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On the validity of Jeffery torques for rigid particles in turbulence

J. Ravnik[+], C. Marchioli[*] and A. Soldati[*]

jure.ravnik@um.si, http://jure.ravnik.si

[+]University of Maribor, Slovenia,

[*]University of Udine, Italy

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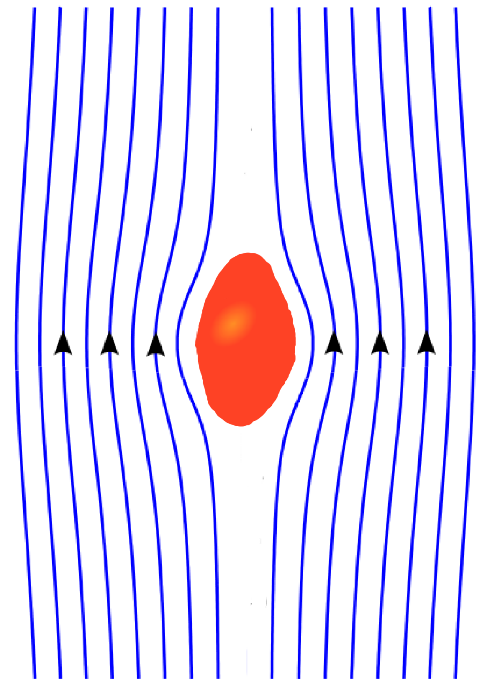
- When λ is large, the particle's length reaches the size of the smallest structures in turbulent flow.
- Thus, we question the validity of governing equations of a Lagrangian approach, which describes the evolution of position, velocity and acceleration of the particle's centre of mass.
- In this presentation, we will present statistics of the error due to the treatment of elongated particles as pointwise.

Particle equations of motion

- Newton's law for a particle states the balance of forces, which are assumed to be linearly additive, as:

$$m_p \frac{d\vec{v}}{dt} = \sum \vec{F} = \vec{F}_{drag} + \vec{F}_g + \vec{F}_b$$

  - Drag force: $\vec{F}_{drag}$
  - Net buoyancy force: $\vec{F}_g + \vec{F}_b$



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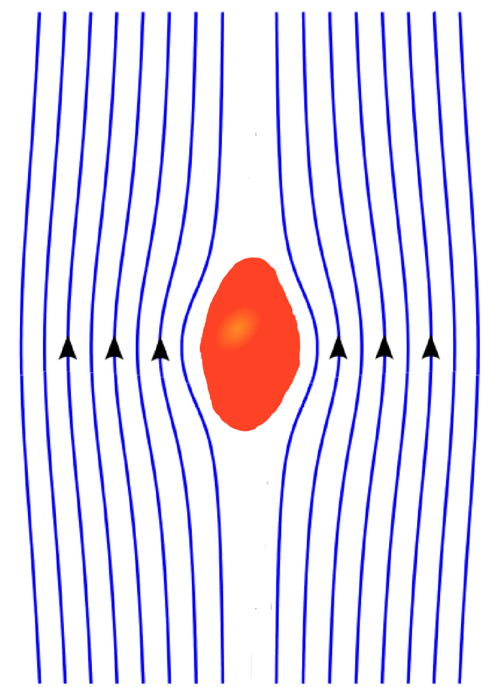
- Particle position

$$\frac{d\vec{x}}{dt} = \vec{v}$$



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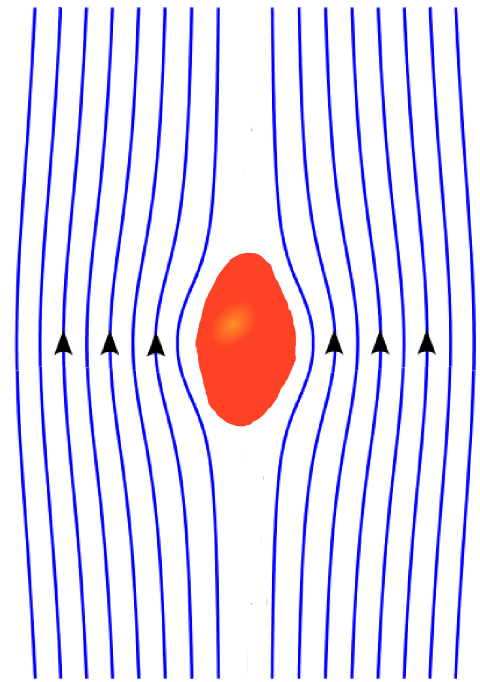
$$\frac{d\vec{x}}{dt} = \vec{v}$$

- Particle orientation and angular velocity

$$\frac{d\vec{e}}{dt} = f(\vec{e}, \vec{\omega}), \qquad \frac{d\vec{\omega}}{dt} = \vec{T}$$



