On the relative motion between rigid fibers and fluid in turbulent channel flow

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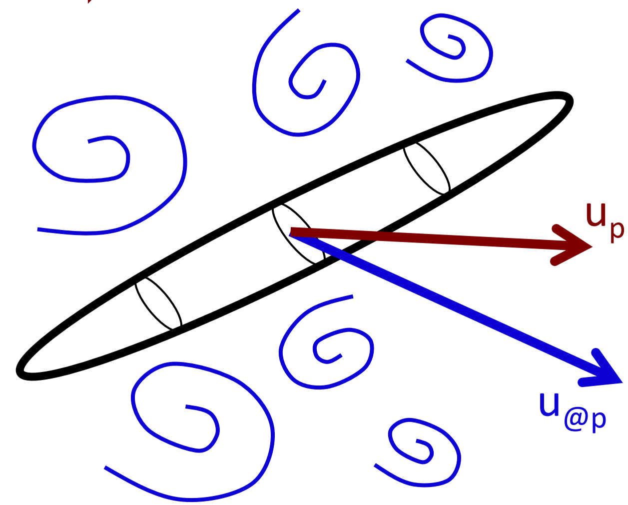
We characterize fiber relative motion with two observables

Relative translational motion

⇨ slip velocity



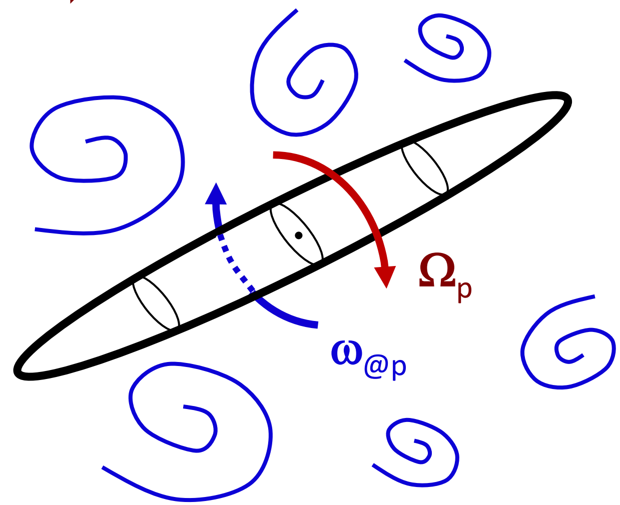
$\mathbf{u}_{@p}$: fluid velocity "seen"

$\mathbf{u}_p$: fiber velocity

$\Delta\mathbf{u}=\mathbf{u}_{@p}- \mathbf{u}_p$: slip velocity

Relative rotational motion

⇨ slip spin



$\boldsymbol{\omega}_{@p}$: fluid angular velocity "seen"

$\boldsymbol{\Omega}_p$: fiber angular velocity

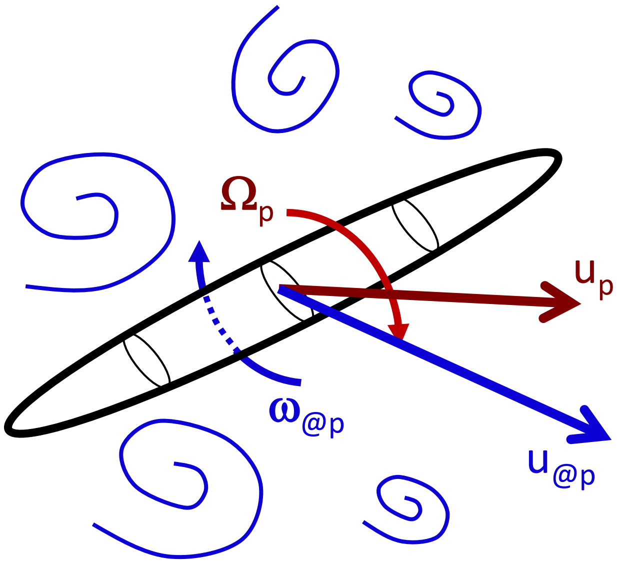
$\Delta\boldsymbol{\omega}=\boldsymbol{\omega}_{@p}- \boldsymbol{\Omega}_p$: slip spin

Refs: Zhao et al. (2014) Phys Fluids, vol 26, 063302
Marchioli et al., J Fluid Mech, In preparation

Motivation: Why are we looking at slip velocity and spin?

Slip velocity and spin are crucial in:

1. Euler-Lagrange simulations:
   - One-way coupling: determine drag and torque experienced by fibers
   - Two-way coupling: determine reaction force/torque from fibers on fluid
2. Two-fluid modeling of particle-laden flows
   - Modeling SGS fiber dynamics in LES flow fields
   - Crossing trajectory effects on time decorrelation tensor of U



