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# On inertial effects of long fibers in wall turbulence: concentration, orientation and fiber stresses

Lihao Zhao[1], Helge Andersson[1] and Jurriaan Gillissen[2]

[1]Dept. of Energy and Process Engineering, NTNU, Norway
[2]Dept. of Chemical Engineering, Delft University of Technology, The Netherlands

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- Mathematical modeling and governing equations
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  - Lagrangian modeling of inertial fibers
  - Lagrangian modeling of massless fibers
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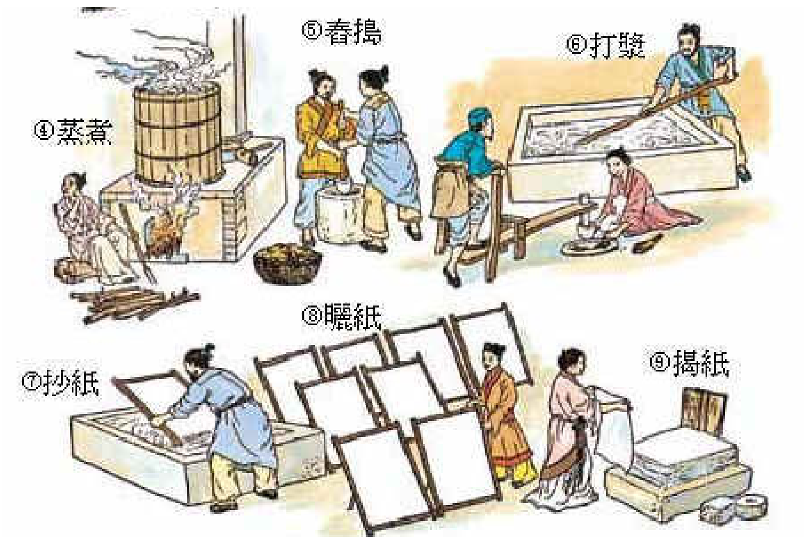
  - Fiber concentration
  - Fiber orientation
  - Fiber stresses
- Concluding remarks

# Introduction

Fiber suspensions are common seen in industry applications




Paper making in ancient China around 2000 years ago

Drag reduction induced by elongated particles in oil or gas transport

# Introduction

A first attempt to bridge the gap between:

- Massless and infinitely long fibers:
  Gillissen, Boersma, Mortensen & Andersson; *Physics of Fluids* 2007

- Inertial finite-length fibers:
  Mortensen, Andersson, Gillissen & Boersma; *Physics of Fluids* 2008

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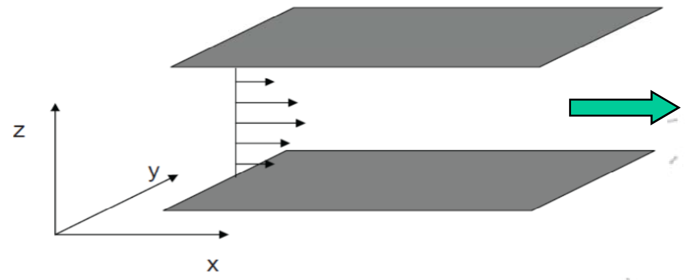
# Eulerian fluid representation

- Incompressible and isothermal Newtonian fluid.
- Frictional Reynolds number: $\mathrm{Re}_\tau = \frac{u_\tau h}{\nu}$
- Governing equations (non-dimensional):
  - Mass balance $\nabla \cdot \mathbf{u} = 0$
  - Momentum balance $\frac{\partial \mathrm{u}}{\partial t} + (\mathrm{u} \cdot \nabla)\mathrm{u} = -\nabla p + \frac{1}{\mathrm{Re}_\tau}\nabla^2 \mathrm{u}$