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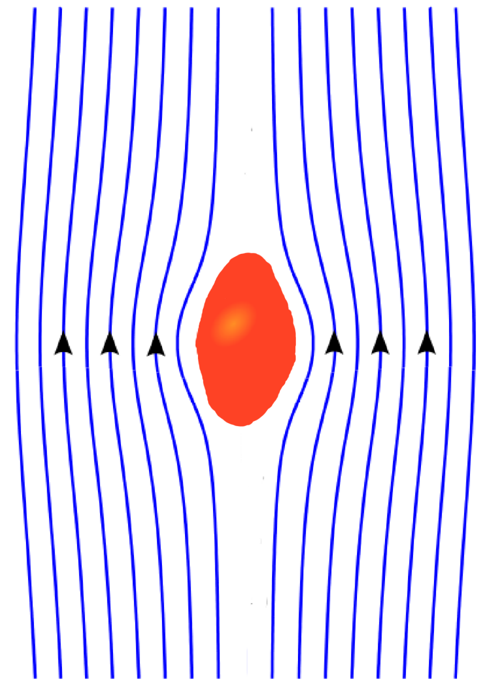
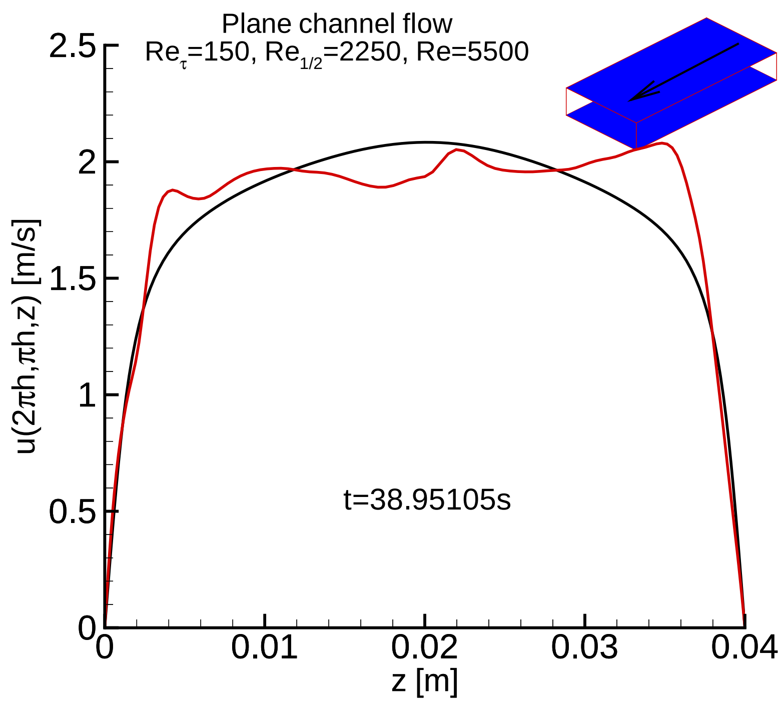
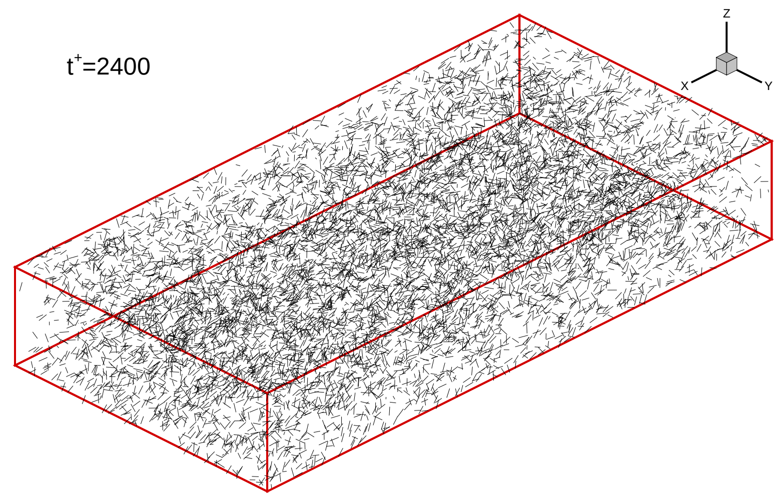
- 13 ODE in total: (3) position, (3) velocity, (3) angular velocity, (4) orientation.

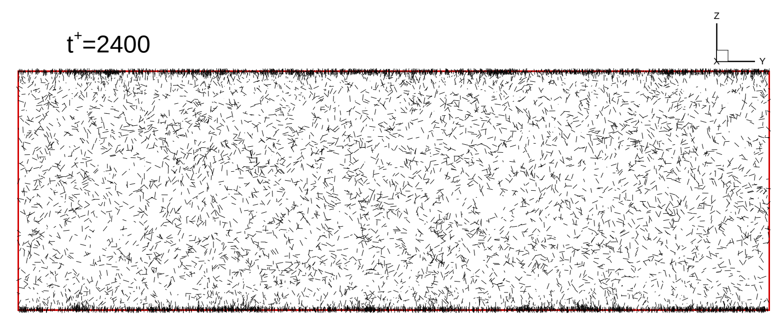


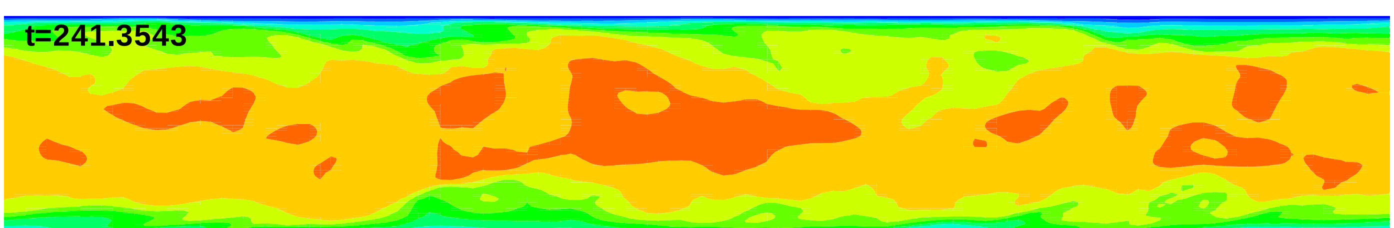
Test cases - turbulent channel flow

- We tracked fibers in $Re_\tau = 150$ turbulent channel
- $St = 1$, $\lambda = 3$, $a^+ = 0.36$.