$\Delta \mathbf{u} = \mathbf{u}_{@p} - \mathbf{u}_p$: slip velocity

$\Delta \omega = \omega_{@p} - \Omega_p$: slip spin

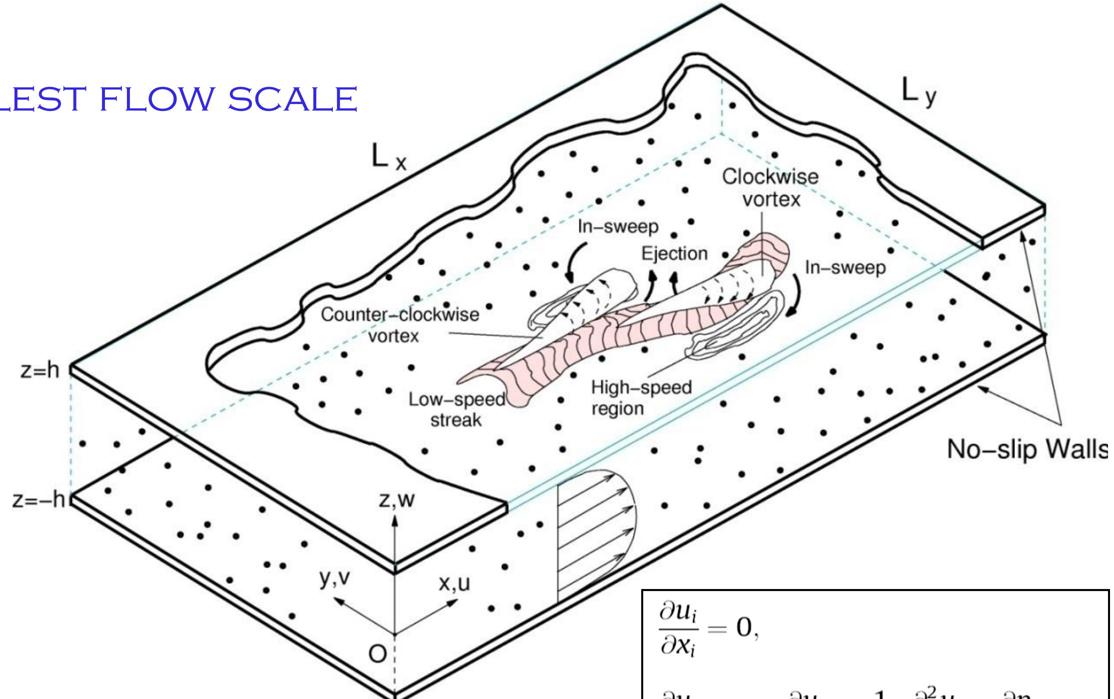
⇨ Aim of this study: statistical characterization of $\Delta \mathbf{u}$ and $\Delta \omega$ at varying fiber inertia and elongation

Problem: Dilute suspension of rigid fibers in turbulent channel

Fibers are modelled as non-deformable prolate ellipsoids evolving in 3D time-dependent fully-turbulent flow (e.g. Marchioli et al, 2010)

Assume:

- Fibers smaller than the smallest flow scale
  - > point-wise
  - > Stokes regime
- Dilute flow
  - > no turbulence modulation
  - > no collisions



$$\frac{\partial u_i}{\partial x_i} = 0,$$

$$\frac{\partial u_i}{\partial t} = -u_j \frac{\partial u_i}{\partial x_j} + \frac{1}{Re_\tau} \frac{\partial^2 u_i}{\partial x_j^2} - \frac{\partial p}{\partial x_i} + \delta_{1,i}$$

Flow solver:

Time-dependent 3D turbulent flow

Channel size: $L_x \times L_y \times L_z = 4\pi h \times 2\pi h \times 2h$

Pseudo-spectral DNS: Fourier modes (1D FFT) in the homogeneous directions,
Chebyshev coefficients in the wall-normal direction

Time integration: Adams-Bashforth (convective terms), Crank-Nicolson (viscous terms)

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Modelling approach: Fiber tracking

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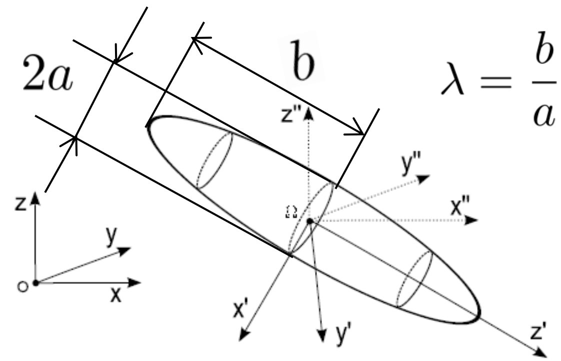
$$
\begin{cases}
\dfrac{\mathrm{d}\mathbf{x}_{p,(G)}}{\mathrm{d}t} = \mathbf{u}_p \\
\dfrac{\mathrm{d}e_0}{\mathrm{d}t} = \dfrac{1}{2}(-e_1\Omega_{x'} - e_2\Omega_{y'} - e_3\Omega_{z'}) \\
\dfrac{\mathrm{d}e_1}{\mathrm{d}t} = \dfrac{1}{2}(e_0\Omega_{x'} - e_3\Omega_{y'} + e_2\Omega_{z'}) \\
\dfrac{\mathrm{d}e_2}{\mathrm{d}t} = \dfrac{1}{2}(e_3\Omega_{x'} + e_0\Omega_{y'} - e_1\Omega_{z'}) \\
\dfrac{\mathrm{d}e_3}{\mathrm{d}t} = \dfrac{1}{2}(-e_2\Omega_{x'} + e_1\Omega_{y'} + e_0\Omega_{z'})
\end{cases}
$$