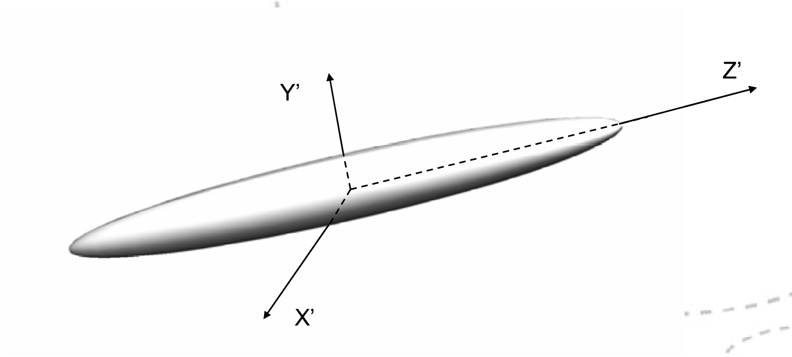
The geometry of the channel flow

- Direct numerical simulations (DNS), i.e. turbulence from first principles

# Lagrangian approach – Inertial fiber

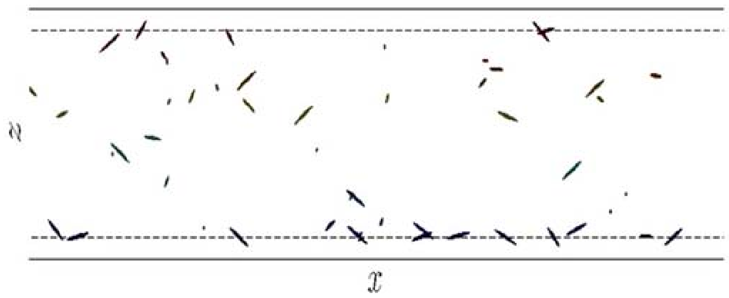
- Inertial fiber aspect ratio
  - a: minor half-axis
  - b: major half-axis

$$\lambda = \frac{b}{a}$$




- Lagrangian approach
  - Translational and rotational motions of each individual fiber

- Finite number of particles




- Point-particle assumption
  - Smaller than Kolmogorov length scale

- Translational response time

$$\tau_p = \frac{2\lambda D a^2}{9\nu} \frac{\ln(\lambda + \sqrt{\lambda^2 - 1})}{\sqrt{\lambda^2 - 1}}$$

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# Lagrangian approach – Inertial fiber

- Prolate spheroid
  - Translational and rotational motions are governed by:

$$m\frac{dv_i}{dt} = F_i, \qquad I'_{ij}\frac{d\omega'_j}{dt} + \epsilon_{ijk}\omega'_j I'_{kl}\omega'_l = N'_i$$

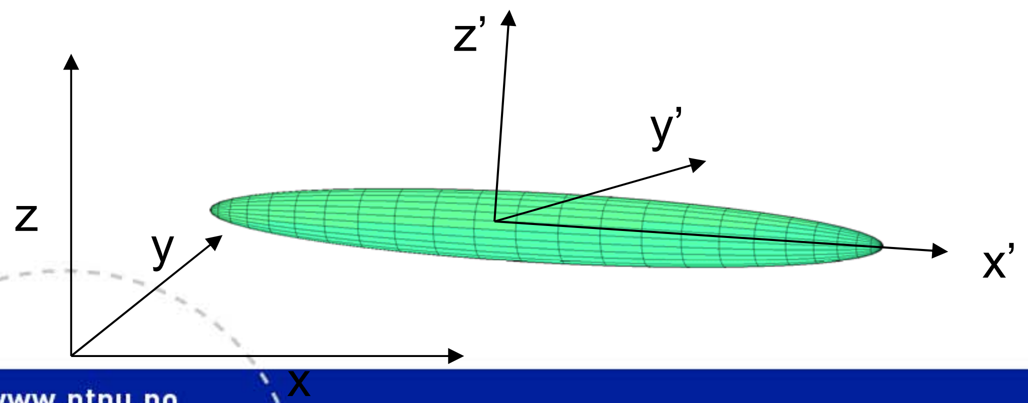
  - The force acting on a particle can be expressed as (Brenner 1964, Harper & Chang 1968):

$$F_i = D_{ij}(u_j - v_j) + \frac{Re_\kappa^{1/2}}{\mu a} D_{ij} L_{jk} D_{kl}(u_l - v_l), \quad D_{ij} = \pi\mu a K_{ij}$$

$$Re_\kappa = \rho\kappa a^2/\mu$$

$K_{ij}$ is the resistance tensor.