Particle size and turbulence

- the particles are ellipsoids, minor axis a^+, major axis λa^+.
- the length of the particles is thus $2\lambda a^+$.
- The minimal size of turbulent flow structures is defined by the Kolmogorov length scale η_K.
- If

$$\frac{2\lambda a^+}{\eta_K} = \begin{cases} < 1 & \text{particle is } \textbf{smaller} \text{ than the smallest turbulent motions} \\ \approx 1 & \text{particle size is } \textbf{comparable} \text{ to the smallest turbulent motions} \\ > 1 & \text{particle is } \textbf{larger} \text{ than the smallest turbulent motions} \end{cases}$$

Particle size and eq. of motion

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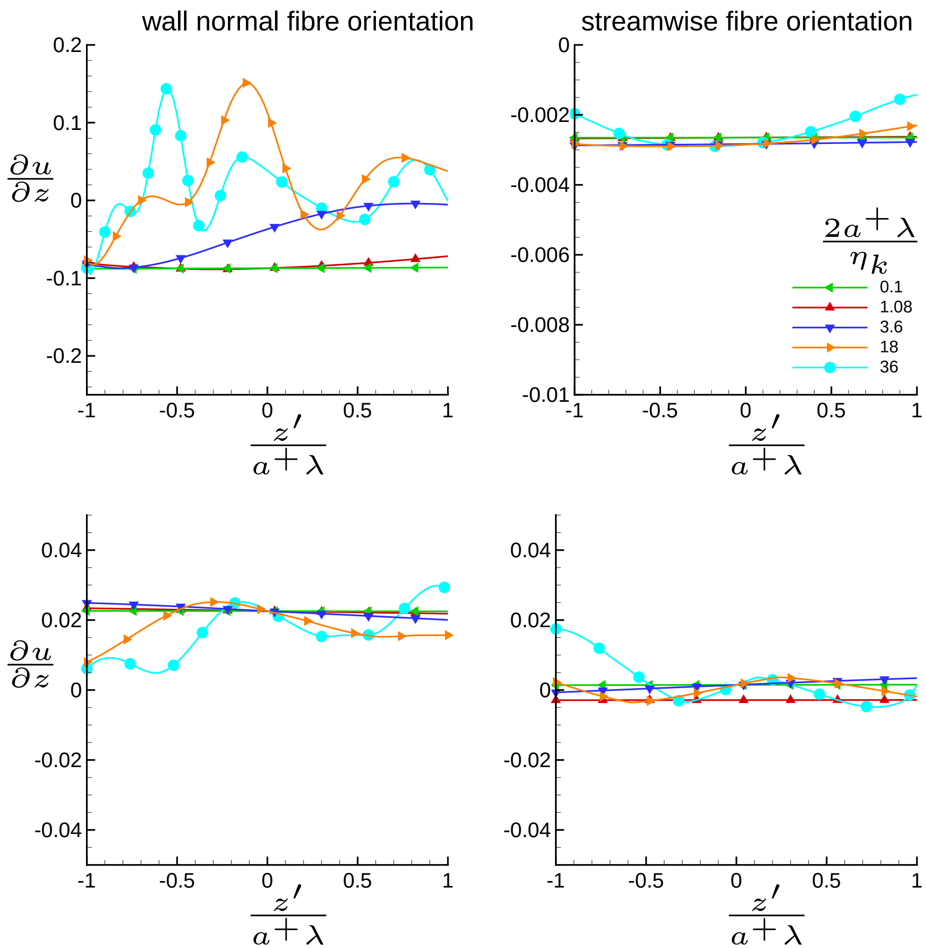
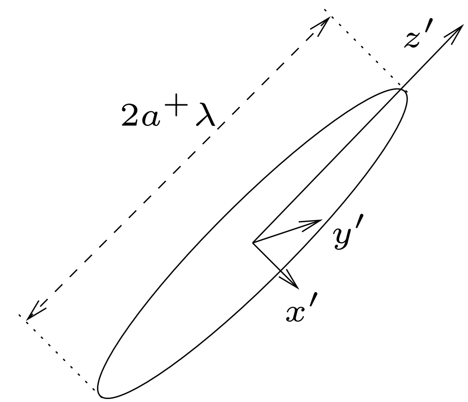
- In other words, this means that the velocity gradient tensor should be constant in space in an area that is larger than the fibre.
- Turbulent flow theory suggest that this assumption is violated when the particle size is comparable or larger than the smallest turbulent structures, which are defined by the Kolomogorov length scale.

