Modelling of shear lift force for non-spherical particles in viscous flows

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Introduction

- Motion of spherical particles in turbulent flow is understood quite well.

Introduction

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- However, vast majority of industrially interesting flows encompass non-spherical particles which often have large aspect ratios.
  - Paper and pulp industry, for example, deals with the transport of fibres in a liquid solution. This can be generalized as the transport of non-spherical particles with a regular shape and with high aspect ratio.
  - Another example is the flow of disc-shaped blood cells in veins and transport of certain food products.
  - Another case is that of the pneumatic transport of powders, typically grinded or crushed materials with a crystalline origin where the shape of the particles tend to be highly irregular and angular.
  - Another problem arises from the toner and printing industry where the assessment of quality is directly related to the eccentricity of the toner or ink particles.

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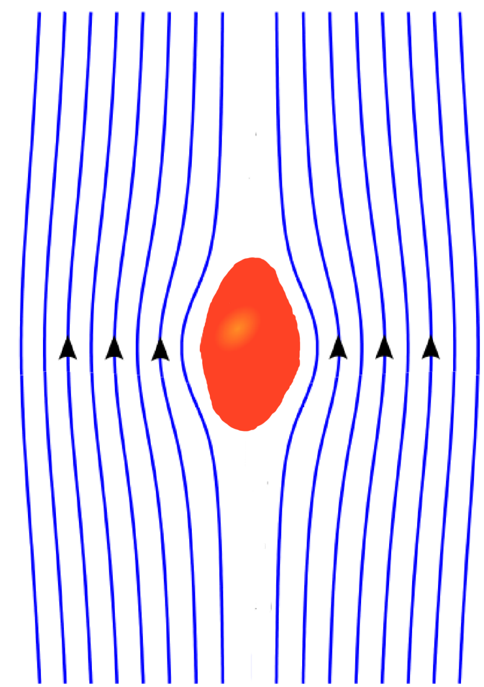
  - Another problem arises from the toner and printing industry where the assessment of quality is directly related to the eccentricity of the toner or ink particles.
- In this work we focus on fibres, modelled as prolate ellipsoids with aspect ratio λ.

Particle equations of motion

- Newton's law for a particle states the balance of forces, which are assumed to be linearly additive, as:

$$m_p \frac{d\vec{v}}{dt} = \sum \vec{F} = \vec{F}_{drag} + \vec{F}_g + \vec{F}_b + \vec{F}_{lift}$$

  - Drag force: $\vec{F}_{drag}$
  - Net buoyancy force: $\vec{F}_g + \vec{F}_b$
  - Lift force, induced by flow shear, $\vec{F}_{lift}$



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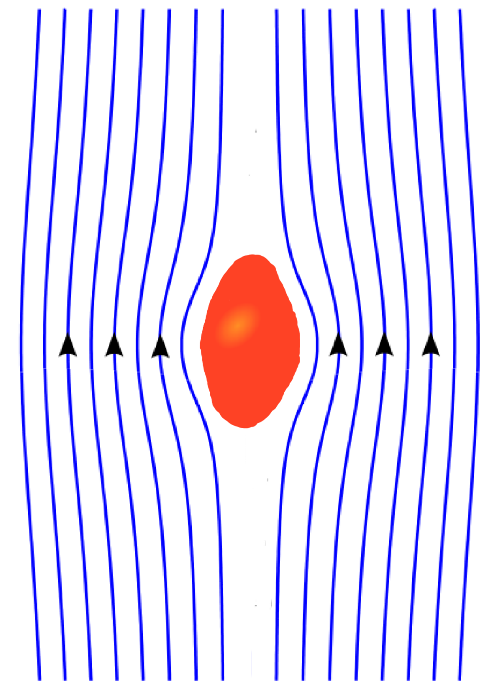
- Particle postion

$$\frac{d\vec{x}}{dt} = \vec{v}$$



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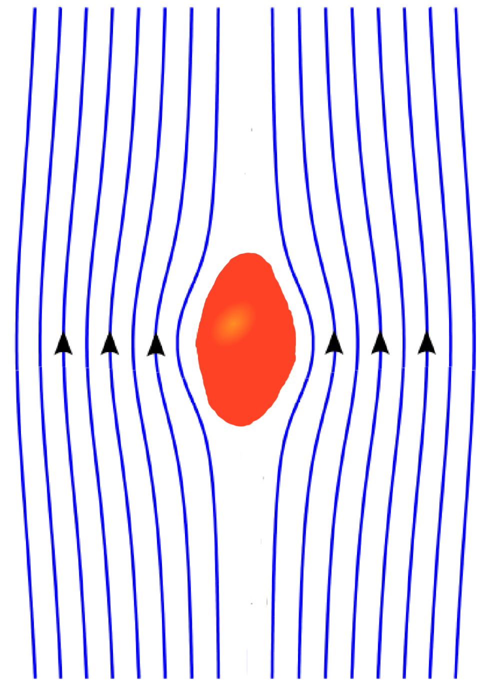
- Particle postion

$$\frac{d\vec{x}}{dt} = \vec{v}$$

- Particle orientation and angular velocity

$$\frac{d\vec{e}}{dt} = f(\vec{e}, \vec{\omega}), \qquad \frac{d\vec{\omega}}{dt} = \vec{T}$$



Particle equations of motion

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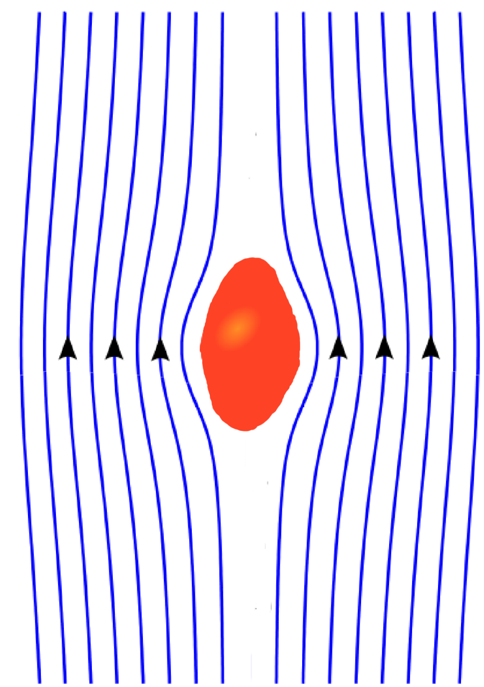

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$$\frac{d\vec{x}}{dt} = \vec{v}$$

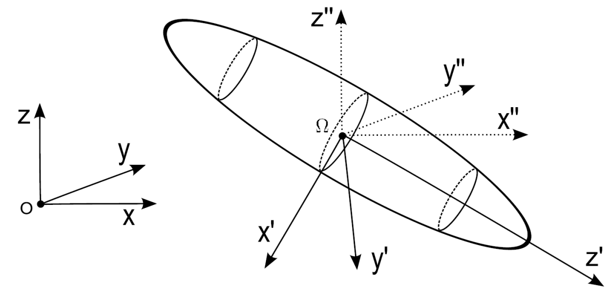
- Particle orientation and angular velocity

$$\frac{d\vec{e}}{dt} = f(\vec{e}, \vec{\omega}), \qquad \frac{d\vec{\omega}}{dt} = \vec{T}$$

- 13 ODE in total: (3) position, (3) velocity, (3) angular velocity, (4) orientation.

The drag force, $\pi a \rho_f \nu \hat{K} \cdot (\vec{u} - \vec{v})$

- The hydrodynamic drag force acting on an ellipsoidal particle under Stokes flow conditions was derived by Brenner (1964). It introduces the resistance tensor, which may be evaluated in the particle frame of reference, where only diagonal components of the resistance tensor $\hat{K}'$ are non-zero.