**NOTE:** The lift tensor $L_{ij} = 0$

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## Lagrangian approach – Massless fibers

- Fiber aspect ratio, λ>>1; the finite-aspect-ratio effect is ignored.
- Fiber is massless
- Point-particle assumption
  - Smaller than Kolmogorov length scale
- Lagrangian approach
  - Translational and rotational motions of each individual fiber governed by:

$$\frac{d\mathbf{x}}{dt} = \mathbf{u}; \qquad \frac{d\mathbf{P}}{dt} = \nabla\mathbf{u}^{T} \cdot \mathbf{P} \cdot (\boldsymbol{\delta} - \mathbf{PP})$$

  - $\boldsymbol{\delta}$ is the unit tensor.
  - This time rate-of-change of the fiber orientation vector **P** is a simplified version of Jeffery's equation derived by Doi & Edwards for aspect ratios $\lambda \geq 100$.

# Lagrangian approach – Fiber stress

- Doi & Edwards expressed the *fiber stress tensor* **τ** in terms of the *fiber orientation vector* **P** as:

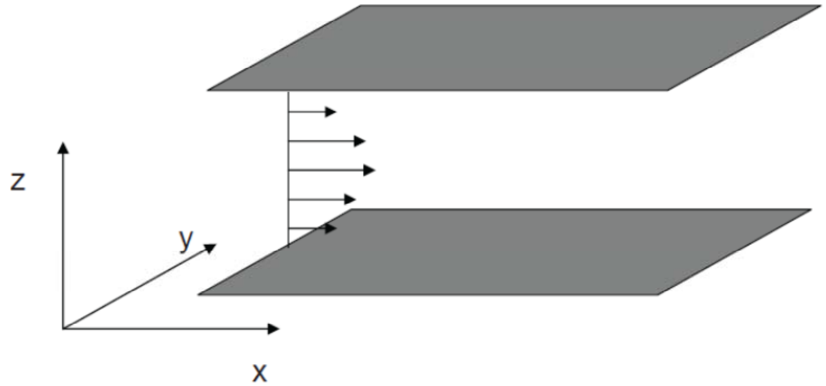
$$\tau = 2\alpha\mu \mathrm{s} : \langle \mathrm{PPPP} \rangle \qquad \alpha = \frac{4\phi\lambda^2}{3(\ln\lambda - 0.8)}$$

;

  - **s** is the rate-of-strain tensor and μ is the fluid dynamic viscosity.
  - <…> signifies an average over a small volume centered around the point where the stress is to be obtained.
  - Φ is particle volume fraction.

# Results – Simulation conditions

- Simulations of fiber suspensions in channel flow (one-way coupling)
  - Frictional Reynolds number 360
  - Computational domain
    - 1.5h*0.75h*h (x*y*z)
  - Mesh size
    - 48*48*192
- Fiber parameters




| Case | St | λ | $N_p$ |
|---|---|---|---|
| A | 10 | 100 | $10^5$ |
| B | 1 | 100 | $10^5$ |
| C | 0.1 | 100 | $10^5$ |
| D | 0 | ∞ | $10^6$ |

