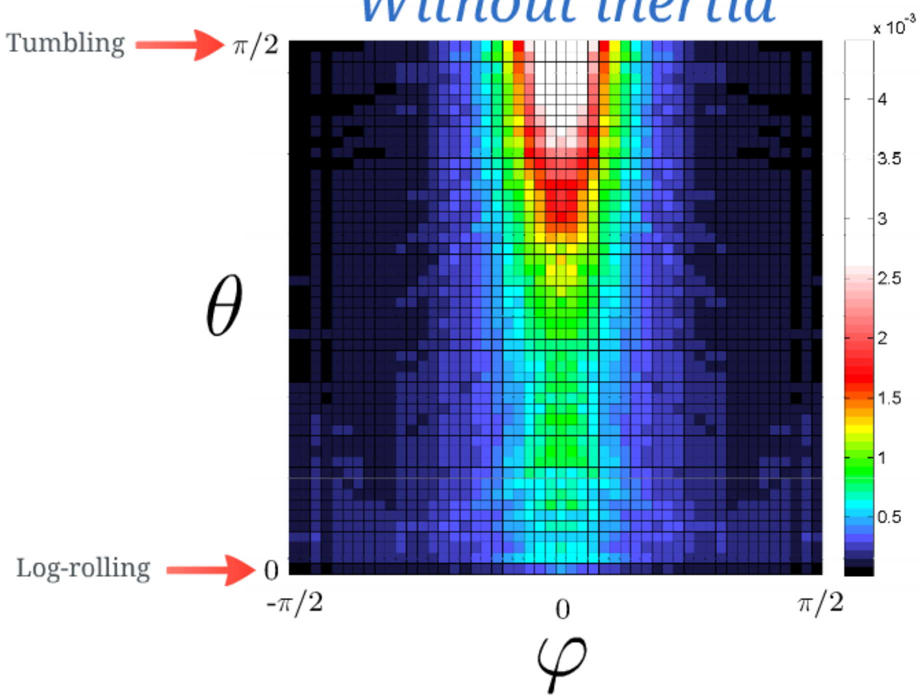
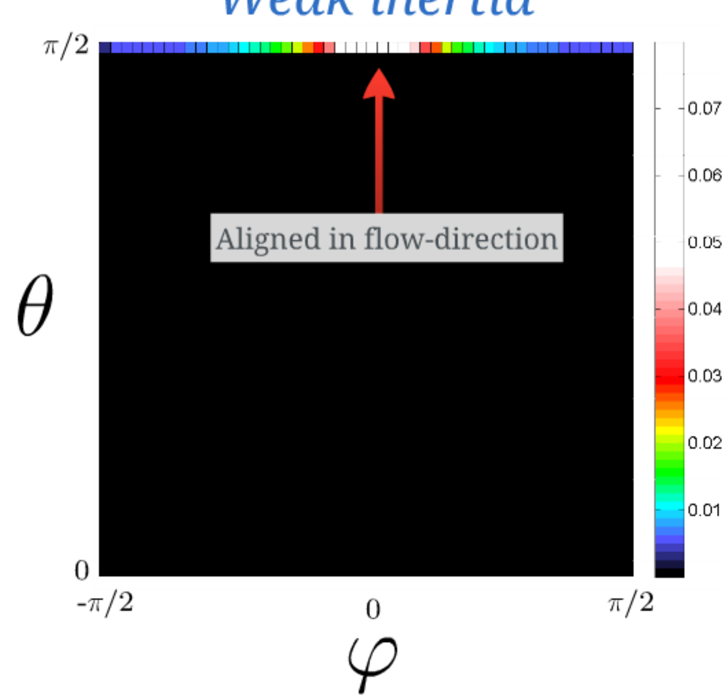