$$
\begin{cases}
\dfrac{\mathrm{d}\mathbf{u}_p}{\mathrm{d}t} = \dfrac{3}{4\lambda S a^2}\mathbf{K}\cdot(\mathbf{u}_{@p} - \mathbf{u}_p) \\
\dfrac{\mathrm{d}\Omega_{x'}}{\mathrm{d}t} = \Omega_{y'}\Omega_{z'}\left(1 - \dfrac{2}{1+\lambda^2}\right) + \dfrac{20\left[(1-\lambda^2)f' + (1+\lambda^2)(\xi' - \Omega_{x'})\right]}{(\beta_0 + \lambda^2\gamma_0)(1+\lambda^2)Sa^2} \\
\dfrac{\mathrm{d}\Omega_{y'}}{\mathrm{d}t} = \Omega_{x'}\Omega_{z'}\left(\dfrac{2}{1+\lambda^2} - 1\right) + \dfrac{20\left[(\lambda^2-1)g' + (\lambda^2+1)(\eta' - \Omega_{y'})\right]}{(\alpha_0 + \lambda^2\gamma_0)(1+\lambda^2)Sa^2} \\
\dfrac{\mathrm{d}\Omega_{z'}}{\mathrm{d}t} = \dfrac{20}{(\alpha_0 + \beta_0)Sa^2}(\chi' - \Omega_{z'})
\end{cases}
$$



$$\lambda = \frac{b}{a}$$

3 frames of reference:

Inertial, **X**=[x,y,z]

Particle, **X'**=[x',y',z']

Co-moving, **X"**=[x",y",z"]

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Problem: Dilute suspension of rigid fibers in turbulent channel

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Simulation parameters:

- Fluid

Re_τ	Fluid	$\rho_F\ [kg/m^3]$	$\nu\ [m^2/s]$	$h\ [cm]$	$u_\tau\ [m/s]$	$\overline{u_x}\ [m/s]$
150	Air	1.3	$1.57 \cdot 10^{-5}$	2.0	0.11775	1.77
150	Water	1000	$1.00 \cdot 10^{-6}$	0.5	0.03000	0.45

- Particles

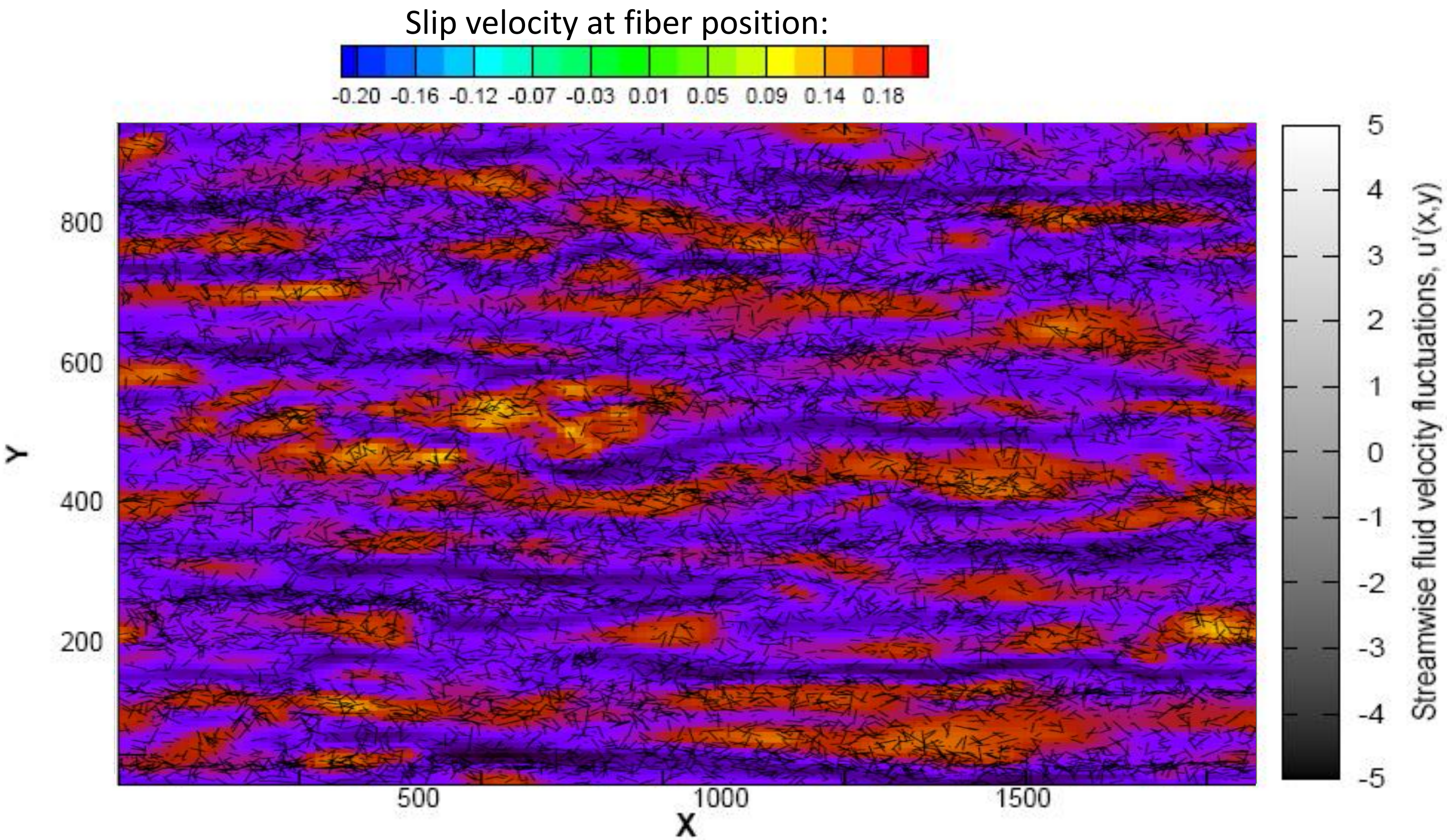
Set	St	λ	S	$2b^+$	(μm)	(kg/m³)
F1-1	1	1.001	34.72	0.72	96.07	45.14
F1-3	1	3	18.57	2.16	287.93	24.14
F1-10	1	10	11.54	7.20	960.09	15.01
F1-50	1	50	7.54	36.00	4800.01	9.80
F5-1	5	1.001	173.60	0.72	96.07	225.68
F5-3	5	3	92.90	2.16	287.93	120.77
F5-10	5	10	57.70	7.20	960.09	75.01
F5-50	5	50	37.69	36.00	4800.01	49.00
F30-1	30	1.001	1041.70	0.72	96.07	1354.21
F30-3	30	3	557.10	2.16	287.93	724.23
F30-10	30	10	346.30	7.20	960.09	450.19
F30-50	30	50	226.15	36.00	4800.01	294.00
F100-1	100	1.001	3472.33	0.72	96.07	4514.03
F100-3	100	3	1857.00	2.16	287.93	2414.10
F100-10	100	10	1154.33	7.20	960.09	1500.63
F100-50	100	50	753.83	36.00	4800.01	979.98

