Mathias Kvick

Effect of fibres on curvature and rotational induced hydrodynamic stability

WWSC is a joint research center at KTH and Chalmers

Outline

- Experiments, setup and results
- Linear stability, method and results
- Comparison

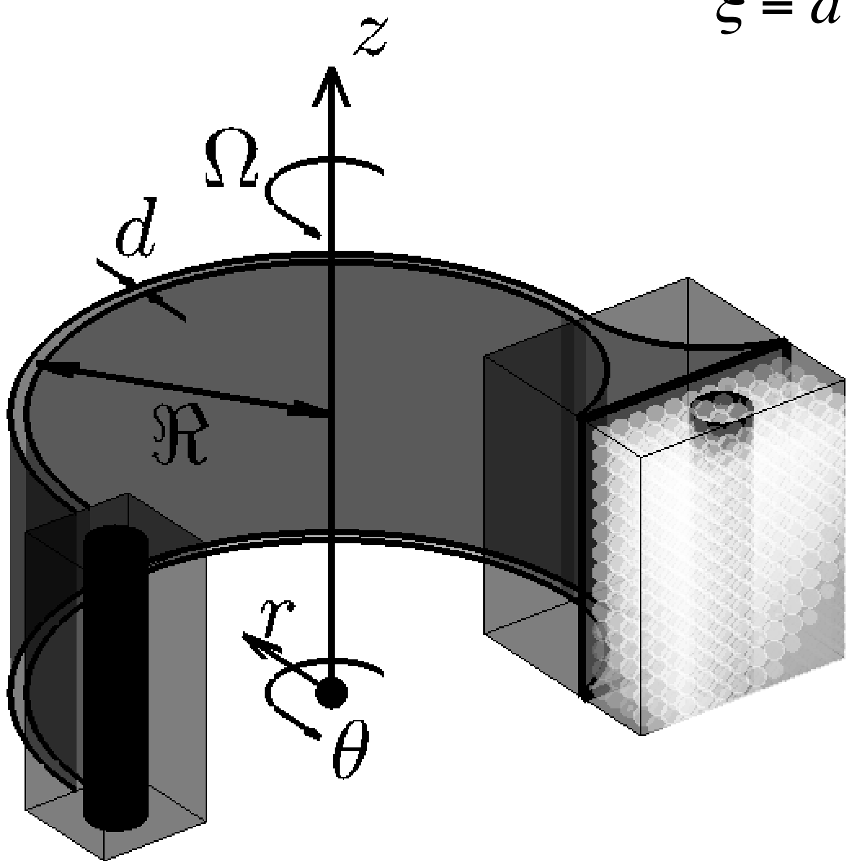
Curved rotating channel

$$\mathrm{Re} = \frac{U_b d}{\nu}$$

$$\mathrm{Ro} = \frac{\Omega d}{U_b}$$

$$\mathrm{De} = \mathrm{Re}\sqrt{\xi}$$

$$\xi = d/\Re << 1$$



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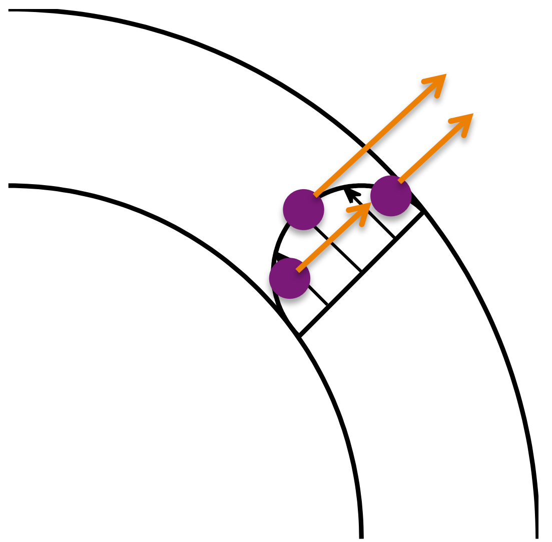
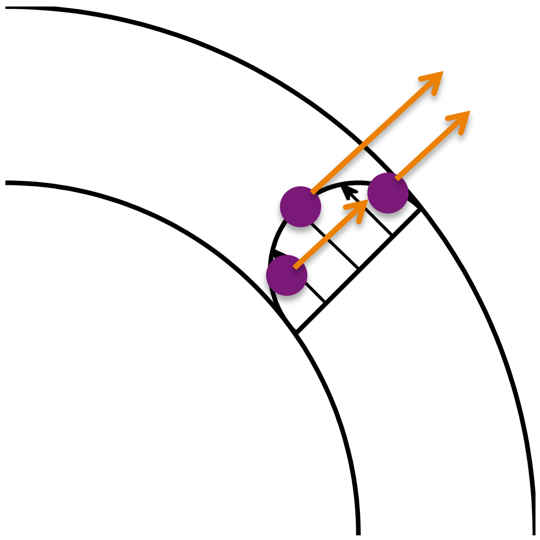
Curvature and rotation induced forces