Velocity gradient along the fibre

- We measured (interpolated from the DNS database) the velocity gradient along the long axis of the particle





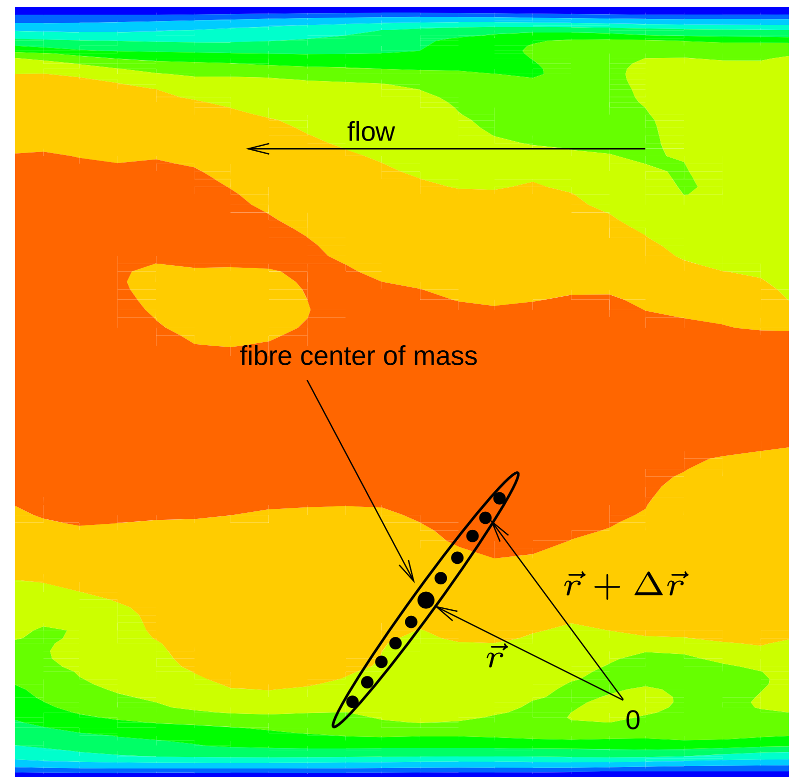
Flow field expansion

- The Jeffrey moments are based on the Taylor expansion of the flow field around an ellipsoidal body. Only leading order is considered, i.e.

$$\vec{u}^J(\vec{r} + \Delta\vec{r}) = \vec{u}(\vec{r}) + \nabla\vec{u} \cdot \Delta\vec{r}.$$

- We compare the flow field components, $\vec{u}^F$ obtained from the DNS database with the Taylor expansion approximation $\vec{u}^J$ for several points along the length of the fibre and express the difference in terms of the relative RMS norm as:

$$||\vec{u}^J - \vec{u}^F|| = \sqrt{\frac{\sum_i (\vec{u}_i^J - \vec{u}_i^F)^2}{\sum_i (\vec{u}_i^F)^2}}.$$



Jeffery moments

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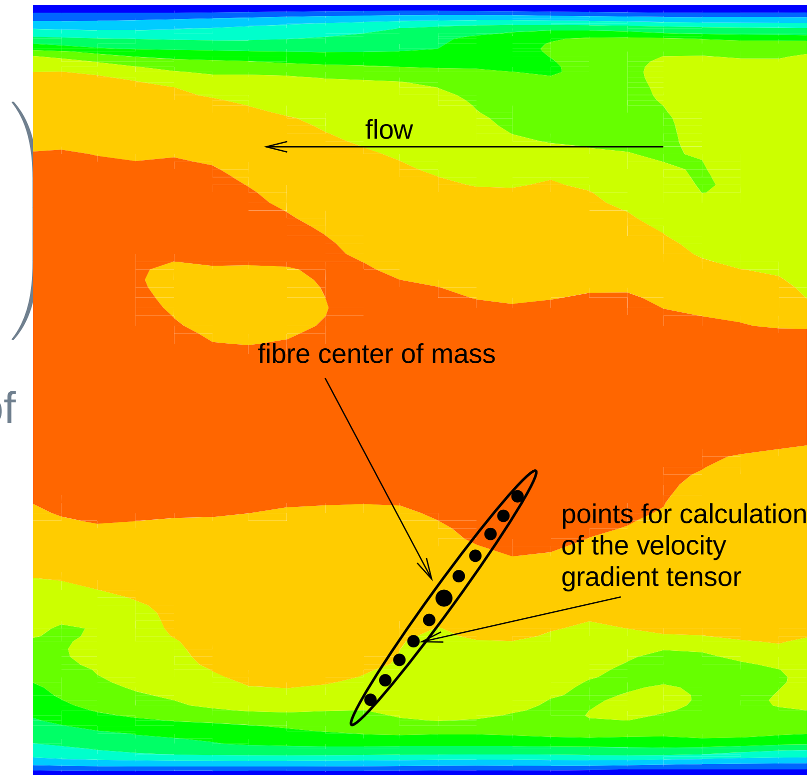
- Jeffery moments divided by fibre moment of inertia were considered

$$\begin{pmatrix} \frac{20}{(\beta_0+\lambda^2\gamma_0)} \frac{\rho_f}{\rho_p} \frac{1}{(a^+)^2} \left[\frac{1-\lambda^2}{1+\lambda^2} f' + (\xi' - \omega_{x'})\right] \\ \frac{20}{(\alpha_0+\lambda^2\gamma_0)} \frac{\rho_f}{\rho_p} \frac{1}{(a^+)^2} \left[\frac{\lambda^2-1}{1+\lambda^2} g' + (\eta' - \omega_{y'})\right] \\ \frac{20}{(\alpha_0+\beta_0)} \frac{\rho_f}{\rho_p} \frac{1}{(a^+)^2} (\chi' - \omega_{z'}) \end{pmatrix}$$

  where f', g', ξ', η' and χ' are the components of the deformation rate tensor and the spin tensor.