$$St = \frac{2Sa^2}{9\nu} f(\lambda)$$

$$f(\lambda) = \frac{\lambda \ln(\lambda + \sqrt{\lambda^2 - 1})}{\sqrt{\lambda^2 - 1}}$$

Results: Near-wall fiber preferential distribution

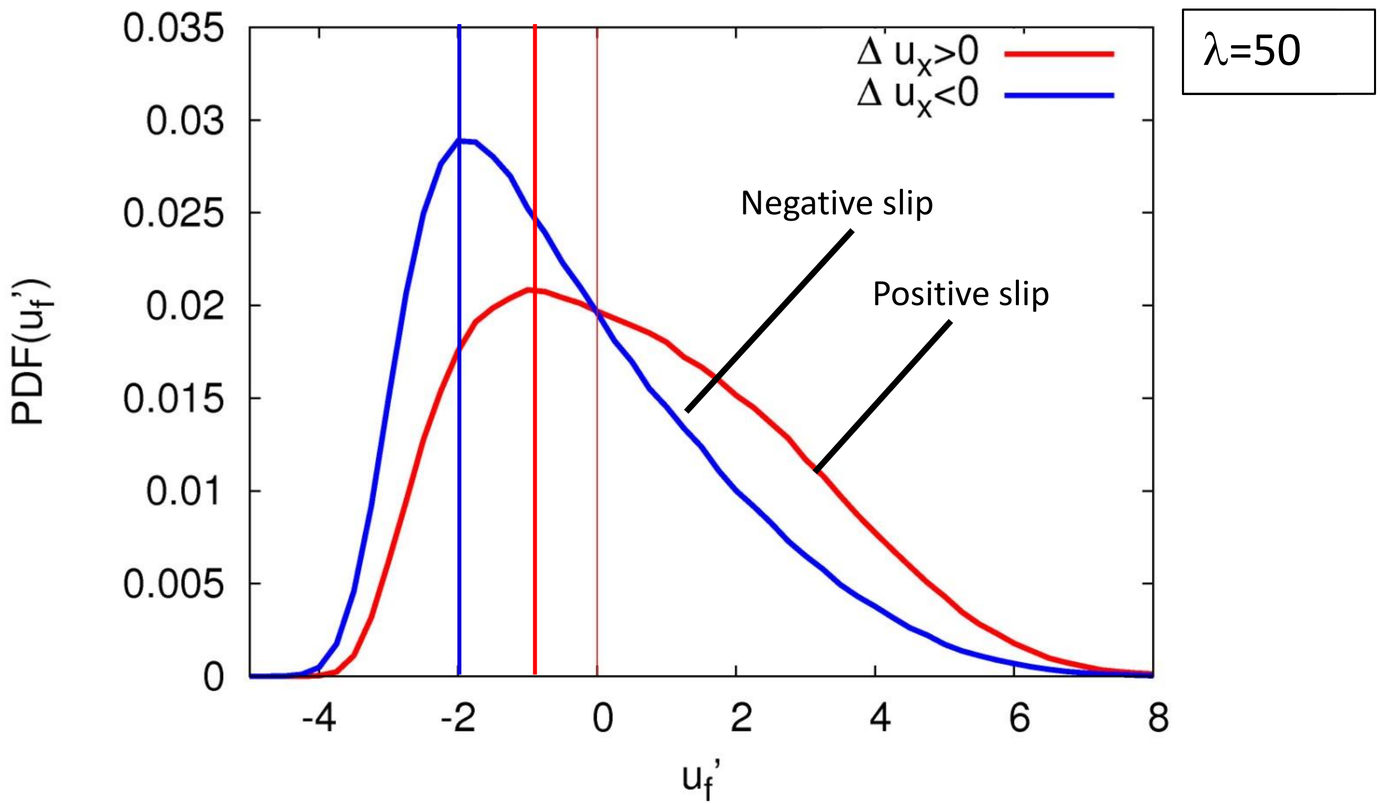
Top view: fibers accumulate in fluid Low-Speed Streaks



