The point particle approximation for fibres

Christopher Nilsen

Department of Energy and Process Engineering, NTNU

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Outline

Objectives

- Present models used in DNS with particles
- Discuss validity of assumptions in the context of fibre suspensions
- Initiate discussion on how DNS results can be used to understand real fibre suspension flows

Direct numerical simulation

- Numerical solution of the Navier-Stokes equations for a turbulent flow
- Assumes Newtonian fluid
- Usually accepted as “numerical experiment” when numerical errors are kept to a minimum

DNS with particles

- DNS instills a notion of exactness that can be misleading
- Particles are either fully resolved or point particles
- Point particle approximation requires a model describing the movement of the particles

The point particle assumptions

The fibres are assumed to be small inertial rigid spheres, ellipsoids or cylinders satisfying:

- $d_p \ll \eta$
- $\left| \underline{x}_p^{(i)} - \underline{x}_p^{(j)} \right| \gg d_p, \quad \forall i, j \quad i \neq j$

This is not generally satisfied for wood fibres in paper production

Basset-Boussinesq-Oseen equation

The motion of a small spherical particle is described by

$$\left(1+\frac{1}{2}\frac{\rho_f}{\rho_p}\right)\frac{\mathrm{d}\underline{v}}{\mathrm{d}t} = \frac{1}{\tau_p}\left(\underline{u}-\underline{v}\right)+\frac{1}{\rho_p}\left(-\nabla p+\nabla \underline{\tau}_i\right)+\frac{1}{2}\frac{\rho_f}{\rho_p}\frac{D\underline{u}}{Dt}+ \sqrt{\frac{9}{2\pi}}\left(\frac{\rho_f}{\rho_p}\right)^{\frac{1}{2}}\frac{1}{\sqrt{\tau_p}}\left(\int_0^t \frac{\frac{D\underline{u}}{Dt}-\frac{d\underline{v}}{dt}}{\sqrt{t-t'}}dt'+\frac{(\underline{u}-\underline{v})_0}{\sqrt{t}}\right) +\underline{g}$$

$$= \frac{1}{\tau_p}\left(\underline{u}-\underline{v}\right)+\frac{3}{2}\frac{\rho_f}{\rho_p}\frac{D\underline{u}}{Dt}+ \sqrt{\frac{9}{2\pi}}\left(\frac{\rho_f}{\rho_p}\right)^{\frac{1}{2}}\frac{1}{\sqrt{\tau_p}}\left(\int_0^t \frac{\frac{D\underline{u}}{Dt}-\frac{d\underline{v}}{dt}}{\sqrt{t-t'}}dt'+\frac{(\underline{u}-\underline{v})_0}{\sqrt{t}}\right) +\left(1-\frac{\rho_f}{\rho_p}\right)\underline{g}$$

Motion of heavy particles

When $\frac{\rho_f}{\rho_p} << 1$ the BBO equation reduces to

$$\frac{d\underline{v}}{dt} = \frac{1}{\tau_p}\left(\underline{u} - \underline{v}\right) + \underline{g}$$

For small ellipsoidal particles we have

$$\frac{d\underline{v}}{dt} = \frac{\mu}{m}\underline{K}\left(\underline{u} - \underline{v}\right) + \underline{g}$$

$\frac{\rho_f}{\rho_p} << 1$ *is not a reasonable assumption for wood fibres in water*

BBO equation for fibres?

$$\left(1+\frac{1}{2}\frac{\rho_f}{\rho_p}\right)\frac{\mathrm{d}\underline{v}}{\mathrm{d}t} = \frac{1}{\tau_p}\left(\underline{u}-\underline{v}\right)+\frac{1}{\rho_p}\left(-\nabla p+\nabla\underline{\tau}_i\right)+\frac{1}{2}\frac{\rho_f}{\rho_p}\frac{D\underline{u}}{Dt}+ \sqrt{\frac{9}{2\pi}}\left(\frac{\rho_f}{\rho_p}\right)^{\frac{1}{2}}\frac{1}{\sqrt{\tau_p}}\left(\int_0^t\frac{\frac{D\underline{u}}{Dt}-\frac{d\underline{v}}{dt}}{\sqrt{t-t'}}dt'+\frac{(\underline{u}-\underline{v})_0}{\sqrt{t}}\right) +\underline{g}$$

What about the torque?