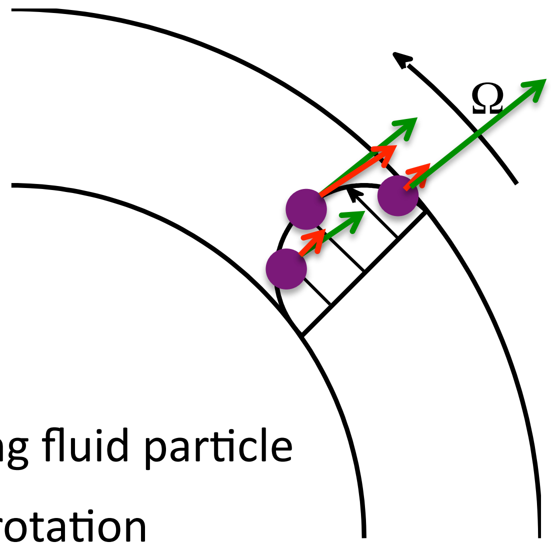


- Centrifugal force following fluid particle

$$\boxed{-\frac{V^2}{r}} = -\frac{1}{\rho}\frac{\partial p}{\partial r}$$

Curvature and rotation induced forces



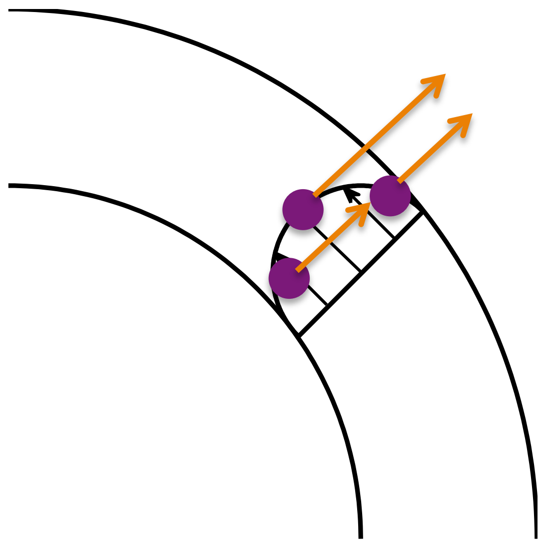


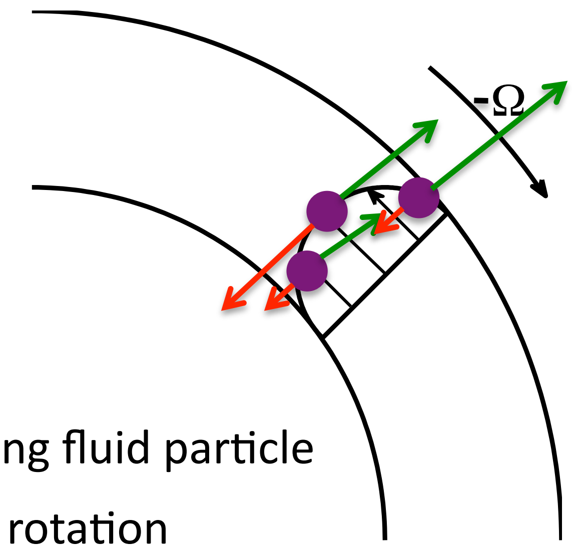
- Centrifugal force following fluid particle
- Centrifugal force due to rotation
- Coriolis force