Sample snapshot for τ^+=30, λ=50 fibers

Results: Using slip velocity to examine fiber accumulation in LSS

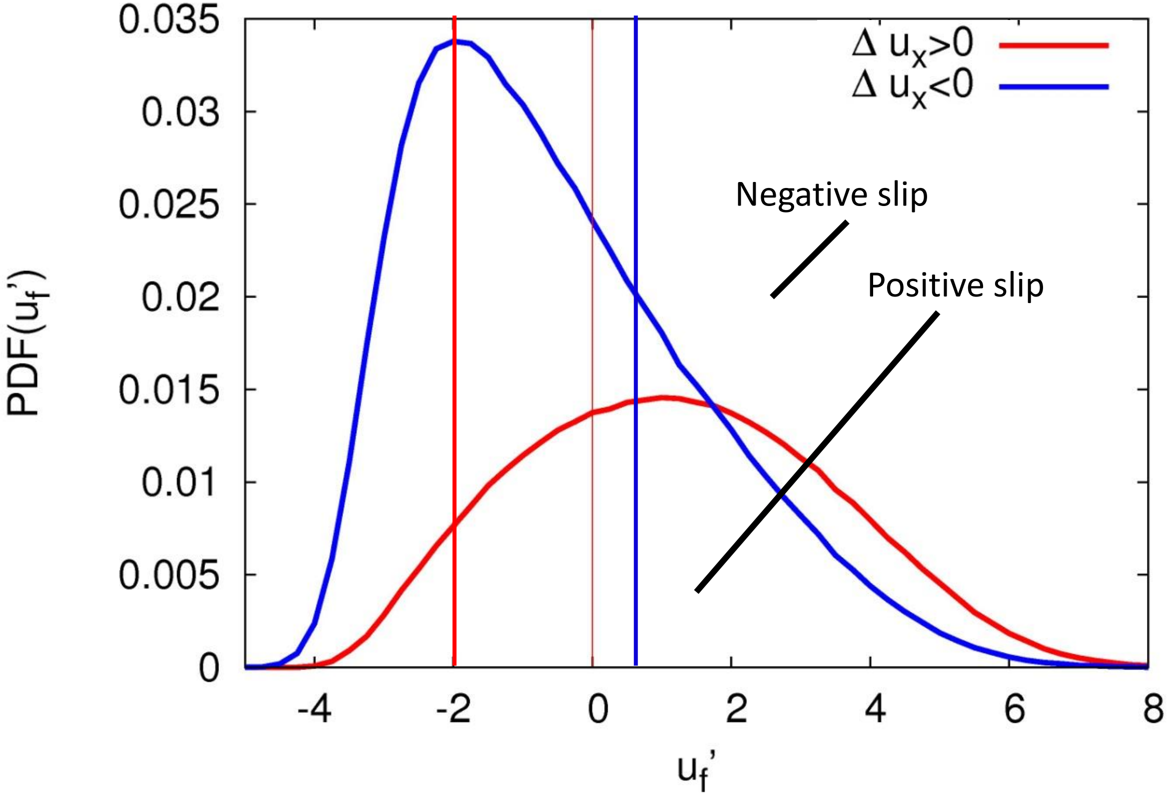
Effect of fiber elongation on conditioned PDF(u_f') – St=30



The influence of λ is not dramatic: only a change in the peak values is observed (no PDF shape change)

Results: Using slip velocity to examine fiber accumulation in LSS

Effect of fiber inertia on conditioned PDF(u_f')



St=100

Significant PDF shape change with curve "inversion" between St=5 and St=30

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Results: Streamwise slip velocity - Mean and RMS values