Spherical particle in uniform flow

Uniform velocity field: $\underline{u} = (U, 0, 0)$

Non-dimensional equation

$$\beta \frac{dv}{dt} = \frac{1}{\mathrm{St}} (1 - v) , \qquad \beta = \left(1 - \frac{1}{2} \frac{\rho_f}{\rho_p} \right)$$

With $v(t = 0) = 0$ we obtain

$$v(t) = 1 - \exp\left(-\frac{t}{\beta \mathrm{St}} \right)$$

Spherical particle in simple vortex 1

Two-dimensional velocity field: $\underline{u} = (-\Gamma y, \Gamma x) = \Gamma \underline{Kx}$

Non-dimensional equation

$$\beta \frac{d\underline{v}}{dt} = \frac{1}{\mathrm{St}} \left(\underline{Kx} - \underline{v} \right) + 3 \left(\beta - 1 \right) \underline{KKx}$$

$$\begin{bmatrix} \underline{\dot{v}} \\ \underline{\dot{x}} \end{bmatrix} = \begin{bmatrix} -\frac{1}{\beta \mathrm{St}} \underline{I} & \frac{1}{\beta} \left(\frac{1}{\mathrm{St}} \underline{K} + 3(\beta - 1) \underline{KK} \right) \\ \underline{I} & \underline{0} \end{bmatrix} \begin{bmatrix} \underline{v} \\ \underline{x} \end{bmatrix} = \underline{A} \begin{bmatrix} \underline{v} \\ \underline{x} \end{bmatrix}$$

Spherical particle in simple vortex 2

The eigenvalues of $\underline{A}$ are

$$\lambda_{1,2,3,4} = -\frac{1}{2\beta \mathrm{St}}\left(1 \pm \sqrt{1 - 12\mathrm{St}^2(\beta - 1)\beta \pm 4\beta \mathrm{St}i}\right)$$

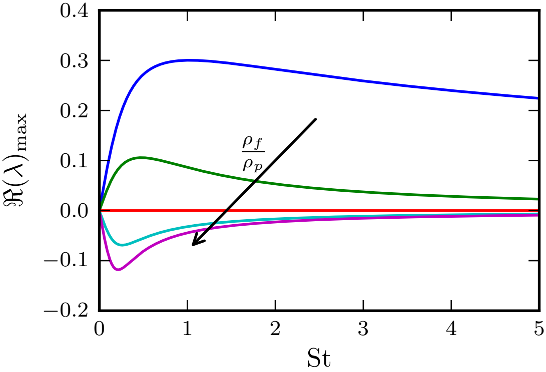
Largest real part $\Re(\lambda)_{\max}$ is negative when

$$\mathrm{St} > 0, \qquad \beta > \frac{3}{2}$$

i.e.

$$\mathrm{St} > 0, \qquad \frac{\rho_f}{\rho_p} > 1$$

Spherical particle in simple vortex 3



Maximum real part for $\frac{\rho_f}{\rho_p} = 0.0,\ 0.5,\ 1.0,\ 1.5,\ 2.0$

Summary of assumptions

DNS with suspended fibres usually requires

- $d_p \ll \eta$
- $\left| \underline{x}_p^{(i)} - \underline{x}_p^{(j)} \right| \gg d_p, \quad \forall i,j \quad i \neq j$
- Rigid ellipsoidal particles
- $\frac{\rho_f}{\rho_p} << 1$

For which parts of the papermaking process are these assumptions reasonable?

Conclusions

- Models should be chosen with the intended engineering application in mind
- It is important to understand how each assumption limits the applicability of the results
- It is not obvious that a DNS will provide better results than a cheaper simulation using a more relevant particle model

Topics for further discussion

- How can we compare DNS results with experimental results?
- What can we learn about general fibre suspensions by studying the cases where the assumptions are valid?
- How can DNS results be used to understand real fibre suspension flows?