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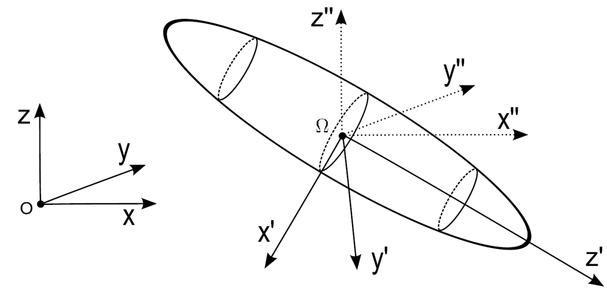

- The resistance tensor $\hat{K}'$ is a function of the aspect ratio λ.

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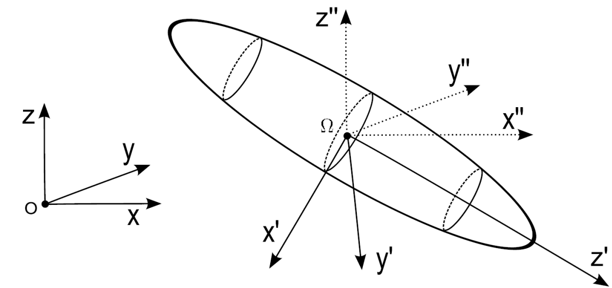

- The resistance tensor $\hat{K}'$ is a function of the aspect ratio λ.
- The resistance tensor in inertial frame of reference is calculated based on the particle orientation, $\hat{K} = V^T \hat{K}' V$ using a rotation matrix V.

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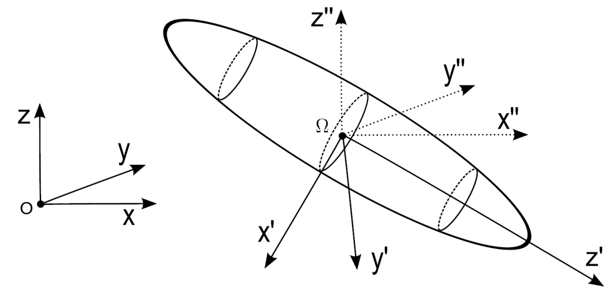

- The resistance tensor $\hat{K}'$ is a function of the aspect ratio λ.
- The resistance tensor in inertial frame of reference is calculated based on the particle orientation, $\hat{K} = V^T \hat{K}' V$ using a rotation matrix V.
- The spherical particle limit renders $\lim_{\lambda \to 1} \hat{K}' = 6\hat{I}d$

The lift force

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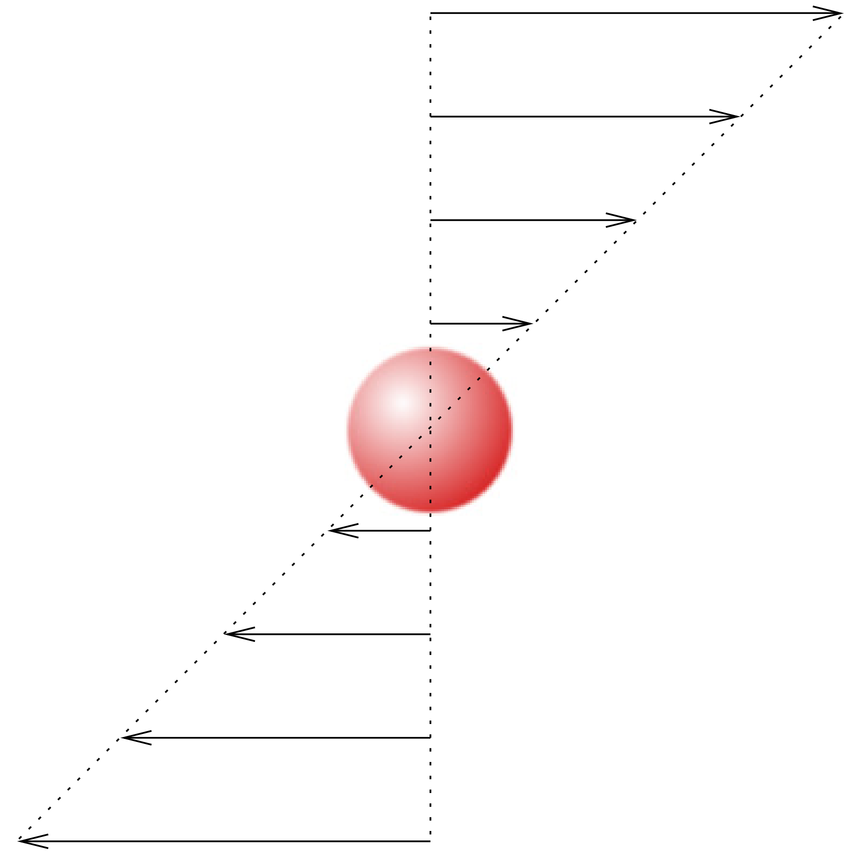
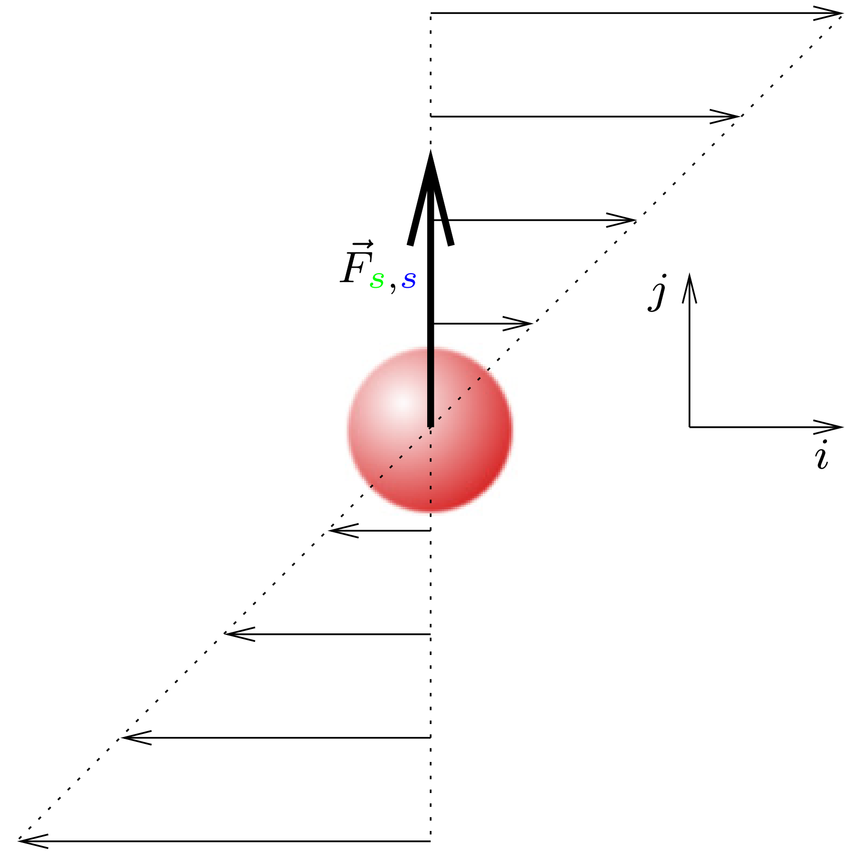
- Saffman derived an expression for shear lift force for a spherical particles in single shear flow.