- We compare the Jeffery moments calculated using velocity gradients at the centre of the fibre with Jeffery moments calculated using the average velocity gradients along the fibre.
- Difference is expressed in terms of the RMS norm.



Torque on a cylinder

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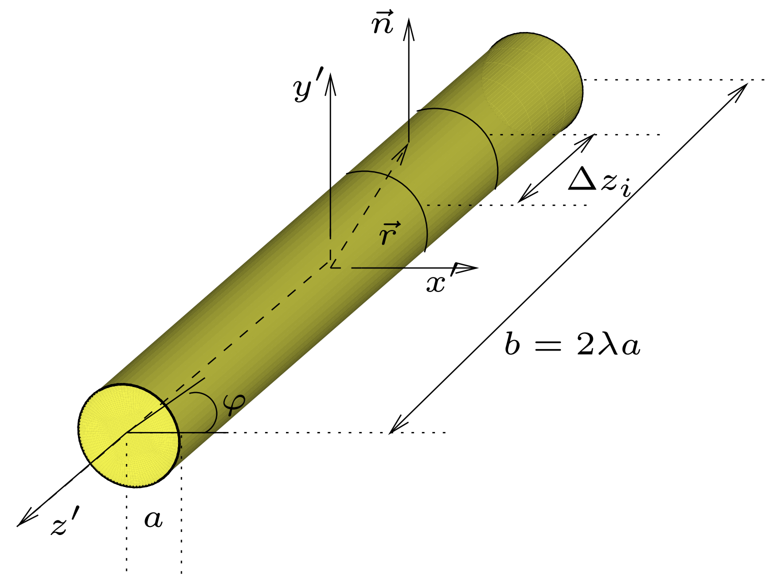

- We calculate the torque exerted on the cylinder by the fluid twice.
  - First, by using the flow properties at the center of mass of the cylinder
  - and second, by discretisizing the cylinder into sections and calculating the contribution of each section by taking into account the flow field at the center of mass of each individual section.
- Let the length of the section be Δz_i and let the velocity gradient measured at the centre of mass in each section be $\hat{G}'_i$. The torque generated by the flow around the i-th section, $\vec{M}_i$, can be written as

$$\vec{M}_i = \mu \int_{\Gamma_i} \vec{r} \times \hat{G}'_i \cdot \vec{n} d\Gamma,$$

- Difference is expressed in terms of the RMS norm; $||\vec{M}_c^m - \vec{M}_c^c||$.

Statistics

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- We randomly inserted 10^5 fibres into the flow.
- Initially the fibre velocity was equal to the fluid velocity at the position of the fibre.
- Before the start of gathering data for statistics, we ran the simulation for $t^+ = 300$, which is longer than ten fibre characteristic time scales
- After this period, statistics were gathered for $\Delta t^+ = 1000$.
- We chose $n = 11$ points along the fibre, where the velocity gradients were evaluated.
- We varied the fiber size to have $\frac{2a^+\lambda}{\eta_k} = 0.1, \ldots, 36$.
- Furthermore, we changed the fibre density, to set the fibre inertia to $St = 1, 5, 25$.