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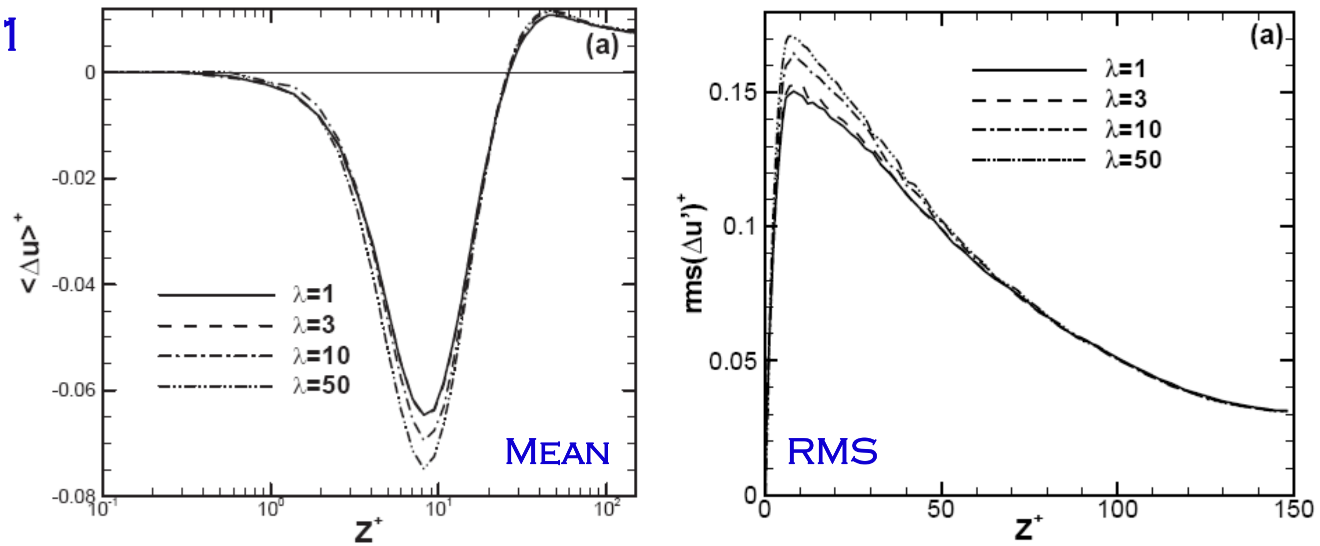
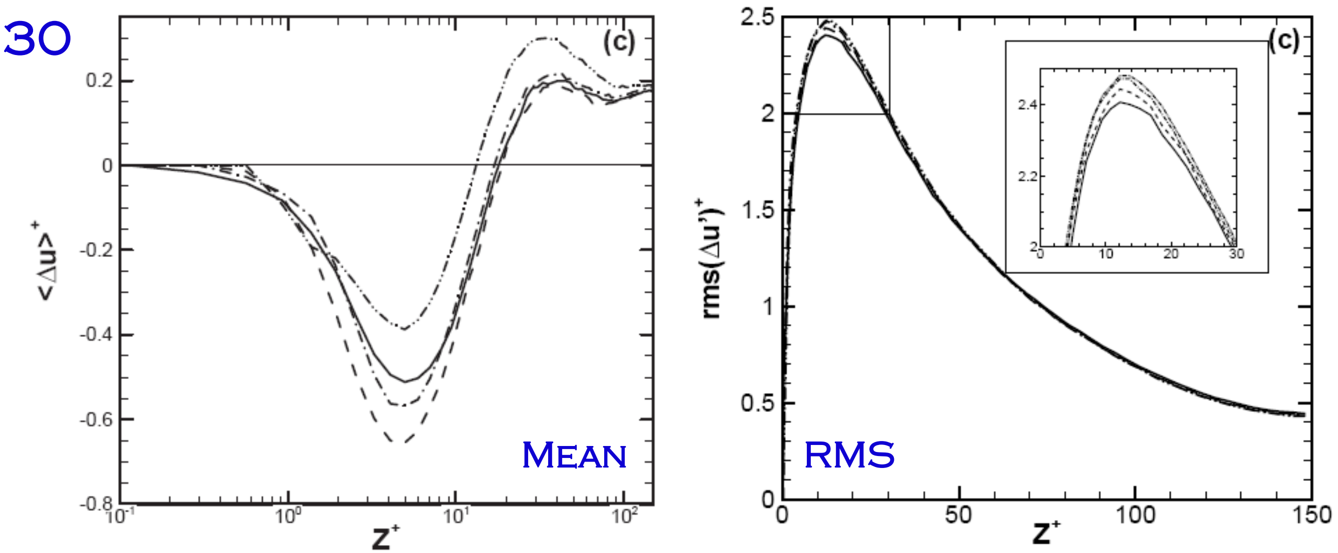
St= 1