The lift force

- Saffman derived an expression for shear lift force for a spherical particles in single shear flow.
- In a $\partial u_i / \partial x_j$ shear we have

$$F_{s,s} = 6.46 \rho_f a^2 \sqrt{\nu} \frac{\partial u_i / \partial x_j}{\sqrt{\partial u_i / \partial x_j}} |\vec{u} - \vec{v}|$$



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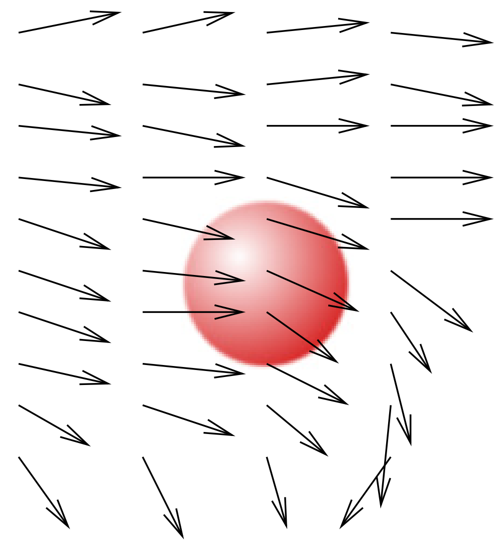
- The Saffman lift force was generalized to vector form for an arbitrary shear flow by introduction of vorticity $\vec{w} = \vec{\nabla} \times \vec{u}$ as

$$\vec{F}_{s,a} = 6.46 \rho_f a^2 \sqrt{\nu} \frac{1}{\sqrt{|\vec{w}|}} \left[(\vec{u} - \vec{v}) \times \vec{w} \right]$$



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The lift force

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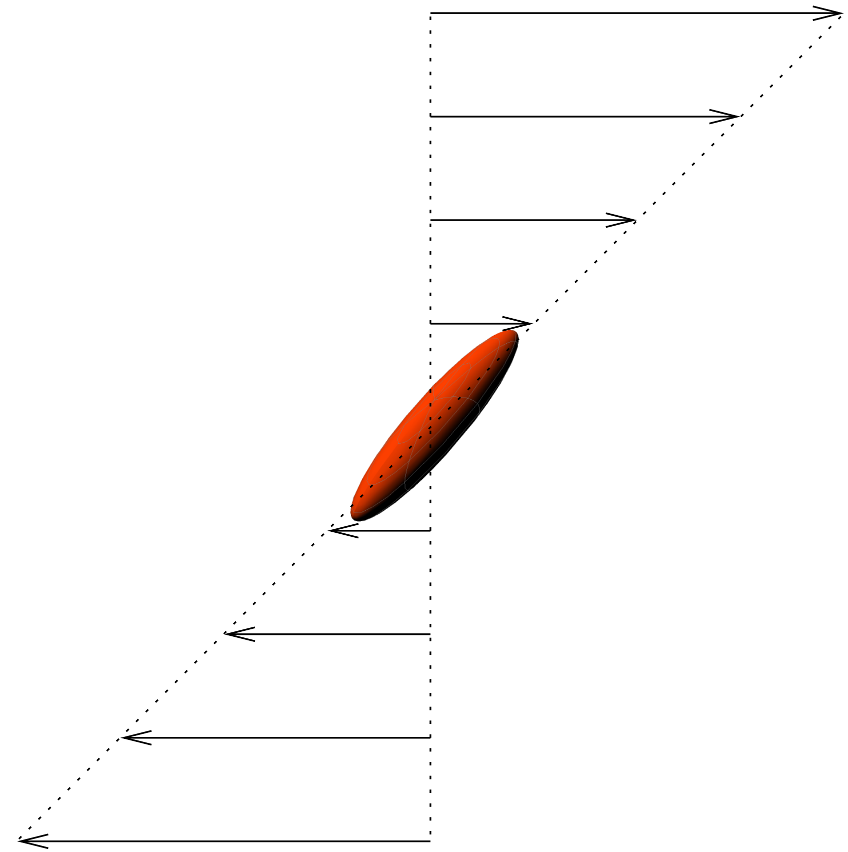
- Harper & Chang and Hogg derived an expression for shear lift force for an arbitrary shaped particle in single shear flow.



The lift force

- Harper & Chang and Hogg derived an expression for shear lift force for an arbitrary shaped particle in single shear flow.
- In a $\partial u_i/\partial x_j$ shear we have

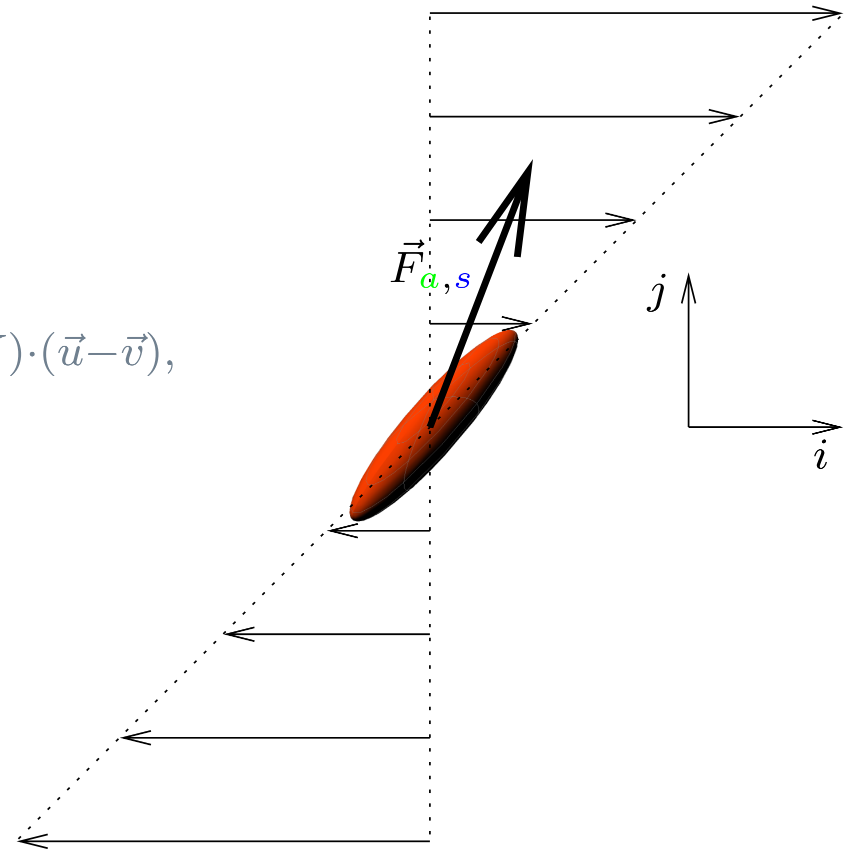
$$\vec{F}_{a,s} = \frac{\pi}{6}\rho_f a^2 \sqrt{\nu}\frac{\partial u_i/\partial x_j}{\sqrt{\partial u_i/\partial x_j}}(\hat{K}\cdot\hat{L}_{ij}\cdot\hat{K})\cdot(\vec{u}-\vec{v}),$$

where $\hat{L}_{ij}$ is the shear lift tensor.