Velocity gradient statistics

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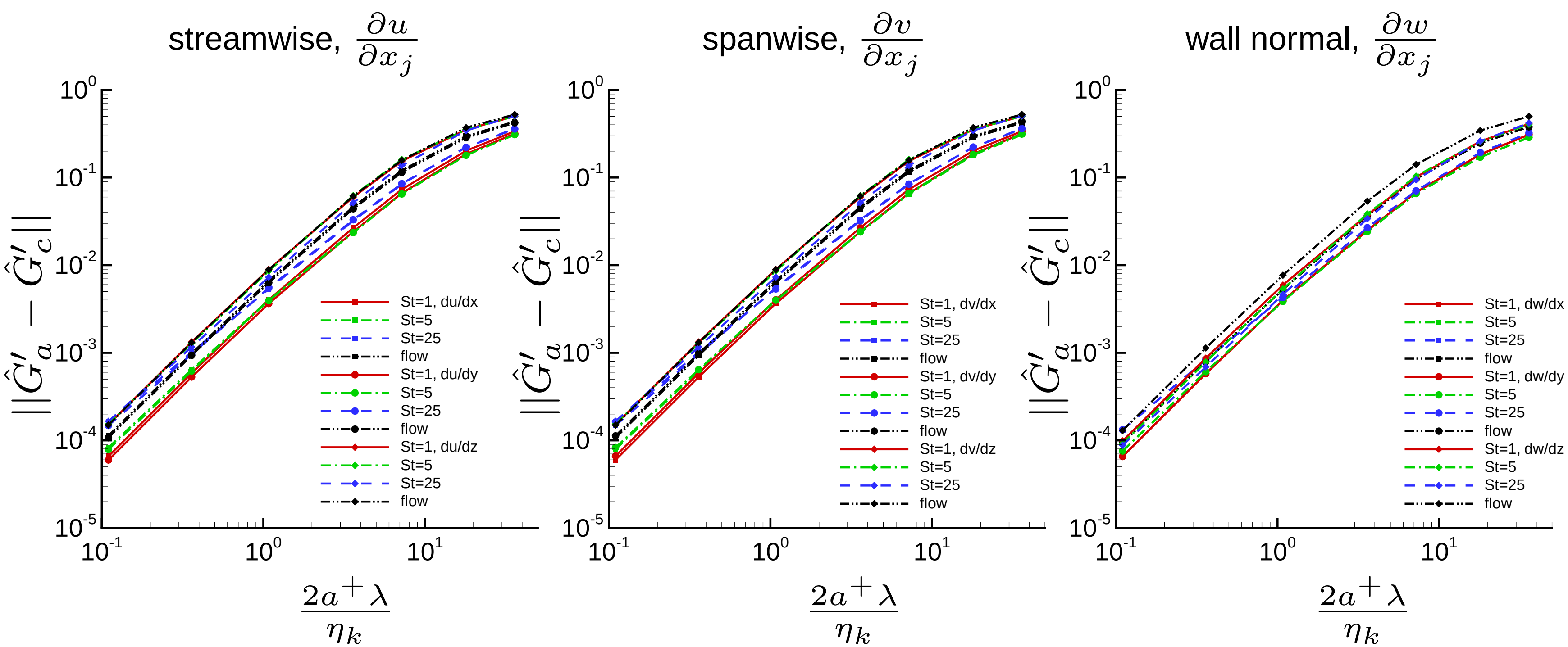
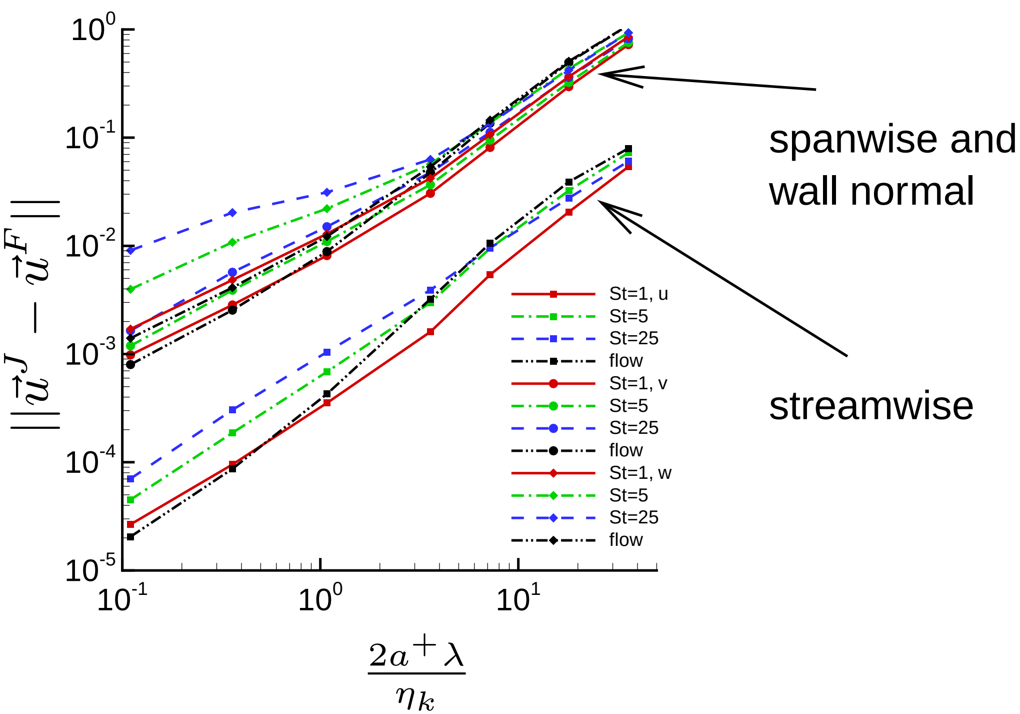
- RMS of velocity gradient components versus the ratio between fibre length and the Kolmogorov length scale.



Velocity around the particle

- RMS difference between the velocity of the DNS database and the velocity obtained by Taylor expansion around the centre of the particle.