Weak inertia



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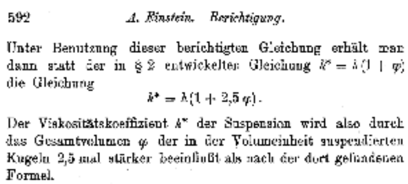
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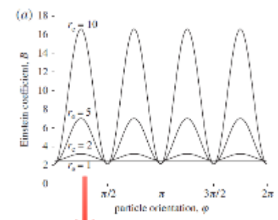

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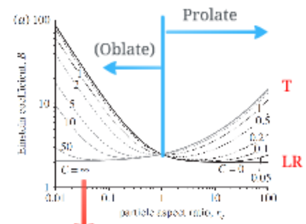

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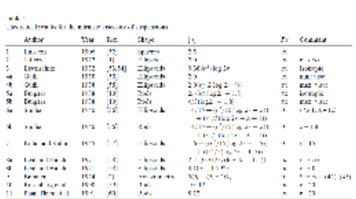

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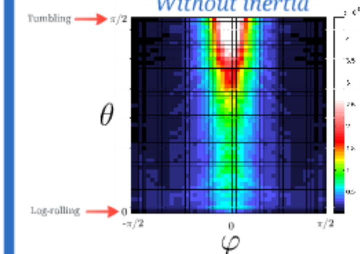

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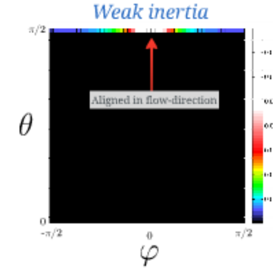
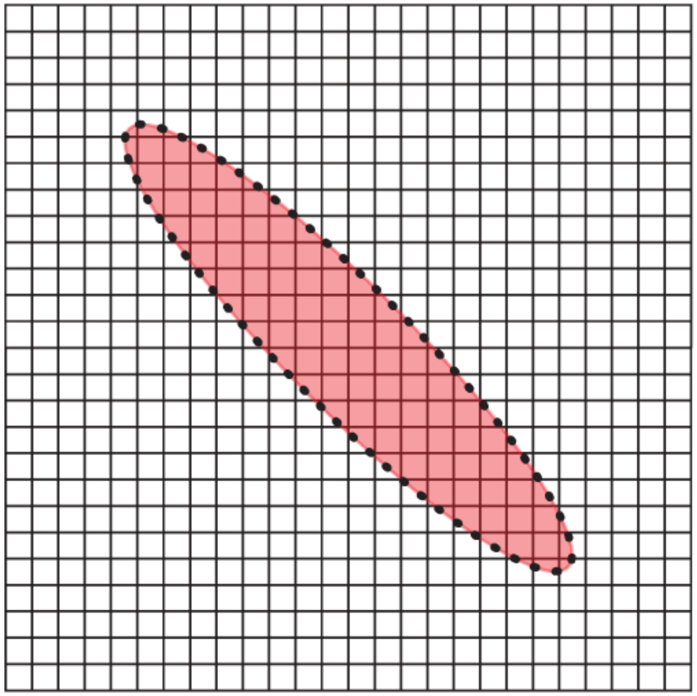


Method

- Lattice Boltzmann Method using External Boundary Force [1].
- Domain size independence ensured
- Shear stress σ evaluated at the wall after convergence.
- Intrinsic viscosity given by relation [2]:

$$\eta = \frac{1}{\Phi}\left(\frac{\langle\sigma\rangle}{\rho\nu_{liq}G} - 1\right)$$



[1] J. Wu and C. K. Aidun, Int. J. Numer. Meth. Fluids 62, pp. 765 (2010)
[2] H. Huang, X. Yang, M, Krafczyk, X-Y Lun, J. Fluid Mech. 692 (2012)