- For example, for $\partial u/\partial y$ shear, the corresponding shear lift tensor $\hat{L}_{xy}$ is

$$\hat{L}_{xy} = \begin{pmatrix} A & B & 0 \\ \mathbf{D} & E & 0 \\ 0 & 0 & C \end{pmatrix}$$

$$A = 6\pi\cdot 0.0501, B = 6\pi\cdot 0.0329, C = 6\pi\cdot 0.0373, D = 6\pi\cdot 0.0182, E = 6\pi\cdot 0.0173.$$



The lift force

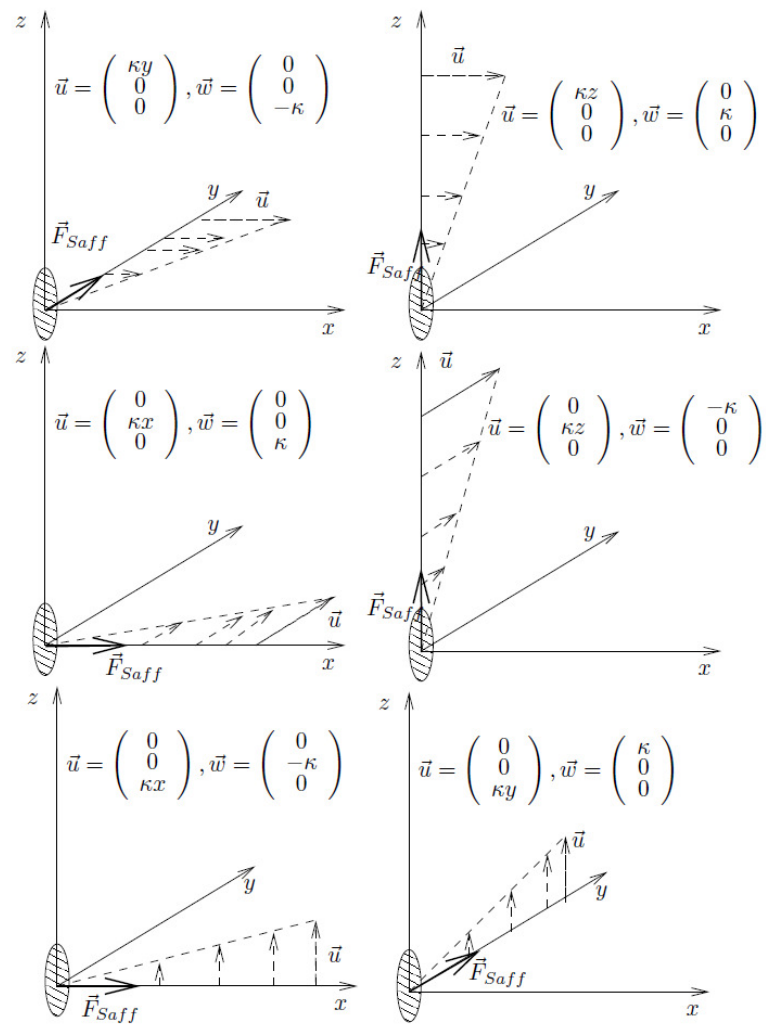
- The are 6 shears as well as 6 permutation of the lift tensor.

$$\hat{L}_{xy} = \begin{pmatrix} A & B & 0 \\ \mathbf{D} & E & 0 \\ 0 & 0 & C \end{pmatrix}, \quad \hat{L}_{xz} = \begin{pmatrix} A & 0 & B \\ 0 & C & 0 \\ \mathbf{D} & 0 & E \end{pmatrix}$$

$$\hat{L}_{yx} = \begin{pmatrix} E & \mathbf{D} & 0 \\ B & A & 0 \\ 0 & 0 & C \end{pmatrix}, \quad \hat{L}_{yz} = \begin{pmatrix} C & 0 & 0 \\ 0 & A & B \\ 0 & \mathbf{D} & E \end{pmatrix}$$

$$\hat{L}_{zx} = \begin{pmatrix} E & 0 & \mathbf{D} \\ 0 & C & 0 \\ B & 0 & A \end{pmatrix}, \quad \hat{L}_{zy} = \begin{pmatrix} C & 0 & 0 \\ 0 & E & \mathbf{D} \\ 0 & B & A \end{pmatrix}$$

- Please note the position of the D value in bold, since it corresponds to the original Saffman result.



Lift force

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	spherical particle	nonspherical particle
single shear flow		
arbitrary shear flow		

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arbitrary shear flow	vorticity based generalization $\vec{F}_{s,a} = = 6.46\rho_f a^2\sqrt{\nu}\frac{1}{\sqrt{\lvert\vec{w}\rvert}}\left[(\vec{u}-\vec{v})\times\vec{w}\right]$	

Lift force

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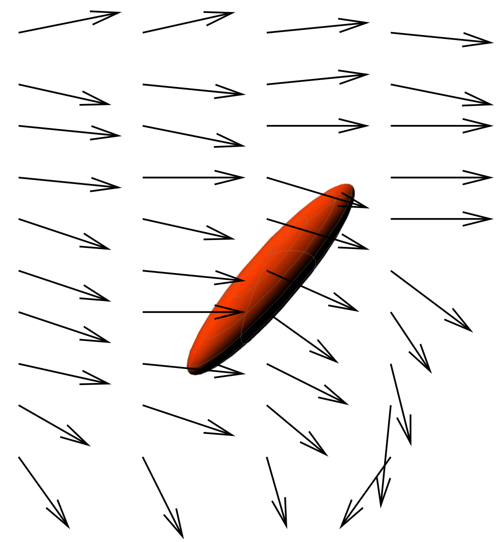
The lift force vector

- We propose the following generalization of the Harper & Chang lift force, which is valid for arbitrary shear and arbitrarily shaped particles.



$$\vec{F}_{a,a} = \frac{\pi}{6}\rho_f a^2 \sqrt{\nu}\frac{\vec{l}}{\sqrt{|\vec{w}|}}$$

where the shear lift vector $\vec{l}$ is defined as

$$\vec{l} = (-w_z \hat{K}\cdot\hat{L}_{xy}\cdot\hat{K} + w_y \hat{K}\cdot\hat{L}_{xz}\cdot\hat{K})\cdot\left[\begin{pmatrix} u_x \\ 0 \\ 0 \end{pmatrix} - \vec{v}\right] +$$

$$(w_z \hat{K}\cdot\hat{L}_{yx}\cdot\hat{K} - w_x \hat{K}\cdot\hat{L}_{yz}\cdot\hat{K})\cdot\left[\begin{pmatrix} 0 \\ u_y \\ 0 \end{pmatrix} - \vec{v}\right] + (-w_y \hat{K}\cdot\hat{L}_{zx}\cdot\hat{K} + w_x \hat{K}\cdot\hat{L}_{zy}\cdot$$

$$\hat{K})\cdot\left[\begin{pmatrix} 0 \\ 0 \\ u_z \end{pmatrix} - \vec{v}\right]$$

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Limit cases

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- for spherical particles, $\lim_{\lambda \to 1} \vec{F}_{a,a} = \vec{F}_{s,a}$
- for single shear, $\lim_{singleshear} \vec{F}_{a,a} = \vec{F}_{a,s}$
- for spherical particles and single shear, $\lim_{singleshear, \lambda \to 1} \vec{F}_{a,a} = \vec{F}_{s,s}$