**Table 1.** Overview of the four different fiber classes. $N_p$ is the number of fibers.

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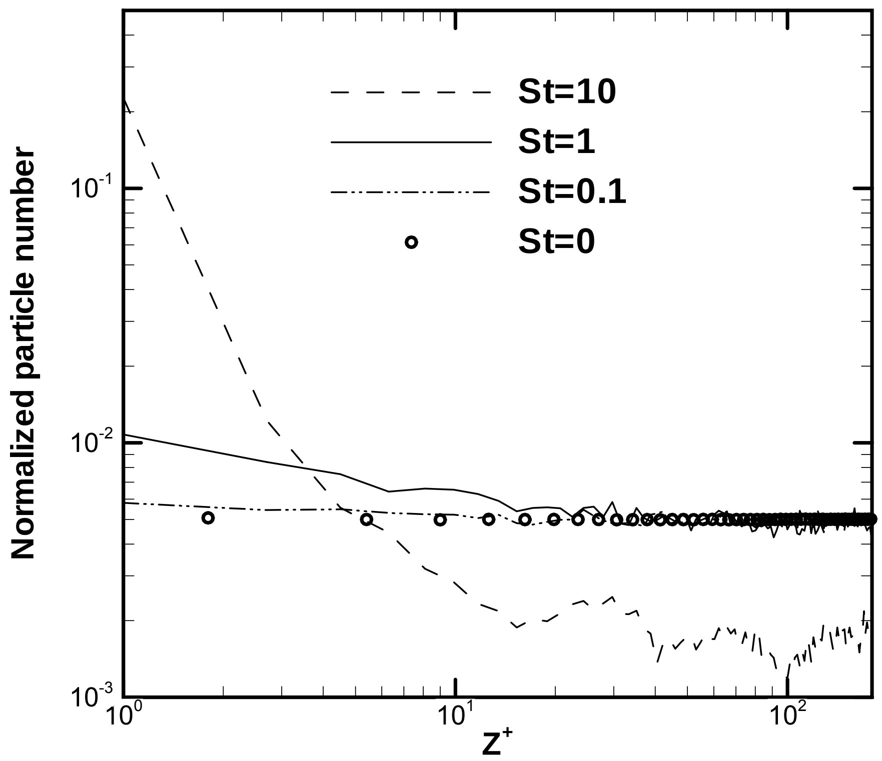
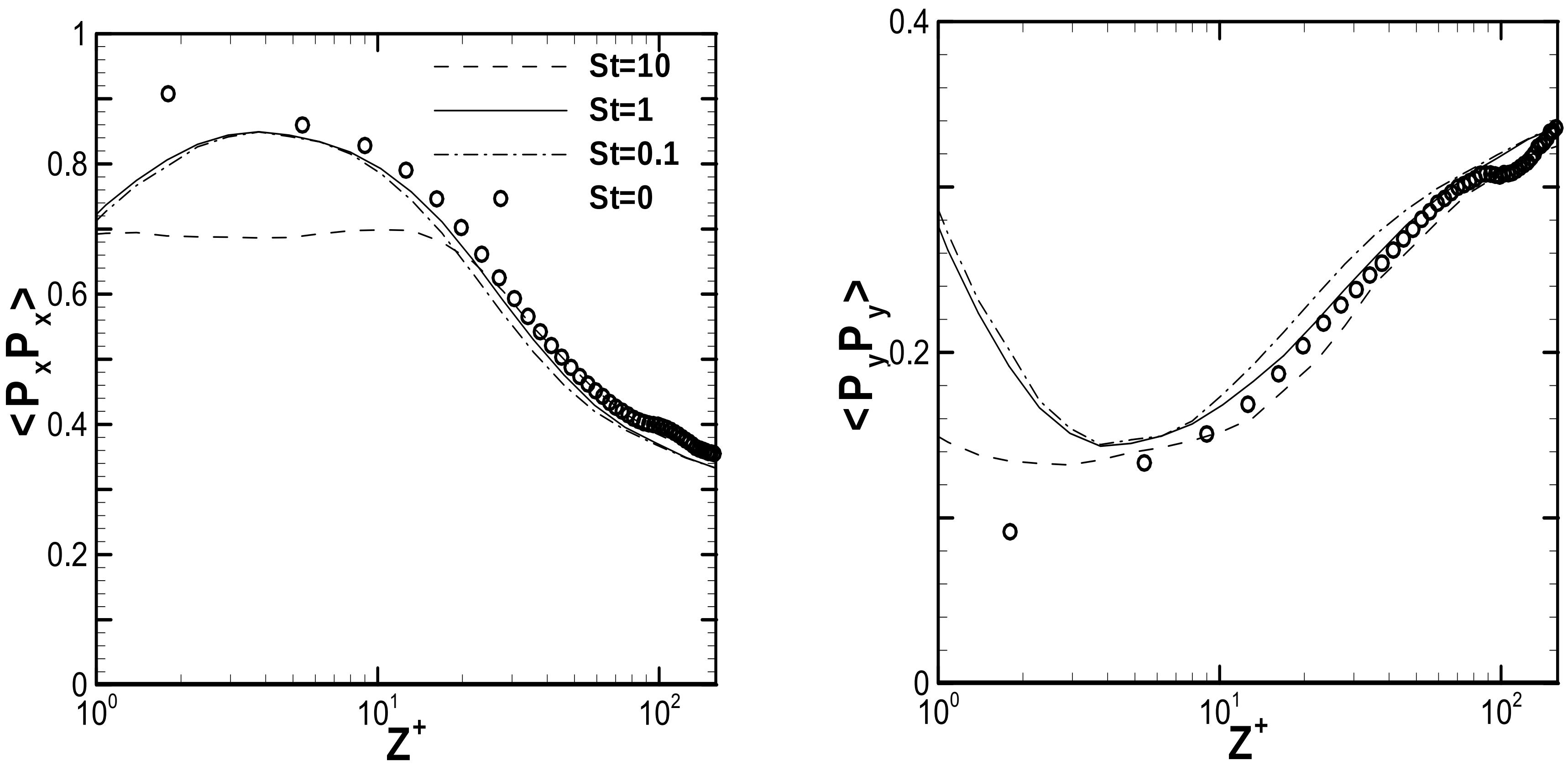
# Results – Fiber concentration