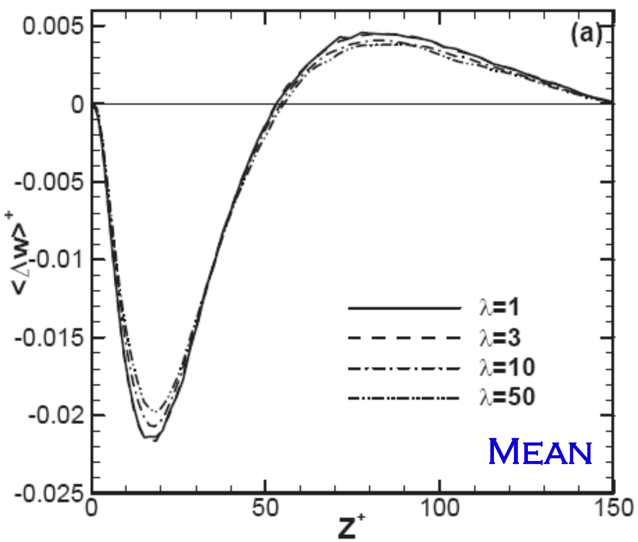


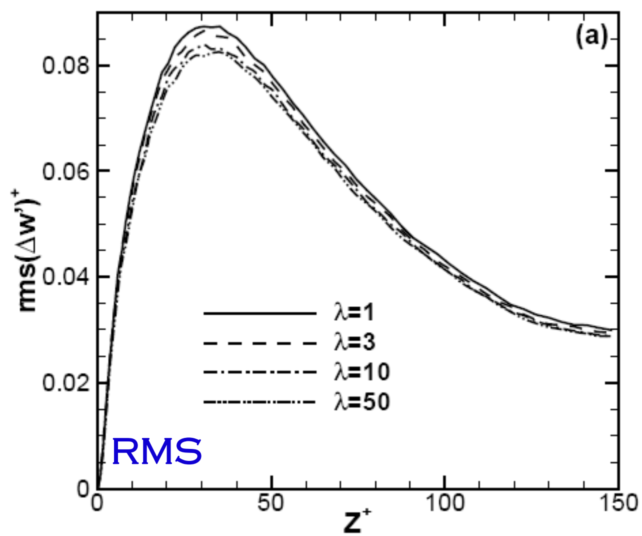
St=30



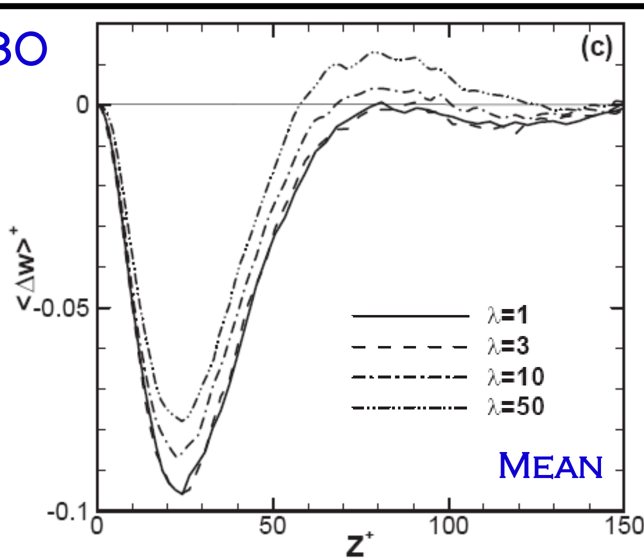
Results: Wall-normal slip velocity - Mean and RMS values