$$-\frac{V^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r}$$

$$-\frac{V^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + 2V\Omega$$

Curvature and rotation induced forces



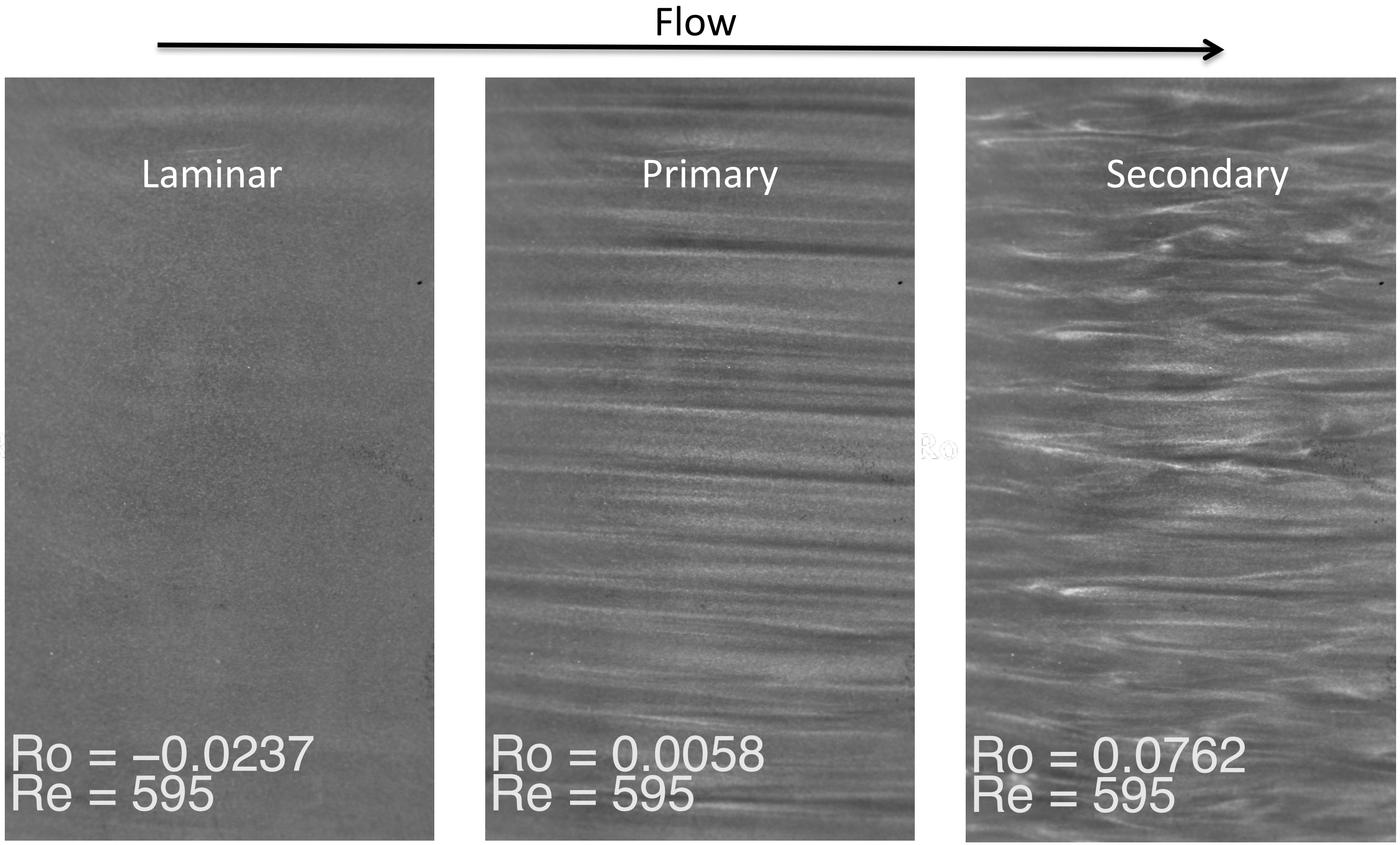


- Centrifugal force following fluid particle (orange)
- Centrifugal force due to rotation (green)
- Coriolis force (red)

$$-\frac{V^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r}$$

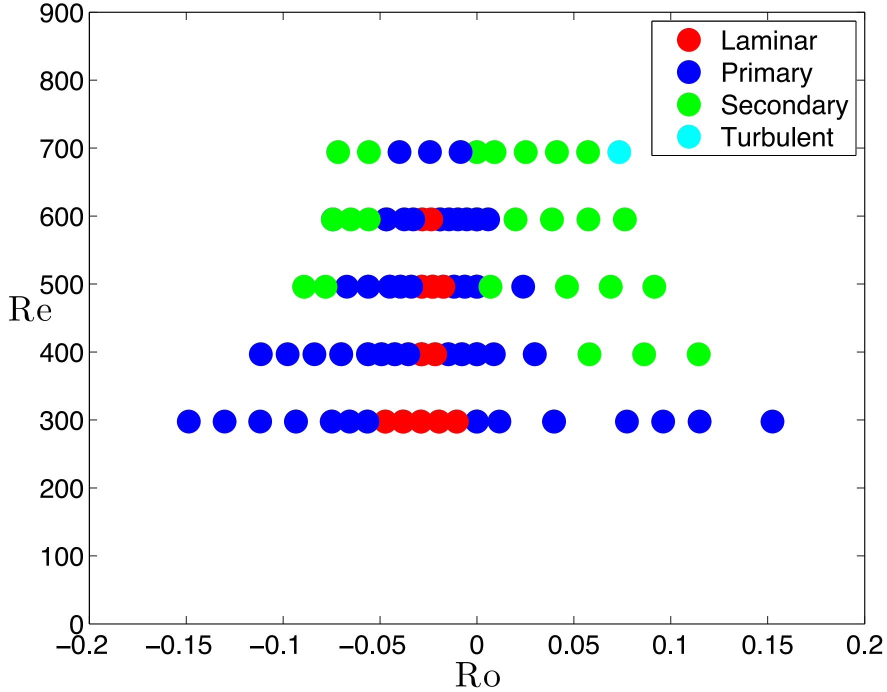
$$-\frac{V^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + 2V\Omega$$

Visualisation of flow structures



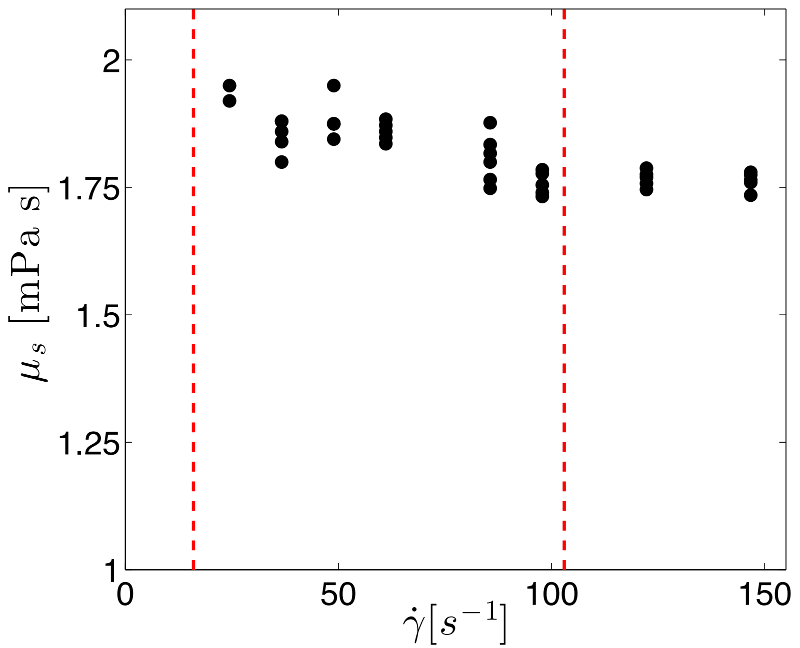
Stability map of water flow

Visual inspection categorises the flow states well.