Limit cases, $\lim_{\lambda \to 1} \vec{F}_{a,a} = \vec{F}_{s,a}$

- In case of spherical particles $\lambda = 1$, thus it holds $(\hat{K} \cdot \hat{L}_{ij} \cdot \hat{K}) = 36\hat{L}_{ij}$. In this limit, the lift force can be written as

$$\vec{F}_{a,a} = 6\pi\rho_f a^2 \sqrt{\nu} \frac{1}{\sqrt{|\vec{w}|}} \cdot \left[D(\vec{u} - \vec{v}) \times \vec{w} + A \begin{pmatrix} w_y - w_z & 0 & 0 \\ & w_z - w_x & \\ 0 & 0 & w_x - w_y \end{pmatrix} \cdot \right.$$

$$\left. \cdot (\vec{u} - \vec{v}) + \begin{pmatrix} E(w_y - w_z) & Bw_z & -Bw_y \\ -Bw_z & E(w_z - w_x) & Bw_x \\ Bw_y & -Bw_x & E(w_x - w_y) \end{pmatrix} \cdot \vec{v} \right],$$

which simplifies exactly to the Saffman result when taking $A = B = C = E = 0$

$$= 6\pi D\rho_f a^2 \sqrt{\nu} \frac{1}{\sqrt{|\vec{w}|}} (\vec{u} - \vec{v}) \times \vec{w} = 6.46\rho_f a^2 \sqrt{\nu} \frac{1}{\sqrt{|\vec{w}|}} (\vec{u} - \vec{v}) \times \vec{w} \, .$$

Limit cases, $\lim_{singleshear} \vec{F}_{a,a} = \vec{F}_{a,s}$

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- In case of a single shear flow $\partial u_x / \partial y$ and a particle travelling in the direction of the flow, e.g.

$$\vec{u} = \begin{pmatrix} u_x(y) \\ 0 \\ 0 \end{pmatrix}, \vec{w} = \begin{pmatrix} 0 \\ 0 \\ -\partial u_x/\partial y \end{pmatrix}, \vec{v} = \begin{pmatrix} v_x \\ 0 \\ 0 \end{pmatrix}.$$

then lift force simplifies to

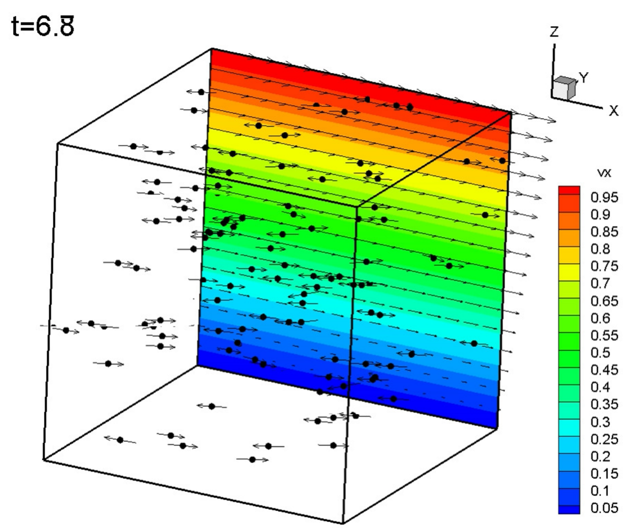
$$\lim_{singleshear} \vec{F}_{a,a} = \vec{F}_{a,s} = \frac{\pi}{6} \rho_f a^2 \sqrt{\nu} \frac{\partial u_x / \partial y}{\sqrt{|\partial u_x / \partial y|}} (\hat{K} \cdot \hat{L}_{xy} \cdot \hat{K}) \cdot (\vec{u} - \vec{v}),$$

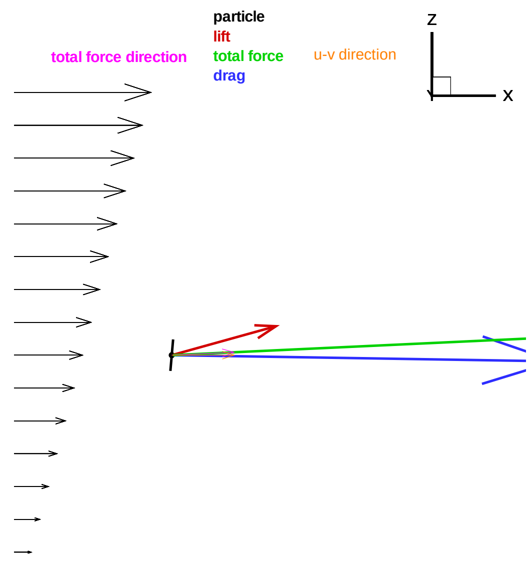
which is equivalent to the Harper & Chang expression.

- Note that under the same conditions ($\vec{u}$ is a single linear shear, $\vec{v}$ is directed in the same direction as the fluid) the vorticity-based expression for spherical particles simplifies to the original Saffman expression.

Test cases - single shear flow

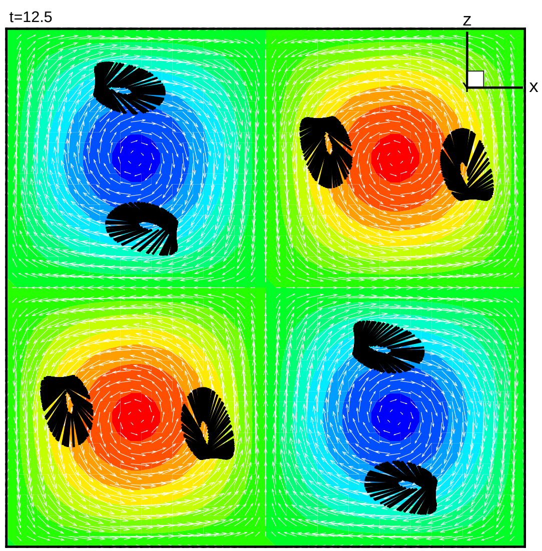
- Fibers in single shear flow orient themselves along streamlines, with the lift force working towards helping them to cross streamlines.

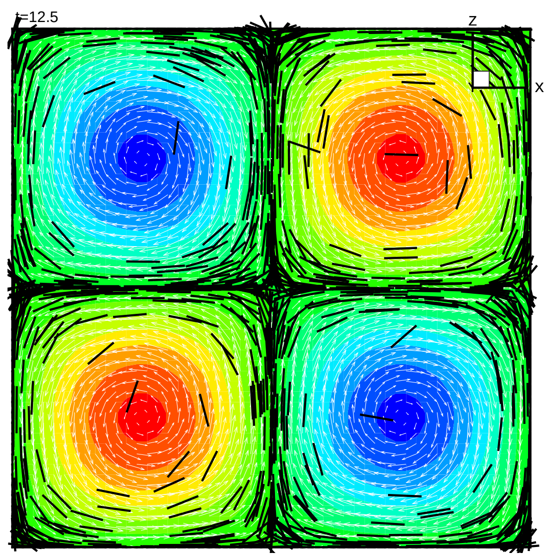


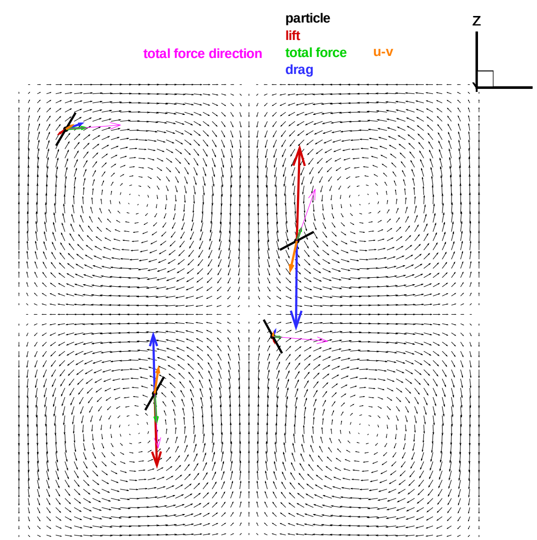


Test cases - cellular flow

- The results show that the lift force may significantly alter the particle behaviour.
- Snapshots of fibre distribution in cellular flow. Colour contours denote vorticity, vectors show the velocity flow field. In the left Figure lift force was taken into account, in the right only drag and gravity are considered.