Jeffery & cylinder moments

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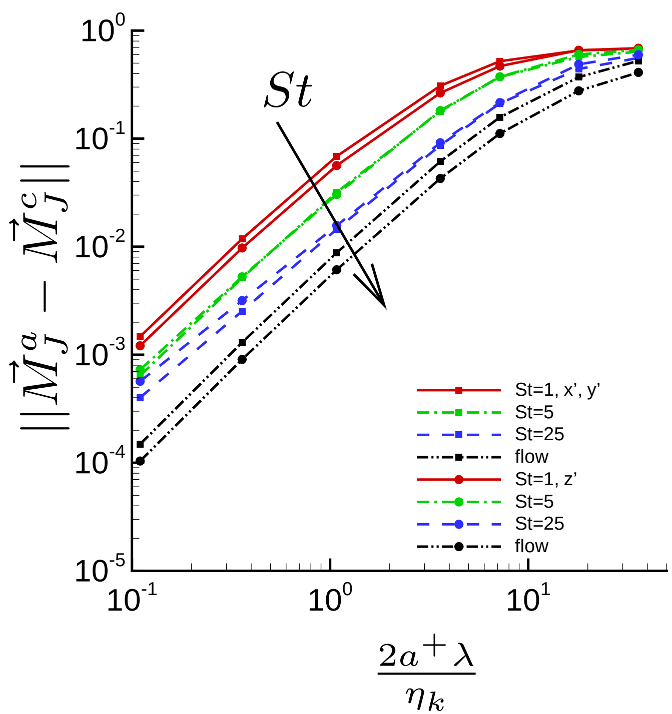
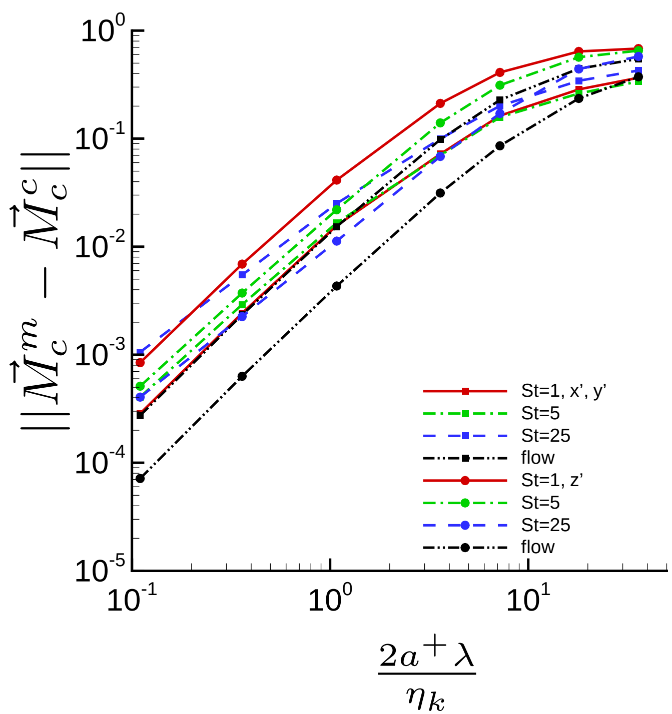
- Left: Jeffery moments calculated from velocity gradients at the centre of the fibre comapred to the average values along the particle length.
- Right: Torque on a cylinder calculated from velocity gradients at the centre of the fibre comared to the torque obtained by dividing the cylinder into sections.