Fibre suspension - NFC

Nano-fibrillated cellulose (NFC)



$$l = 1 - 3\mu\text{m}$$

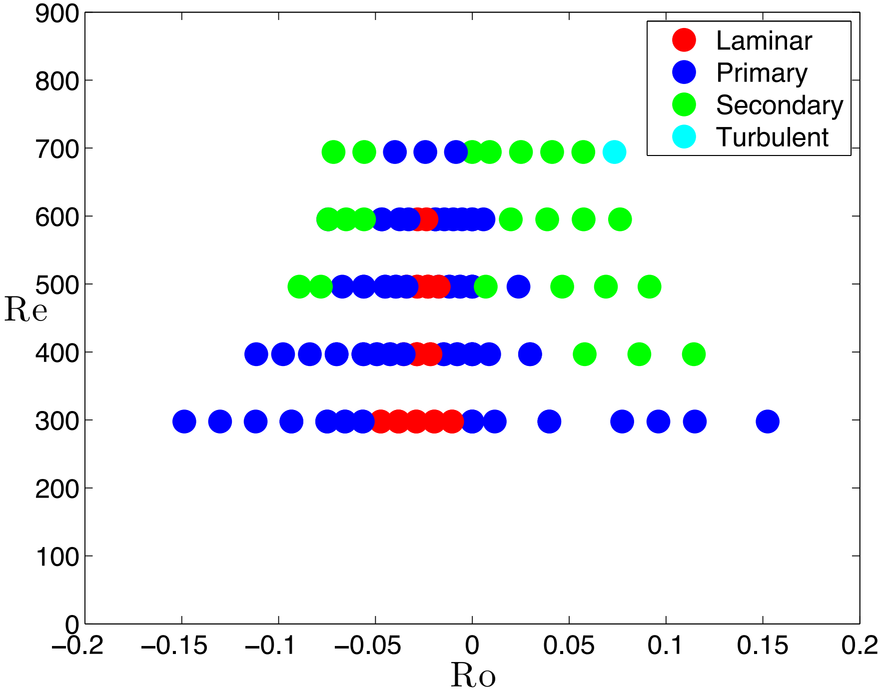
$$d = 20 - 40\text{nm}$$

$$\Phi = 308\text{ppm}$$

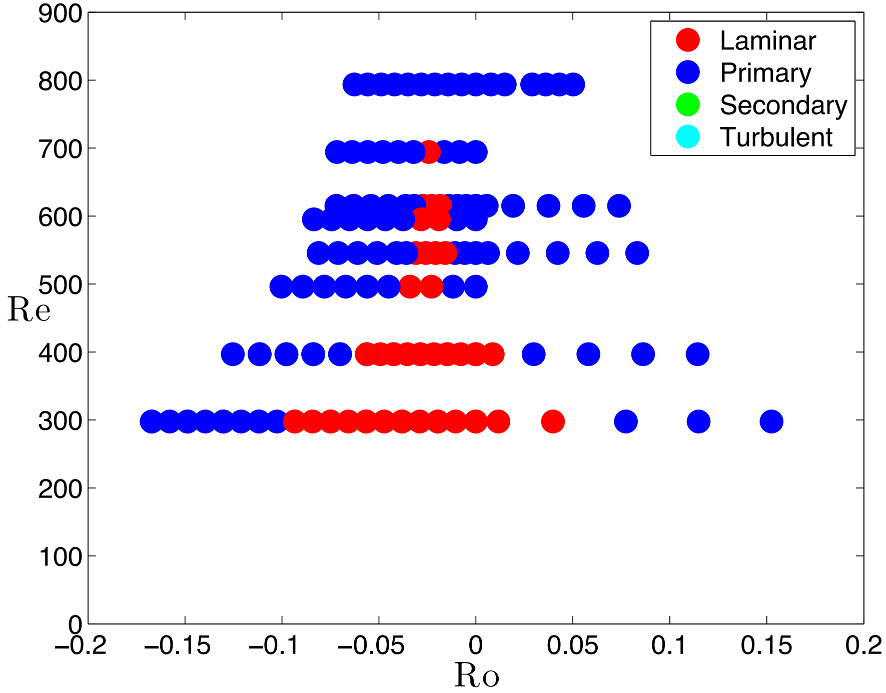
$$\mu_{NFC} \approx 1.8\mu_{H_2O}$$

Stability of fibre suspension

Water



NFC-suspension



NFC is stabilising when considering Re based on viscosity of water.

