On the relative motion
between rigid fibers and fluid
in turbulent channel flow

C. Marchioli¹, L. Zhao², H.I. Andersson²

¹Dept. Elec., Manag. & Mechanical Engineering, University of Udine
²Department of Energy & Process Engineering, NTNU
We characterize fiber relative motion with two observables.

Relative translational motion

- **Slip velocity**
  - $u_{@p}$: fluid velocity “seen”
  - $u_p$: fiber velocity
  - $\Delta u = u_{@p} - u_p$: slip velocity

Relative rotational motion

- **Slip spin**
  - $\omega_{@p}$: fluid angular velocity “seen”
  - $\Omega_p$: fiber angular velocity
  - $\Delta \omega = \omega_{@p} - \Omega_p$: slip spin

Refs:
- Zhao et al. (2014) Phys Fluids, vol 26, 063302
- Marchioli et al., J Fluid Mech, In preparation
Motivation: Why are we looking at slip velocity and spin?

Slip velocity and spin are crucial in:

1. Euler-Lagrange simulations:
   - **One-way coupling**: determine drag and torque experienced by fibers
   - **Two-way coupling**: determine reaction force/torque from fibers on fluid

2. Two-fluid modeling of particle-laden flows
   - **Modeling SGS fiber dynamics in LES flow fields**
   - **Crossing trajectory effects on time decorrelation tensor of u**

Aim of this study: statistical characterization of $\Delta u$ and $\Delta \omega$ at varying fiber inertia and elongation

$\Delta u = u_{@p} - u_p$ : slip velocity

$\Delta \omega = \omega_{@p} - \Omega_p$ : slip spin
Fibers are modelled as non-deformable prolate ellipsoids evolving in 3D time-dependent fully-turbulent flow (e.g. Marchioli et al, 2010)

Assume:
• Fibers smaller than the smallest flow scale
  > point-wise
  > Stokes regime
• Dilute flow
  > no turbulence modulation
  > no collisions

Flow solver:
Time-dependent 3D turbulent flow
Channel size: \( L_x \times L_y \times L_z = 4\pi h \times 2\pi h \times 2h \)

Pseudo-spectral DNS: Fourier modes (1D FFT) in the homogeneous directions, Chebyshev coefficients in the wall-normal direction

Time integration: Adams-Bashforth (convective terms), Crank-Nicolson (viscous terms)
Modelling approach:
Fiber tracking

\[ \frac{d\mathbf{x}_{p(G)}}{dt} = \mathbf{u}_p \]

\[ \frac{de_0}{dt} = \frac{1}{2} \left(-e_1 \Omega_{x'} - e_2 \Omega_{y'} - e_3 \Omega_{z'}\right) \]

\[ \frac{de_1}{dt} = \frac{1}{2} \left(e_0 \Omega_{x'} - e_3 \Omega_{y'} + e_2 \Omega_{z'}\right) \]

\[ \frac{de_2}{dt} = \frac{1}{2} \left(e_3 \Omega_{x'} + e_0 \Omega_{y'} - e_1 \Omega_{z'}\right) \]

\[ \frac{de_3}{dt} = \frac{1}{2} \left(-e_2 \Omega_{x'} + e_1 \Omega_{y'} + e_0 \Omega_{z'}\right) \]

\[ \frac{d\mathbf{u}_p}{dt} = \frac{3}{4\lambda Sa^2} \mathbf{K} \cdot (\mathbf{u}_{@p} - \mathbf{u}_p) \]

\[ \frac{d\Omega_{x'}}{dt} = \Omega_{y'} \Omega_{z'} \left(1 - \frac{2}{1 + \lambda^2}\right) + \frac{20 \left[(1 - \lambda^2) f' + (1 + \lambda^2) (\xi' - \Omega_{x'})\right]}{(\beta_0 + \lambda^2 \gamma_0) (1 + \lambda^2) Sa^2} \]

\[ \frac{d\Omega_{y'}}{dt} = \Omega_{x'} \Omega_{z'} \left(\frac{2}{1 + \lambda^2} - 1\right) + \frac{20 \left[(\lambda^2 - 1) g' + (\lambda^2 + 1) (\eta' - \Omega_{y'})\right]}{(\alpha_0 + \lambda^2 \gamma_0) (1 + \lambda^2) Sa^2} \]

\[ \frac{d\Omega_{z'}}{dt} = \frac{20}{(\alpha_0 + \beta_0) Sa^2} (\chi' - \Omega_{z'}) \]

3 frames of reference:
Inertial, \( \mathbf{X} = [x, y, z] \)
Particle, \( \mathbf{X}' = [x', y', z'] \)
Co-moving, \( \mathbf{X}'' = [x'', y'', z''] \)
Problem: Dilute suspension of rigid fibers in turbulent channel