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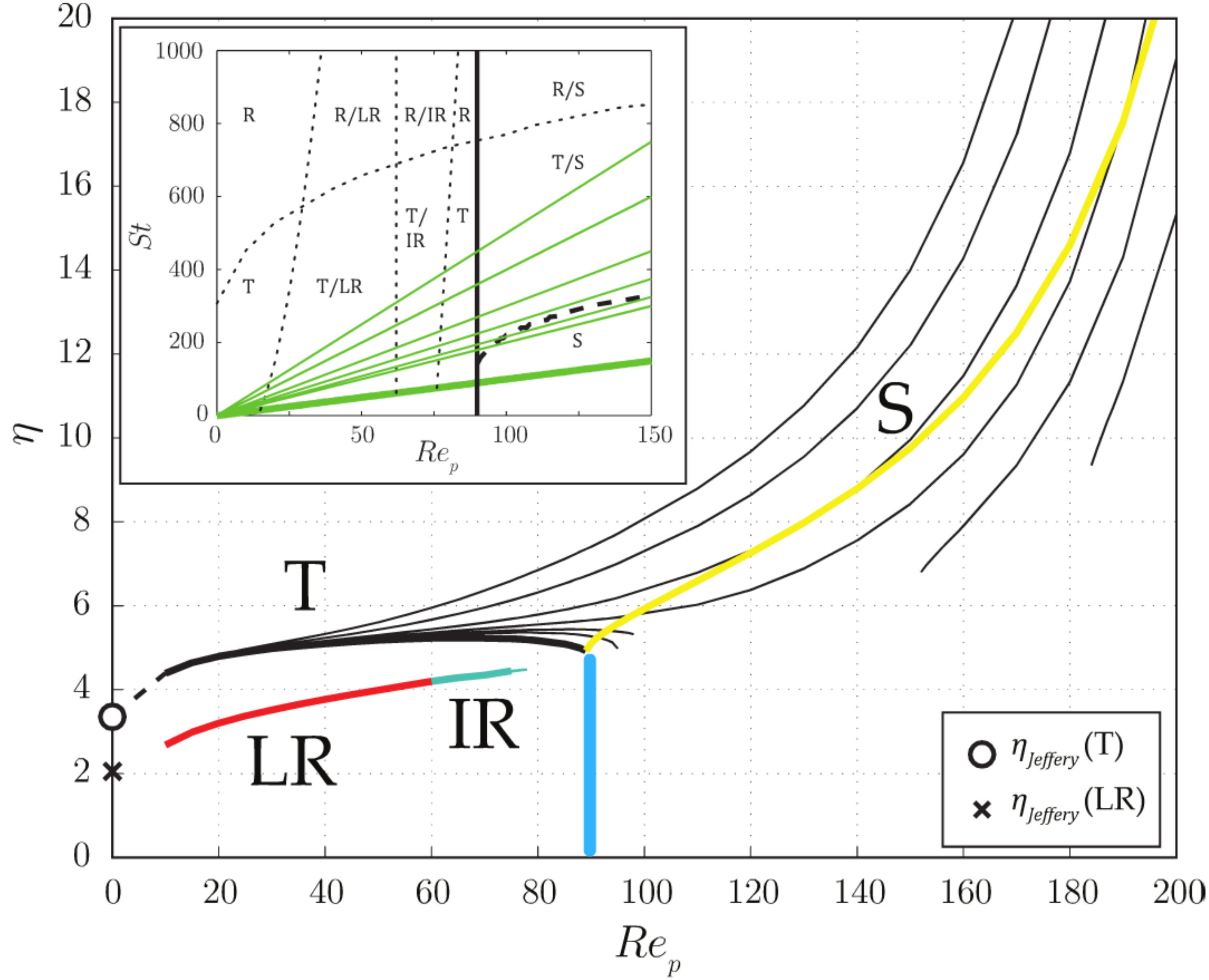