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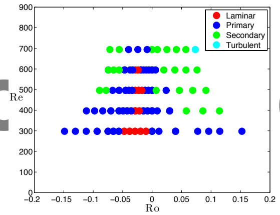
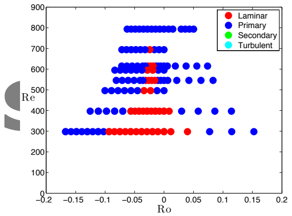
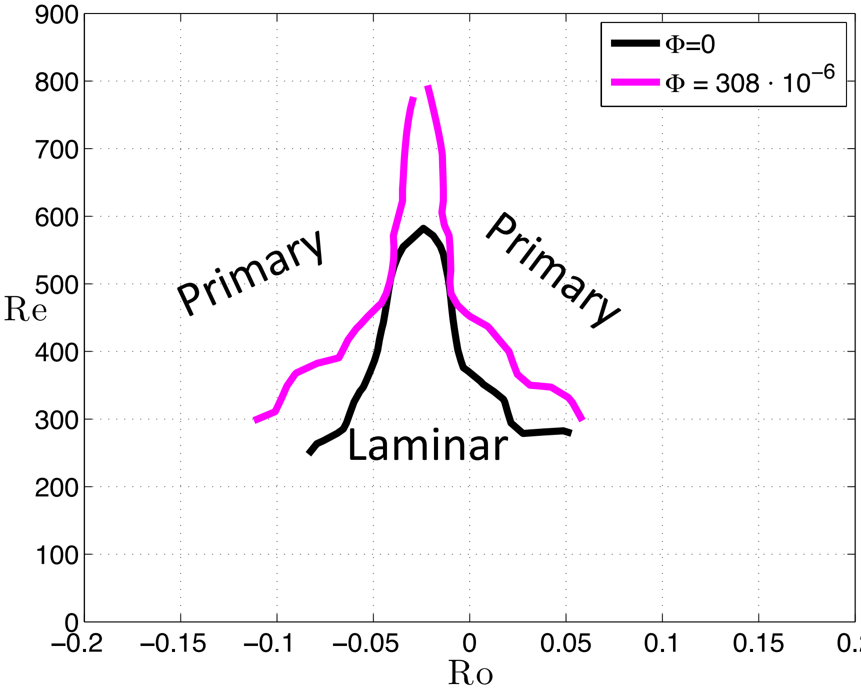
Stability c e s nsion

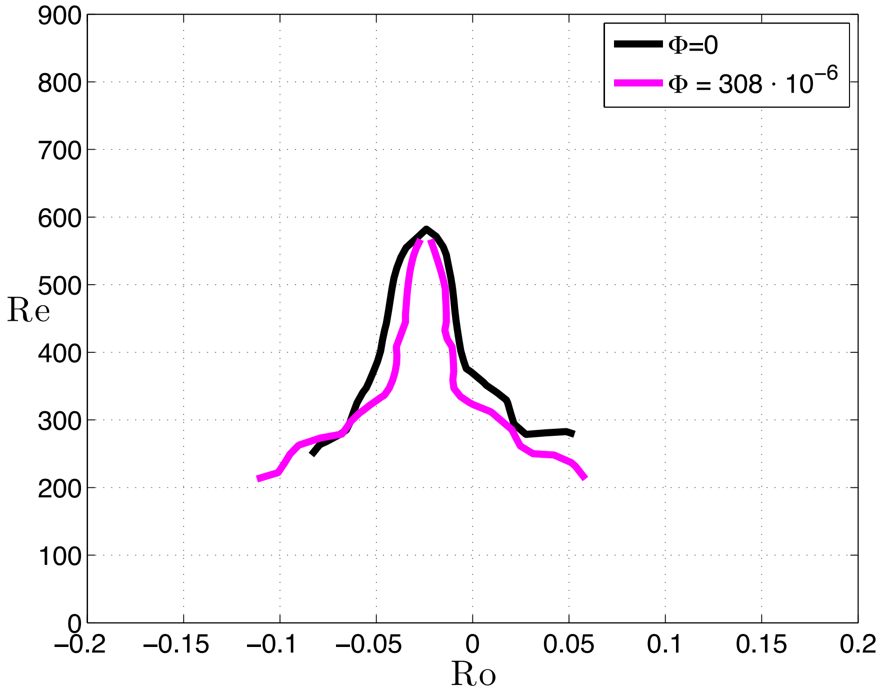




Contours of transition



Scaled by 1.4



$$\mathrm{Re} = \frac{U_b d}{\nu} \qquad \mathrm{Ro} = \frac{\Omega d}{U_b}$$

$$\mathrm{Re}_{CR}^{NFC} \approx 1.4\,\mathrm{Re}_{CR}^{H_2O}$$

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Governing equations

- The flow is governed by Navier-Stokes eqns. in cylindrical coordinates.

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) = -\nabla p + \nabla\cdot\sigma^{tot}$$

$$\sigma^{tot} = \sigma^{Newtonian} + \sigma^{fibrils}$$

$$\sigma^{fibrils} = \mu A \Phi\left(\varepsilon : a_4\right)$$

μ - Fluid viscosity

$A = \dfrac{r_p^2}{3\ln\left(\sqrt{2\pi/\Phi}\right)}$ - Rheological parameter[1]

Φ - Volume concentration

ε - Strain rate

$a_4 = < pppp >$ - Orientation tensor

[1] Batchelor, G. K., JFM **46**(04) 813-829, 1971

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Orientation closure

<u>Closure problem</u>

$$a_4 = f(a_6)$$
$$a_6 = f(a_8)$$
$$\vdots$$

Solved by →

Assume Jeffery Orbits

$$\dot{\varphi} = -\frac{\dot{\gamma}}{r_p^2 + 1}\left(r_p^2 \sin^2\varphi + \cos^2\varphi\right)$$

↓

Orientation distribution calculated through Smoluchowski eq.

$$\frac{\partial}{\partial\varphi}(\dot{\varphi}\Psi - C_l\dot{\gamma}\frac{\partial\Psi}{\partial\varphi}) = 0$$

↓

a_4 is obtained assuming constant orientation distribution

$$a_{ijkl} = \left\langle p_i p_j p_k p_l \right\rangle$$

Normal mode analysis

$$\overline{U} = \left(u'_r, V(r) + u'_\theta, u'_z\right)$$

$$u'_r = R(r)e^{i(\beta z + \alpha\theta - \omega t)} \qquad u'_\theta = \Theta(r)e^{i(\beta z + \alpha\theta - \omega t)}$$

$$u'_z = Z(r)e^{i(\beta z + \alpha\theta - \omega t)} \qquad p' = P(r)e^{i(\beta z + \alpha\theta - \omega t)}$$

- Into Navier-Stokes
- Linearise equation
- Subtract mean eqns.
- Discretize eqns.