Simulation parameters:

- Fluid

<table>
<thead>
<tr>
<th>$Re_{\tau}$</th>
<th>Fluid</th>
<th>$\rho_F$ [kg/m³]</th>
<th>$\nu$ [m²/s]</th>
<th>$h$ [cm]</th>
<th>$u_{\tau}$ [m/s]</th>
<th>$\bar{u}_x$ [m/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>150</td>
<td>Air</td>
<td>1.3</td>
<td>$1.57 \cdot 10^{-5}$</td>
<td>2.0</td>
<td>0.11775</td>
<td>1.77</td>
</tr>
<tr>
<td>150</td>
<td>Water</td>
<td>1000</td>
<td>$1.00 \cdot 10^{-6}$</td>
<td>0.5</td>
<td>0.03000</td>
<td>0.45</td>
</tr>
</tbody>
</table>

- Particles

\[ St = \frac{2Sa^2}{9\nu}f(\lambda) \]

\[ f(\lambda) = \frac{\lambda \ln(\lambda + \sqrt{\lambda^2 - 1})}{\sqrt{\lambda^2 - 1}} \]
Results: Near-wall fiber preferential distribution

Top view: fibers accumulate in fluid Low-Speed Streaks

Sample snapshot for $\tau^+=30$, $\lambda=50$ fibers
Results: Using slip velocity to examine fiber accumulation in LSS

The influence of $\lambda$ is not dramatic: only a change in the peak values is observed (no PDF shape change)

Effect of fiber elongation on conditioned PDF($u_f'$) — St=30
Results: Using slip velocity to examine fiber accumulation in LSS.

Effect of fiber inertia on conditioned PDF($u_f'$)

Significant PDF shape change with curve “inversion” between $St=5$ and $St=30$. 

- Positive slip
- Negative slip

PDF($u_f'$)

$\Delta u_x > 0$ (red)
$\Delta u_x < 0$ (blue)

St=100
Results: Streamwise slip Velocity - Mean and RMS values

**St = 1**

[Graph showing mean and RMS values for St = 1 across different values of \( \lambda \).]

**St = 30**

[Graph showing mean and RMS values for St = 30 across different values of \( \lambda \).]
RESULTS: WALL-NORMAL SLIP VELOCITY - MEAN AND RMS VALUES

**St = 1**

- **Mean**
  - \( \lambda = 1 \)
  - \( \lambda = 3 \)
  - \( \lambda = 10 \)
  - \( \lambda = 50 \)

- **RMS**
  - \( \lambda = 1 \)
  - \( \lambda = 3 \)
  - \( \lambda = 10 \)
  - \( \lambda = 50 \)

**St = 30**

- **Mean**
  - \( \lambda = 1 \)
  - \( \lambda = 3 \)
  - \( \lambda = 10 \)
  - \( \lambda = 50 \)

- **RMS**
  - \( \lambda = 1 \)
  - \( \lambda = 3 \)
  - \( \lambda = 10 \)
  - \( \lambda = 50 \)
Results: Spanwise slip spin - Mean values

- **St=1**
  - \( \lambda = 1, 3, 10, 50 \)

- **St=5**
  - \( \lambda = 1, 3, 10, 50 \)

- **St=30**
  - \( \lambda = 1, 3, 10, 50 \)

- **St=100**
  - \( \lambda = 1, 3, 10, 50 \)
Results: Spanwise slip spin - Correlation w wall-normal vel.

Spanwise slip spin $\Delta \omega_y$ versus fiber wall-normal velocity $W_p$ conditionally sampled at the position of the $St=100$ fibers in the region $10 < z+ < 30$
Results: Spanwise slip spin - RMS values

- $\text{St}=1$
- $\text{St}=5$
- $\text{St}=30$
- $\text{St}=100$
Results: Spanwise slip spin vs Streamwise slip velocity

Spanwise slip spin $\Delta \omega_y$ versus Streamwise slip velocity $\Delta u_x$ conditionally sampled at the position of the St=30 fibers in the viscous region $3 < z^+ < 7$. 

$\lambda=1$, $\lambda=3$, $\lambda=10$, $\lambda=50$. 
Concluding remarks

Slip velocity and spin are useful measures of fibers-turbulence interaction in wall-bounded flows: their statistics provide useful indications for modeling turbulent fiber dispersion.

Slip velocity statistics depend both on fiber elongation (quantitatively) and fiber inertia (also qualitatively!)

RMS exceeds the corresponding mean value by roughly 3 to 5 times: the instantaneous slip velocity may thus frequently change sign.

Slip spin is significantly influenced by fiber elongation ("more" than the slip velocity) but inertia has a relatively weak effect on it ("less" than the slip velocity).

The two quantities seem correlated only for small inertia (both translational and rotational).