Jeffery moments

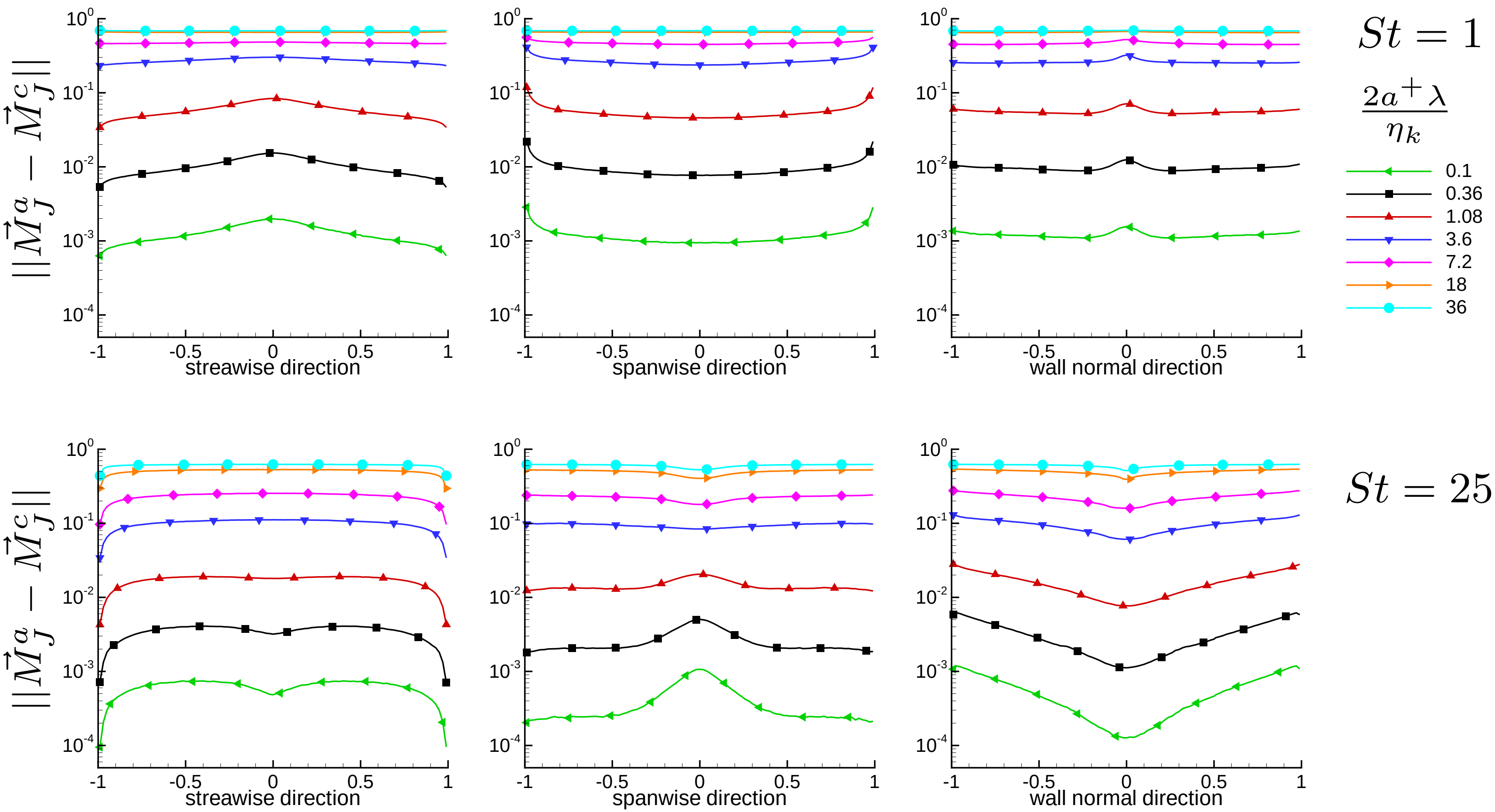


Torque on a cylinder



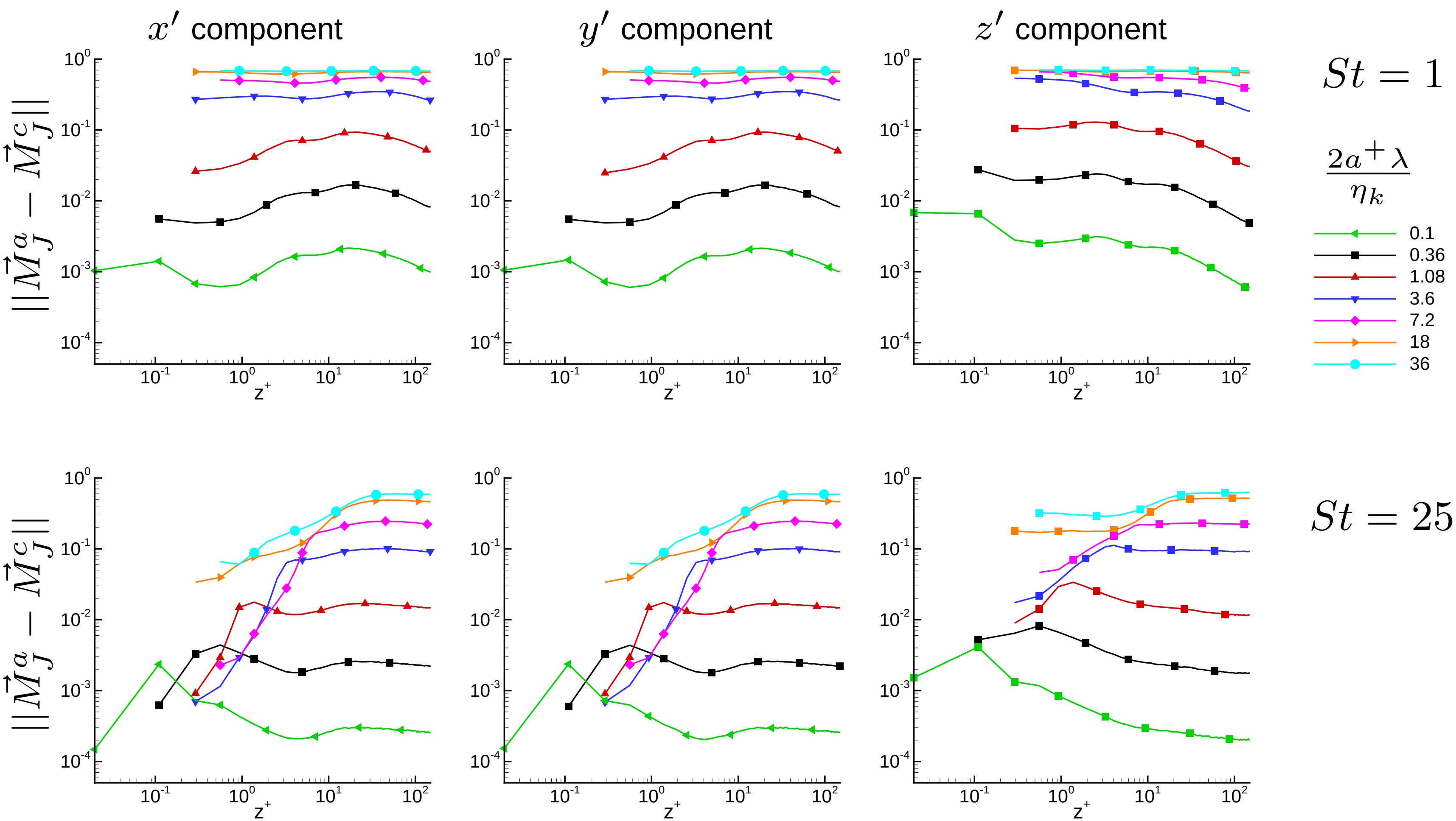
Fibre orientation

- z' Jeffery moment for different fibre direction cosines.



Fibre wall normal distance

- Jeffery moments for different fibre wall normal distances.



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  - for fibres smaller than the Kolmogorov length scale.
  - The error is smaller when fibers are oriented into the streamwise direction.
- When comparing fibres at difference distances from the wall, we observed
  - no difference for low inertia fibres.
  - High inertia fibers exhibit smaller error when they are located very close to the wall and when they are close to the centre of the channel.