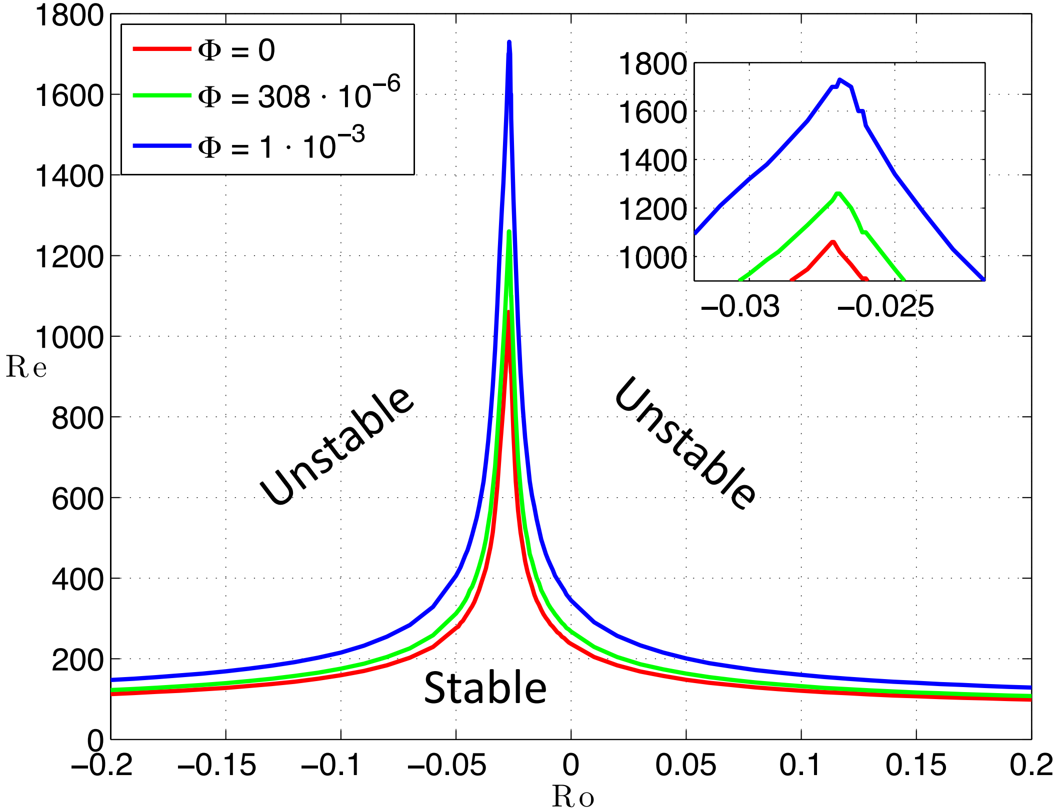
$$\mathbf{A}\begin{pmatrix} R \\ \Theta \\ Z \\ P \end{pmatrix} = -i\omega\mathbf{B}\begin{pmatrix} R \\ \Theta \\ Z \\ P \end{pmatrix}$$

~~$a'_4 = A_4(r)e^{i(\beta z + \alpha\theta - \omega t)}$~~

Stationary orientation distribution

Neutral stability curves

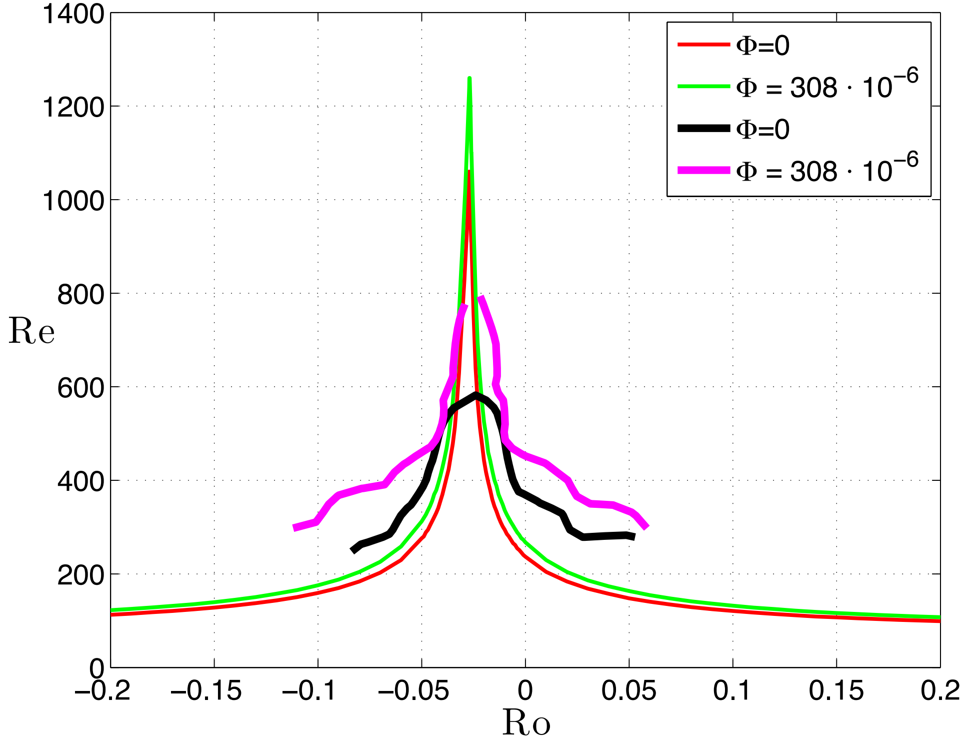
Addition of fibres has a consistent effect in the linear stability analysis.



$$\mathrm{Re}_{CR}^{NFC} \approx 1.2\,\mathrm{Re}_{CR}^{H_2O}$$

Stability of fibre suspension

Experiments and calculations show similarities.