$r_p = 4.0$

$\alpha = 1.0$

2.0

2.2

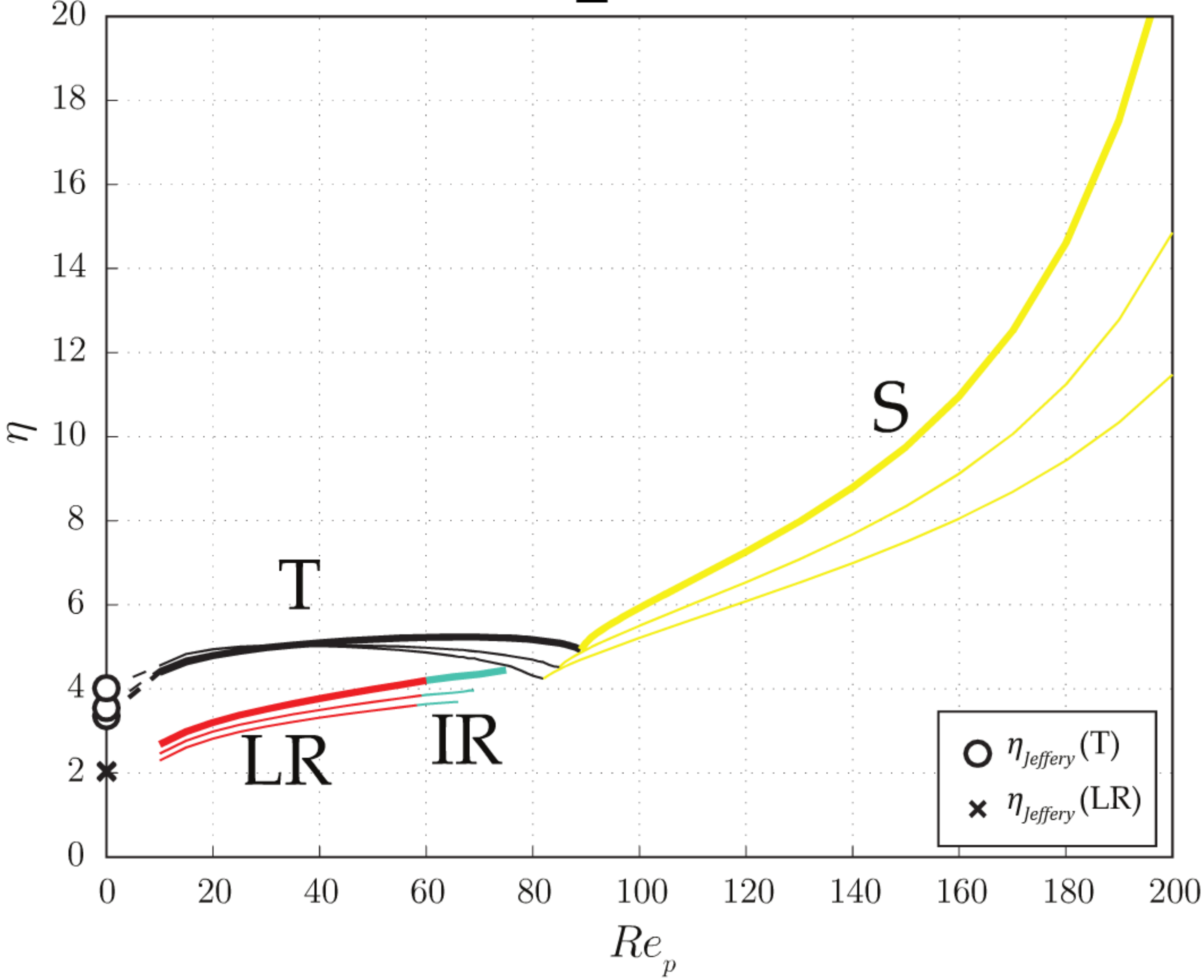
2.5

3.0

4.0

5.0

Aspect ratio



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5.0

6.0

Conclusions

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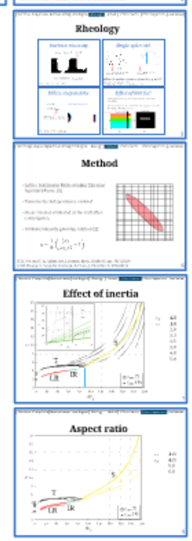

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