Test cases - lid driven cavity flow

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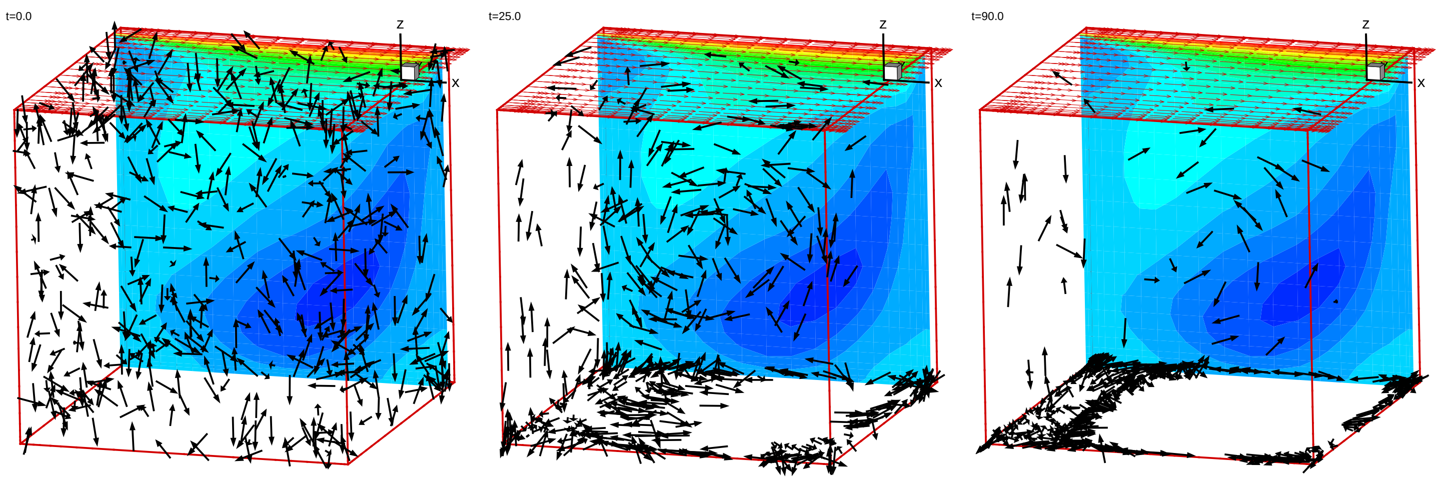
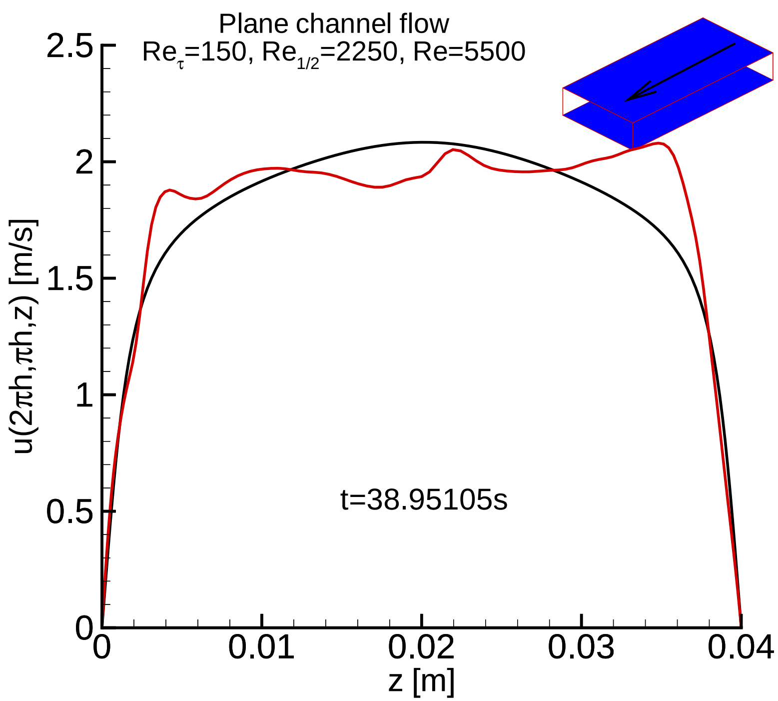
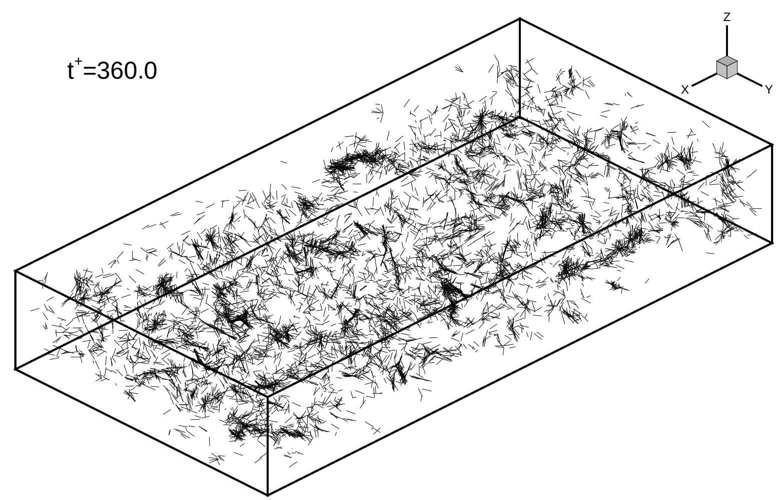
- The primary vortex in the lid driven cavity flow captures the fibers and they follow the vortex until they are eventually pushed out of the vortex and settle at the bottom; $Re = 1000$, $\lambda = 28$.

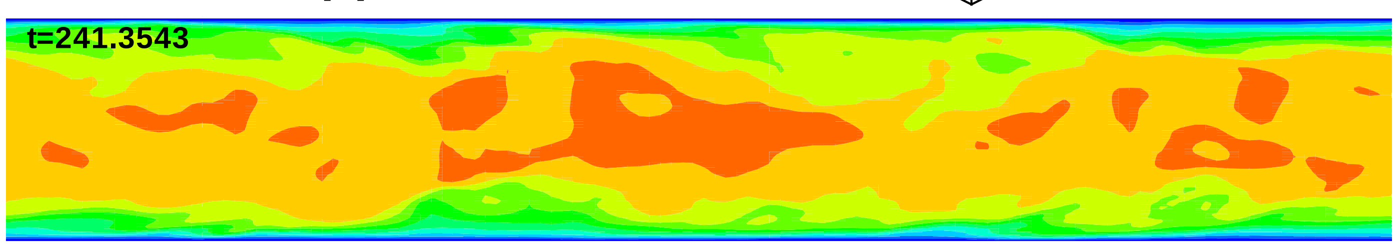


Test cases - turbulent channel flow

- We tracked fibers in $Re_\tau = 150$ turbulent channel flow.
- $St = 1$, $\lambda = 3$, $a^+ = 0.36$.