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Stability of fibre suspension

Experiments

- Viscometer $\mu_{NFC} \approx 1.8 \mu_{H_2O}$
- Stability $\mathrm{Re}_{CR}^{NFC} \approx 1.4 \mathrm{Re}_{CR}^{H_2O}$

$$\frac{\mu_{NFC} / \mu_{H_2O}}{\mathrm{Re}_{CR}^{NFC} / \mathrm{Re}_{CR}^{H_2O}} = 1.3$$

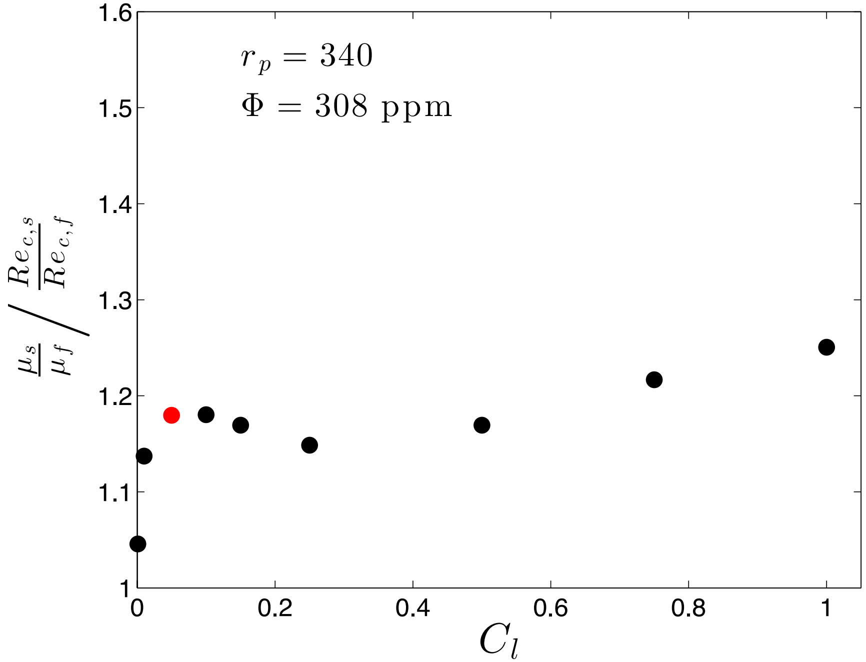
Effects on bulk viscosity larger than on stability

Stability of fibre suspension

Suspension viscosity

$$\mu_{NFC} = \left(1 + 2A\Phi a_{rr\theta\theta}\right)\mu_{H_2O}$$

$$\frac{\mu_{NFC} / \mu_{H_2O}}{\mathrm{Re}_{CR}^{NFC} / \mathrm{Re}_{CR}^{H_2O}} = 1.3$$



Effects on bulk viscosity larger than on stability

Stability can not be understood based only on shear viscosity

Conclusions

- Primary instability is stabilised by addition of NFC.
- Effects on bulk viscosity is larger compared to effect on Re_{CR}
  - ✓ Both in experiments and linear stability theory.
- Theory underpredicts effects on viscosity as well as Re_{CR}.

Thank you

Equations

$$\left[4\Pi D^2 + 2\xi\Pi D - \beta^2\Pi^2 - \alpha^2 - i\alpha \operatorname{Re} V\Pi\right]R + \left[2\xi \operatorname{Re} V\Pi + 2\operatorname{Re} Ro\Pi^2 - 2i\xi\alpha\right]\Theta - 2\operatorname{Re}\Pi^2 DP +$$

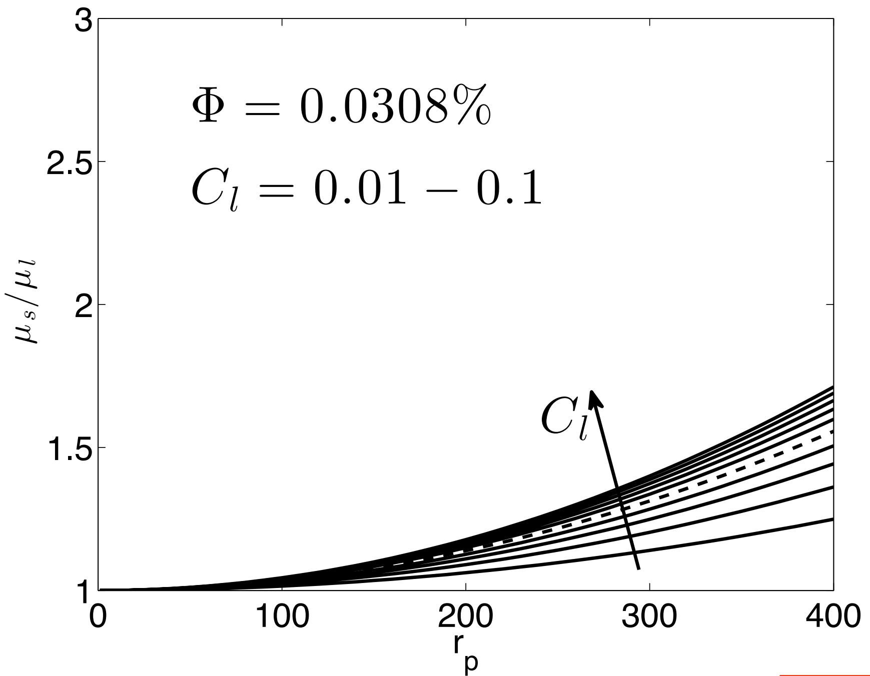
$$+A\Phi\Big(\left[4a_{rrrr}\Pi^2 D^2 + 2\xi a_{rrrr}\Pi D + 4i\alpha a_{rrr\theta}\Pi D\right]R +$$