Particle concentration normalized by the total number of particles Np

# Results – Fiber orientations




Inertial fibers $St > 0$ (lines) and massless fibers $St = 0$ (symbols).

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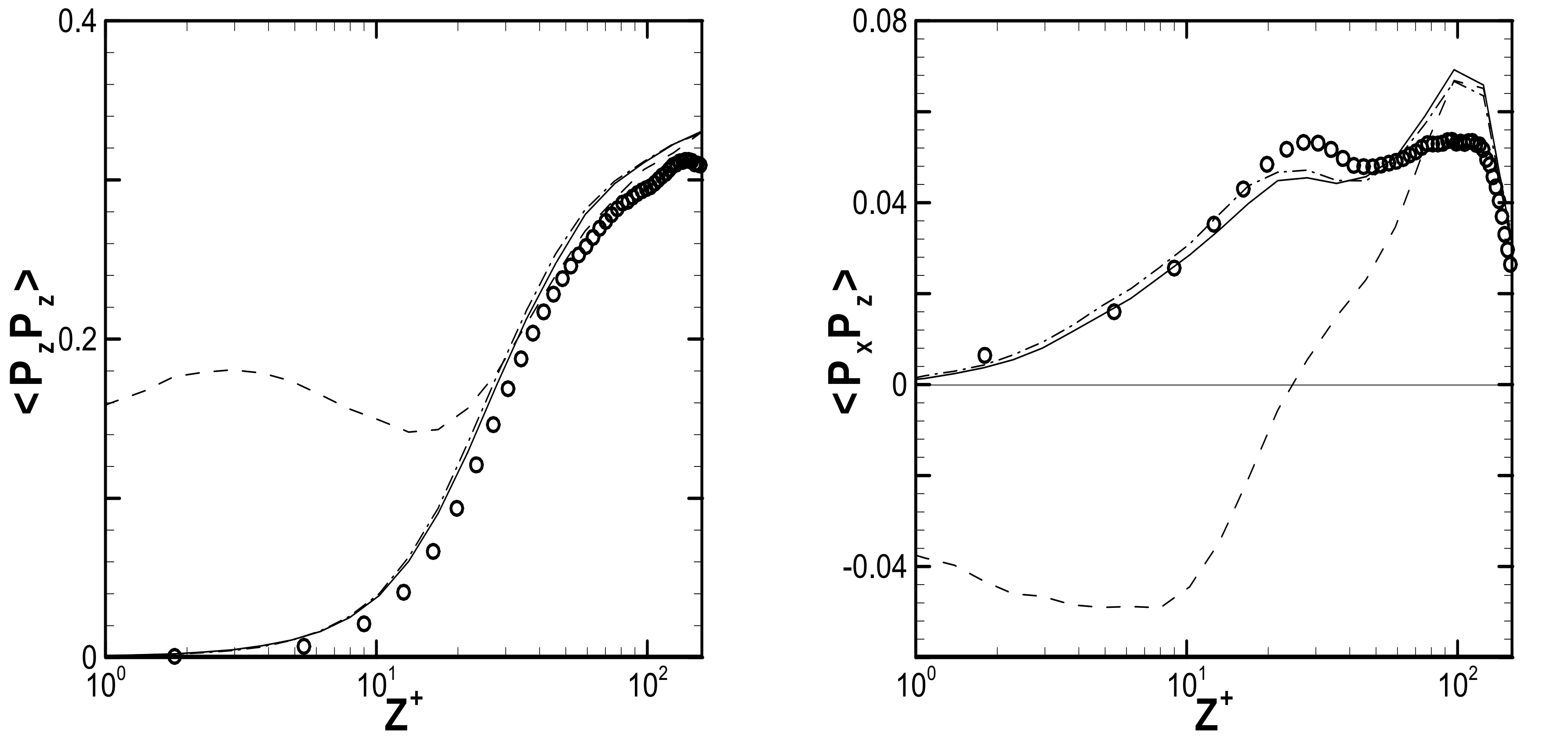
# Results – Fiber orientations




Inertial fibers $St > 0$ (lines) and massless fibers $St = 0$ (symbols).