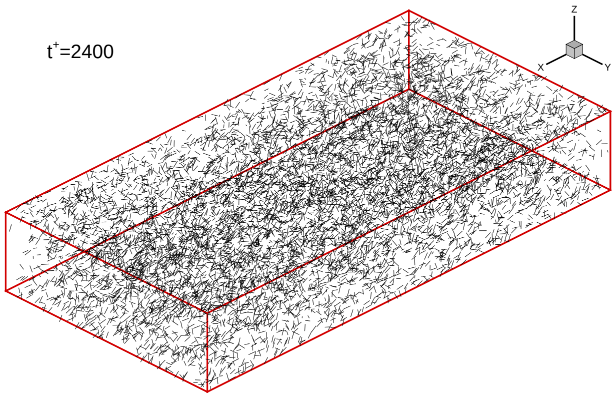


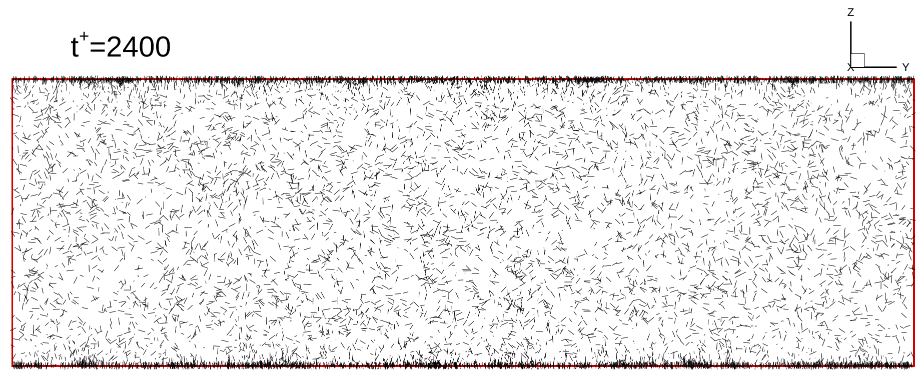




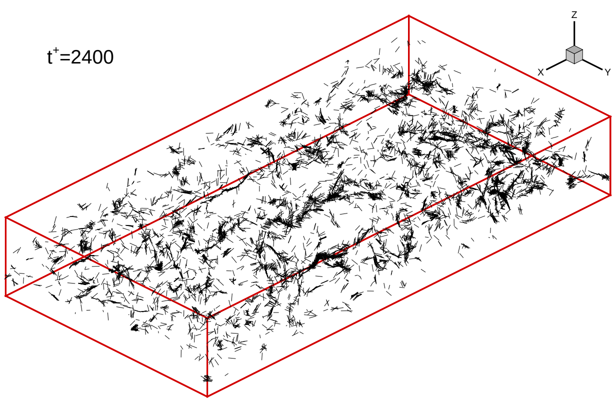
$St = 5, \lambda = 3, a^+ = 0.36$

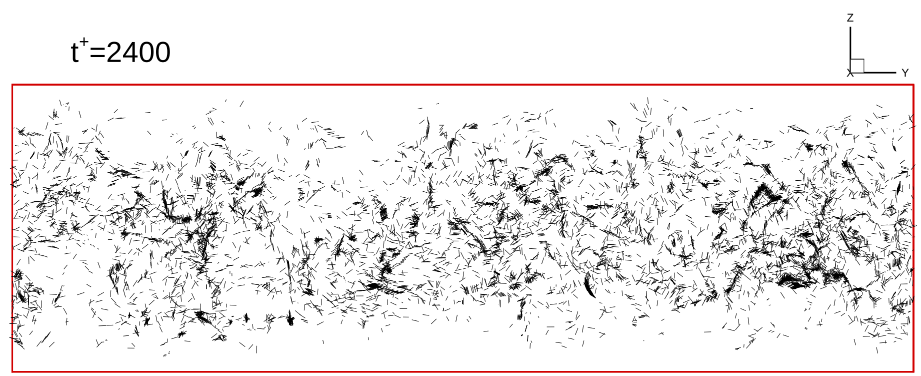
- drag only





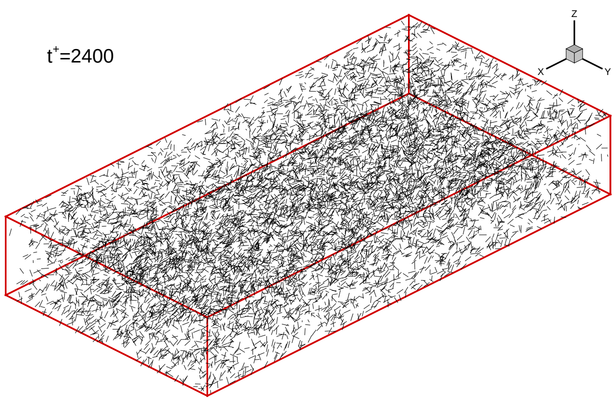
- drag and lift

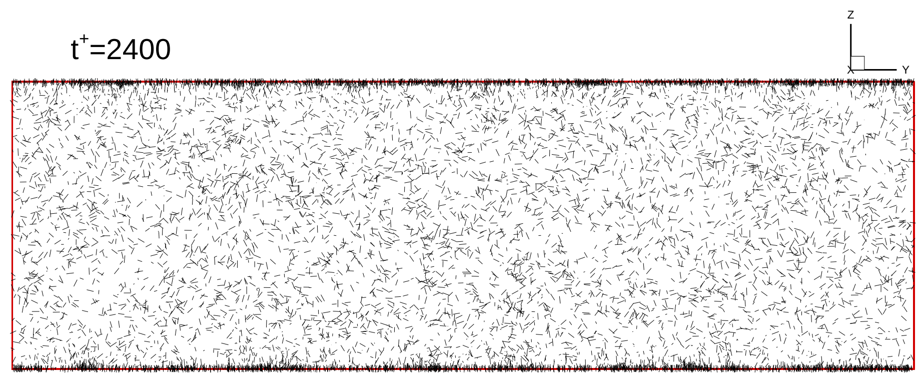




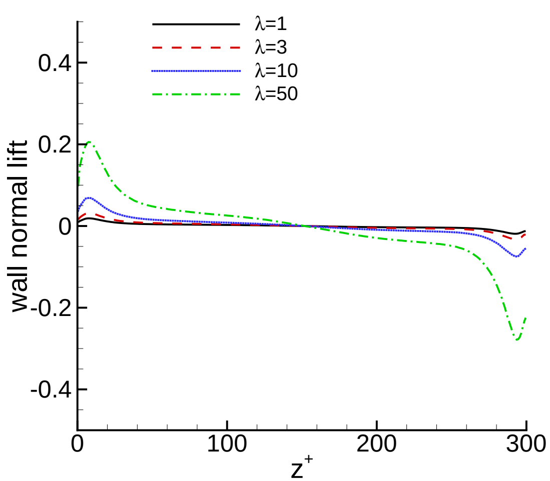
$St = 5, \lambda = 3, a^+ = 0.36$

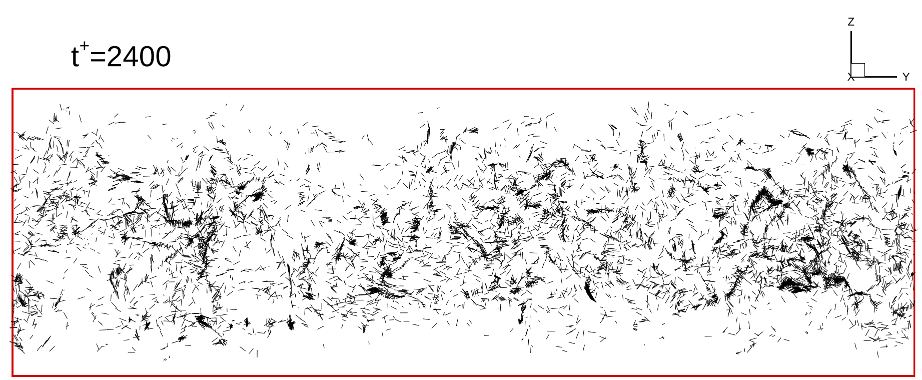
- drag only





- drag and lift





$\lambda = 10, a^+ = 0.36$

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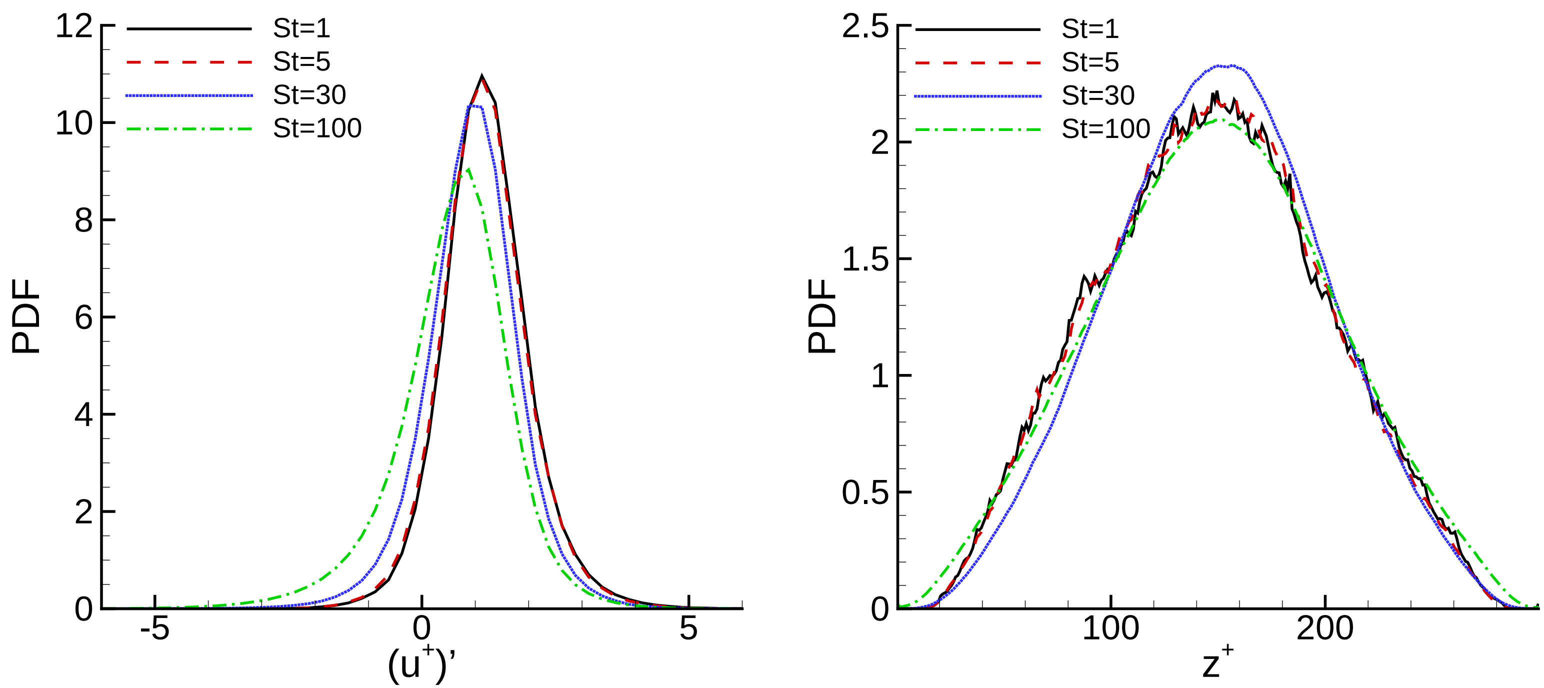
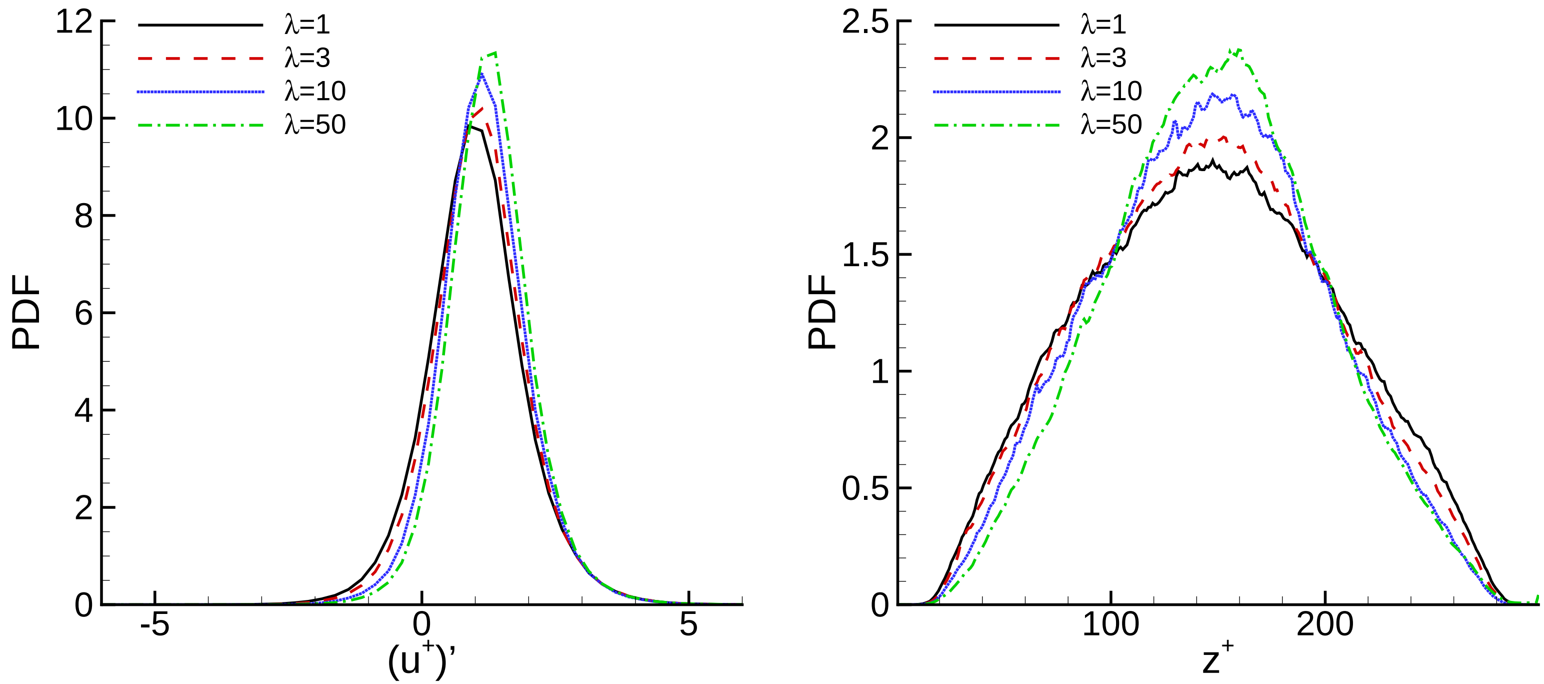
- Effect of varying fiber inertia ($St = 1 \ldots 100$)



$St = 5, a^+ = 0.36$

- Effect of varying fiber elongation ($\lambda = 1 \ldots 50$)



Conclusions

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- We have proposed a lift force for arbitrarily shaped particles in shear flow
- We have shown that the proposed force reverts back to known expression under limit cases
- We have developed a Lagrangian tracking algorithm and tested it in laminar and turbulent flows.
- In the future work, we will gather additional statistics translational, orientational and space distribution behaviour of fibers to quantify the relative contribution of lift and drag and to determine the influence parameters.
- Channel turbulence will be the test example studied.