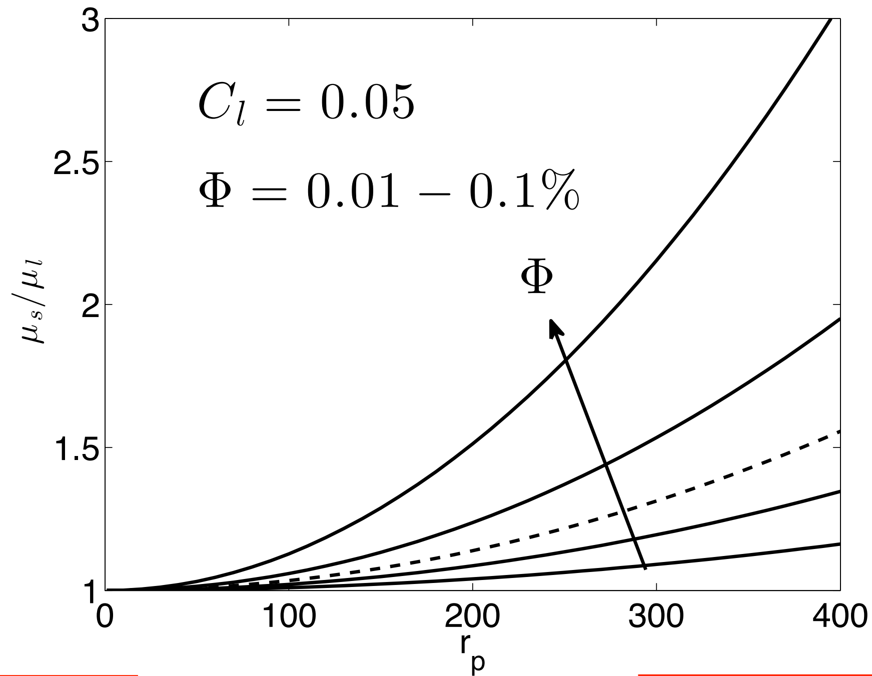
$$+\left[4a_{rrr\theta}\Pi^2 D^2 + 4i\alpha a_{rr\theta\theta}\Pi D - i\xi\alpha a_{rr\theta\theta} - \alpha^2 a_{r\theta\theta\theta} - 2\xi a_{r\theta\theta\theta}\Pi D - i\xi\alpha a_{\theta\theta\theta\theta}\right]\Theta\Big) =$$

$$= -i\omega \operatorname{Re}\Pi^2 R$$

Predicted suspension rheology

Jeffery	Smoluchowski	Suspension viscosity
$\dot{\varphi} = -\frac{\dot{\gamma}}{r_p^2+1}\left(r_p^2 \sin^2\varphi + \cos^2\varphi\right)$	$\frac{\partial}{\partial\varphi}(\dot{\varphi}\Psi - C_l\dot{\gamma}\frac{\partial\Psi}{\partial\varphi}) = 0$	$\mu_s = 1 + 2A\Phi a_{rr\theta\theta}$





$r_p = 340$ $C_l = 0.05$

$\mu_s \approx 1.4\mu_l$

Rheometer: $\mu_s \approx 1.8\mu_l$

Normal mode analysis

$$u'_r = R(\eta)e^{i(\beta z+\alpha\theta-\omega t)} \qquad u'_z = Z(\eta)e^{i(\beta z+\alpha\theta-\omega t)}$$

$$u'_\theta = \Theta(\eta)e^{i(\beta z+\alpha\theta-\omega t)} \qquad p' = P(\eta)e^{i(\beta z+\alpha\theta-\omega t)}$$

$$a'_4 = A_4(\eta)e^{i(\beta z+\alpha\theta-\omega t)}$$

Stationary orientation distribution

$$\left[4\Pi D^2 + 2\xi\Pi D - \beta^2\Pi^2 - \alpha^2 - i\alpha \mathrm{Re}\, V\Pi\right]R + \left[2\xi \mathrm{Re}\, V\Pi + 2\mathrm{Re}\, Ro\Pi^2 - 2i\xi\alpha\right]\Theta - 2\mathrm{Re}\,\Pi^2 DP +$$

$$+A\Phi\Big(\left[4a_{rrrr}\Pi^2D^2 + 2\xi a_{rrrr}\Pi D + 4i\alpha a_{rrr\theta}\Pi D\right]R +$$

$$+\left[4a_{rrr\theta}\Pi^2D^2 + 4i\alpha a_{rr\theta\theta}\Pi D - i\xi\alpha a_{rr\theta\theta} - \alpha^2 a_{r\theta\theta\theta} - 2\xi a_{r\theta\theta\theta}\Pi D - i\xi\alpha a_{\theta\theta\theta\theta}\right]\Theta\Big) =$$

$$= -i\omega \mathrm{Re}\,\Pi^2 R$$