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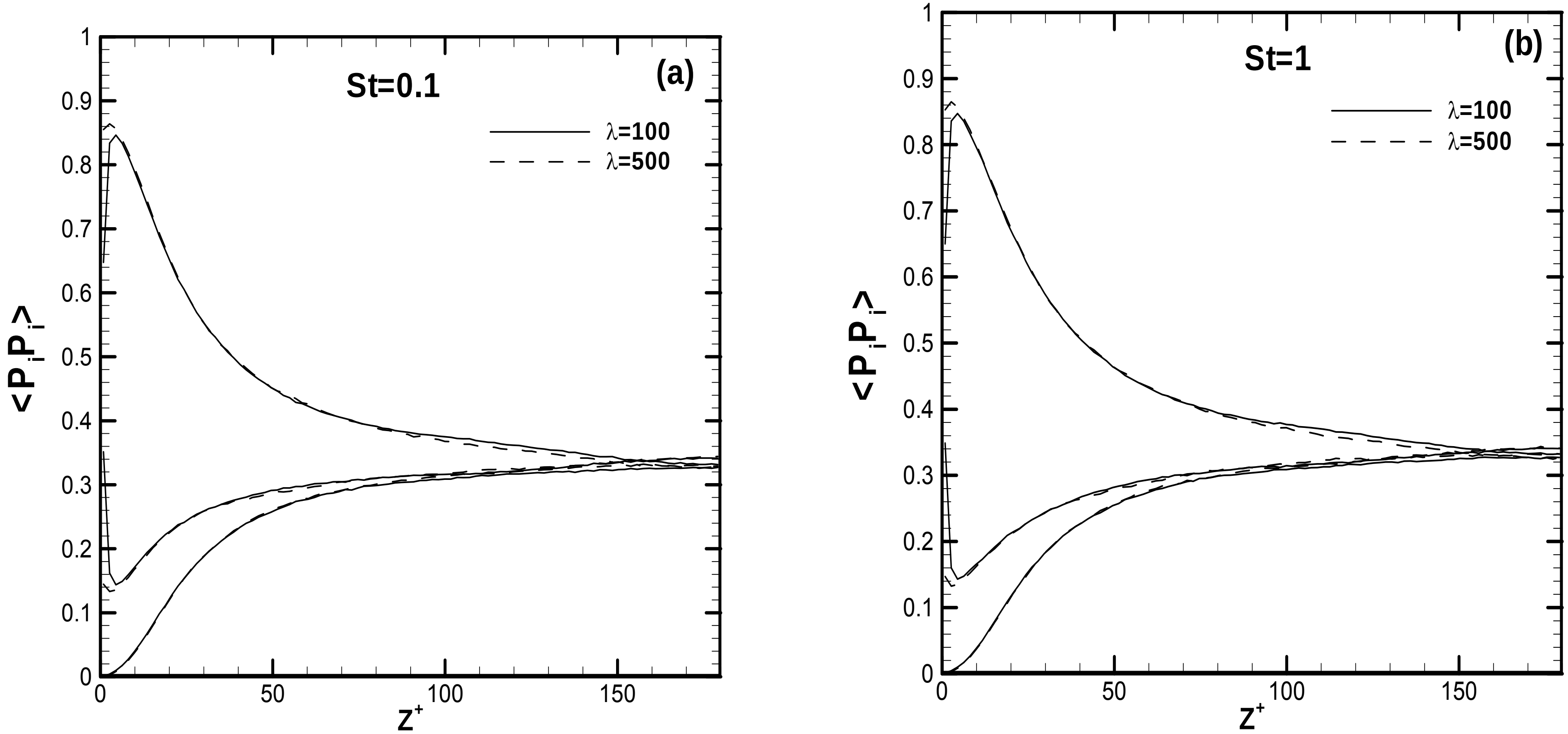
# Results – Fiber orientation: effect of aspect ratio

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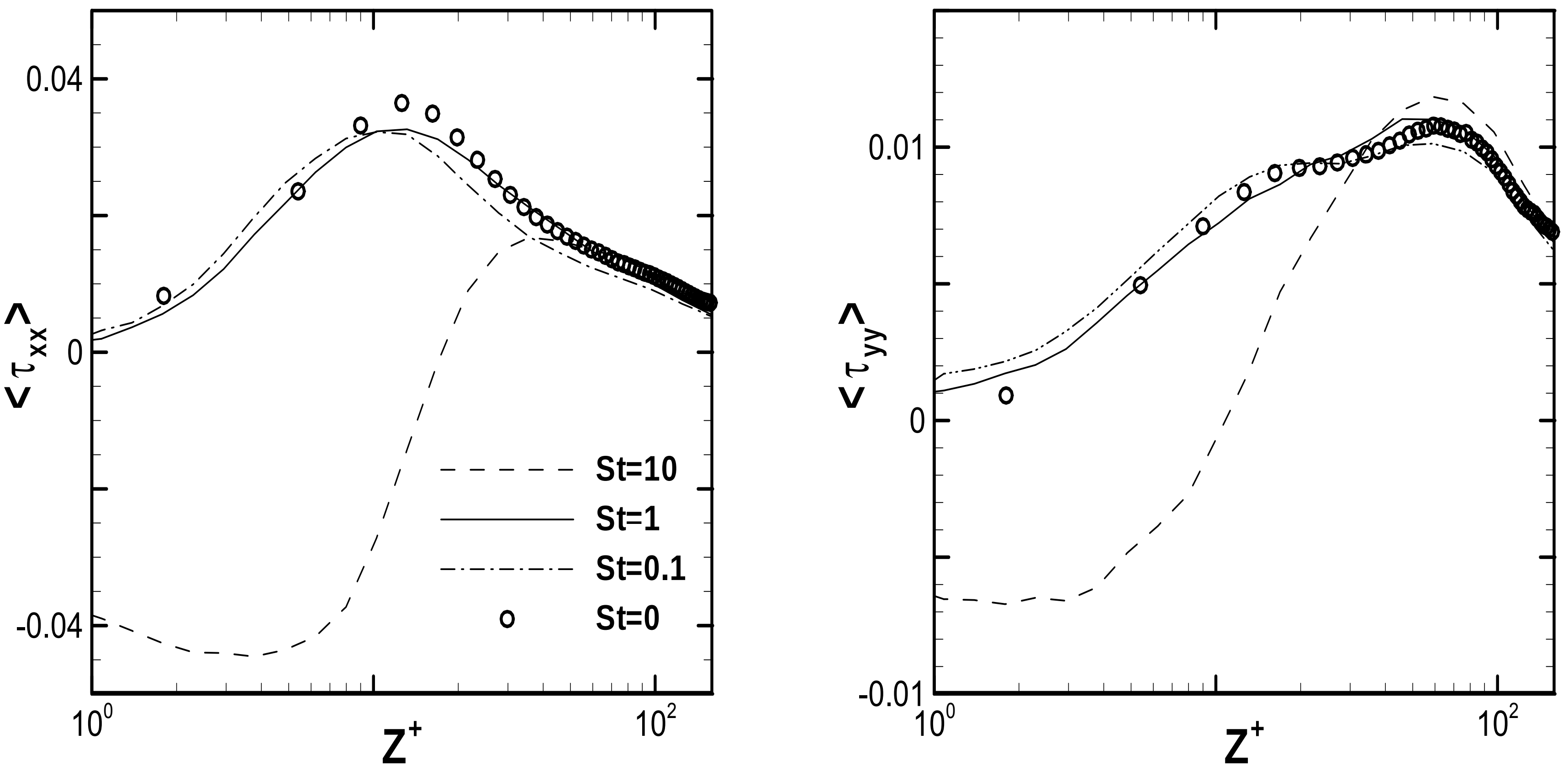
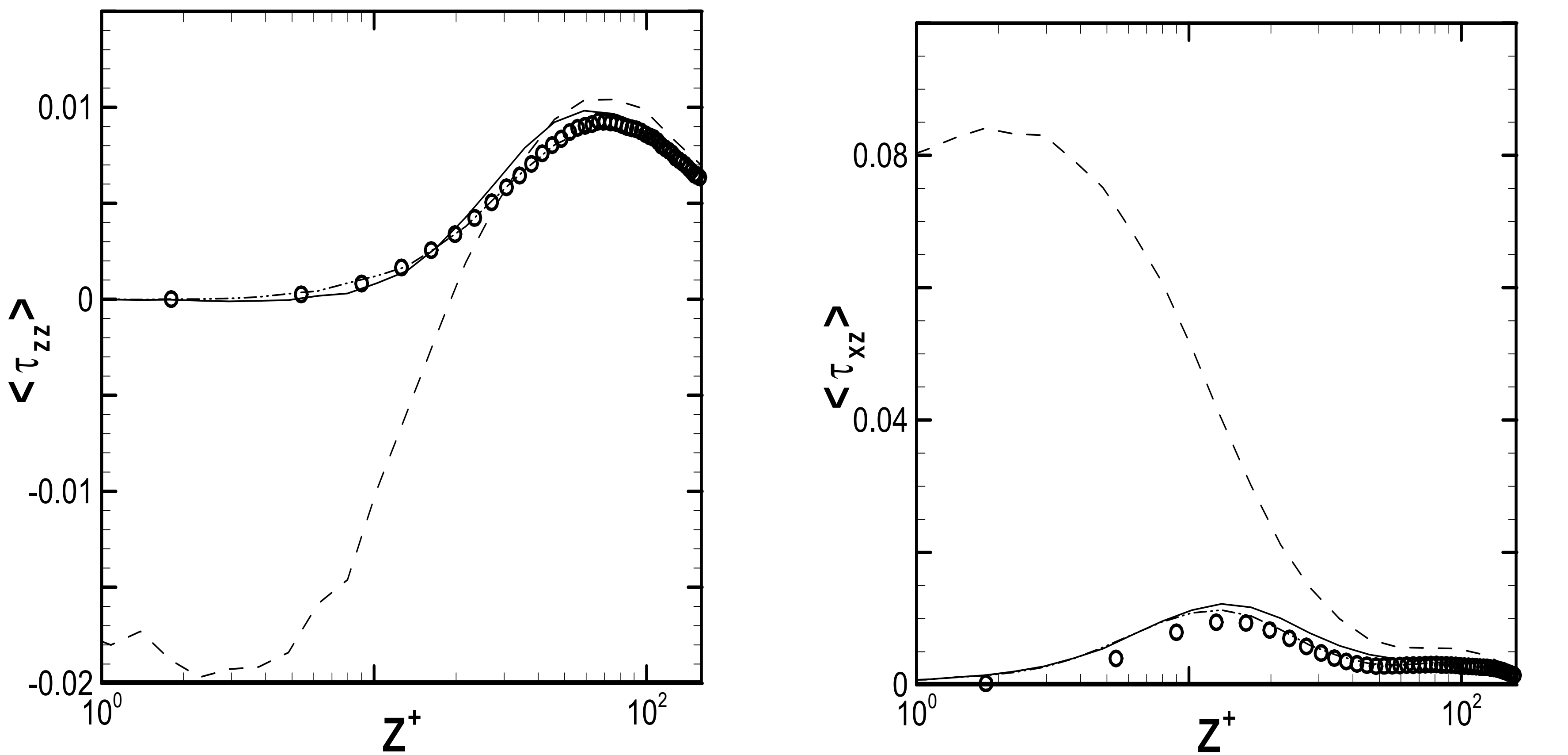
# Results – Fiber stresses




Inertial fibers $St > 0$ (lines) and massless fibers $St = 0$ (symbols).

# Results – Fiber stresses




Inertial fibers $St > 0$ (lines) and massless fibers $St = 0$ (symbols).

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# Concluding remarks

- One-way coupled simulations are performed for inertial fibers with Stokes number St = 10, 1.0 and 0.1 and thereafter for massless fibers which correspond to St = 0.
- The fiber orientation statistics and the normal components of the fiber stress tensor turned out to be almost independent of the fiber inertia all the way from the channel wall to the center for St ≤ 1.
- Present study suggests that fiber inertia plays a negligible role for Stokes number below unity and the gap between inertial fibers and massless fibers has been bridged.
- The effect of aspect ratio on the fiber orientation is negligible for aspect ratios larger than 100.

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