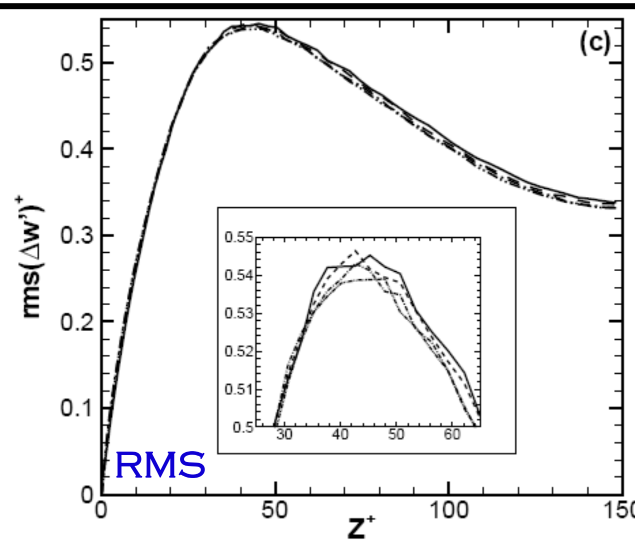
St=1



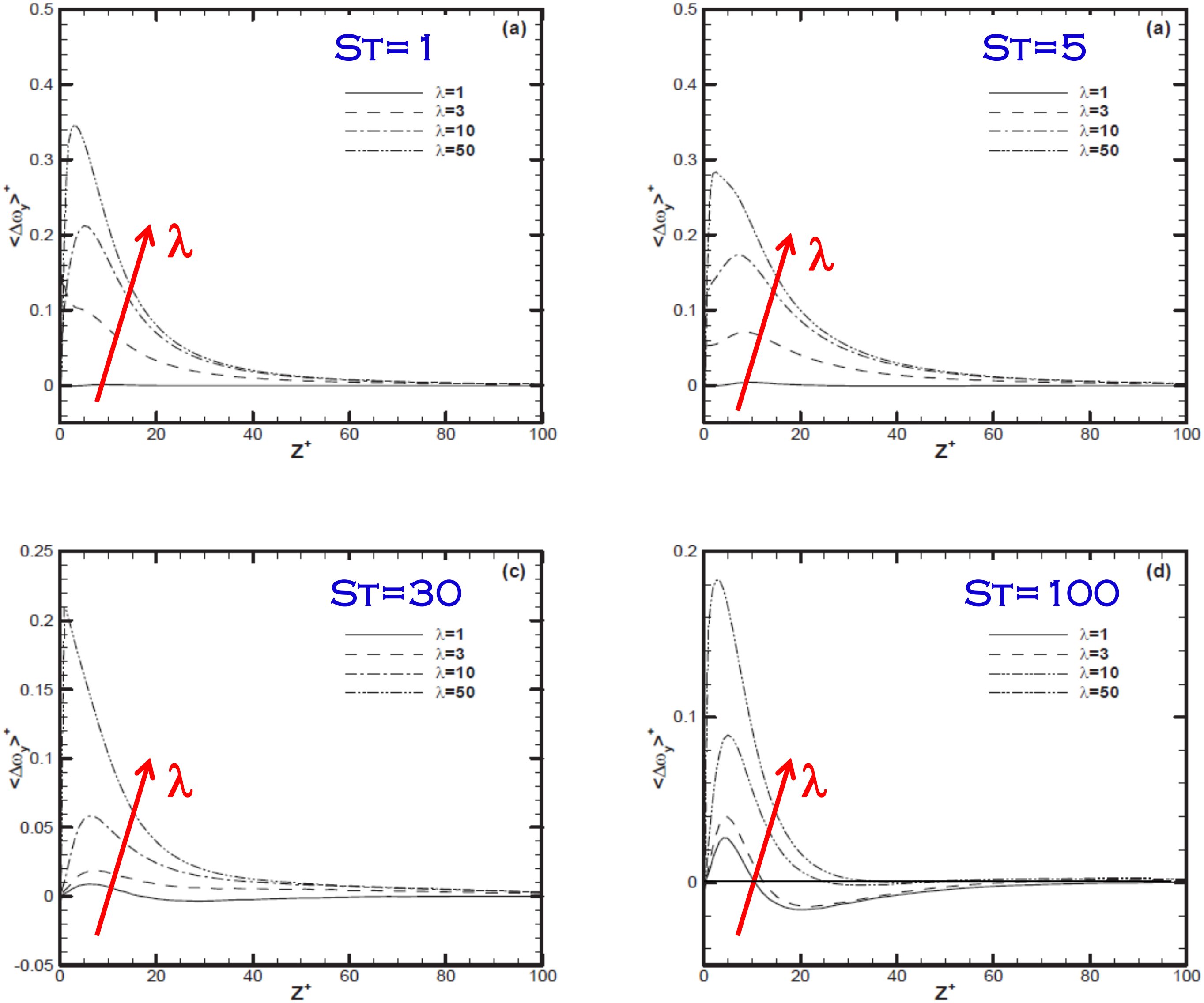


St=30

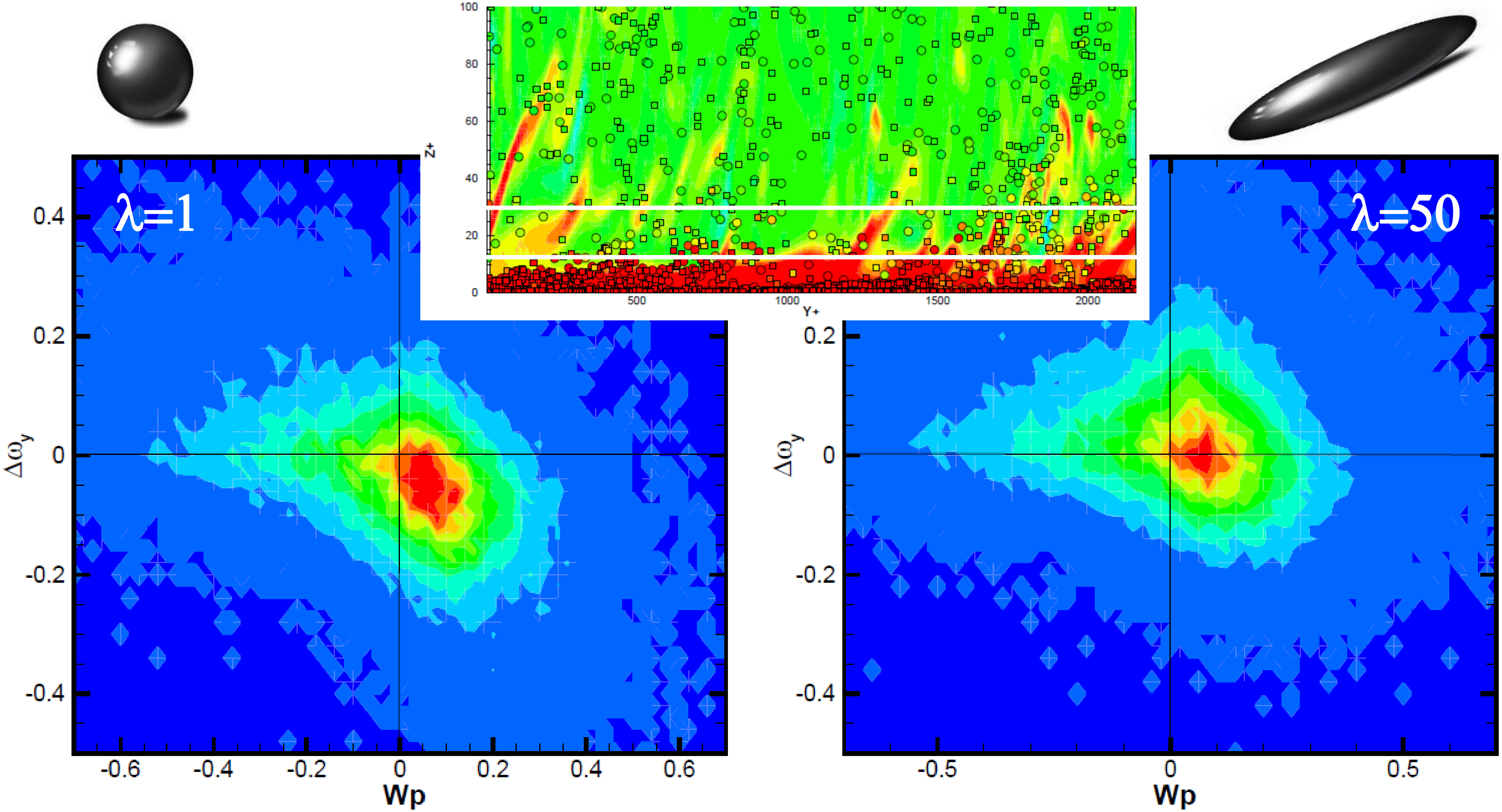




Results: Spanwise slip spin - Mean values

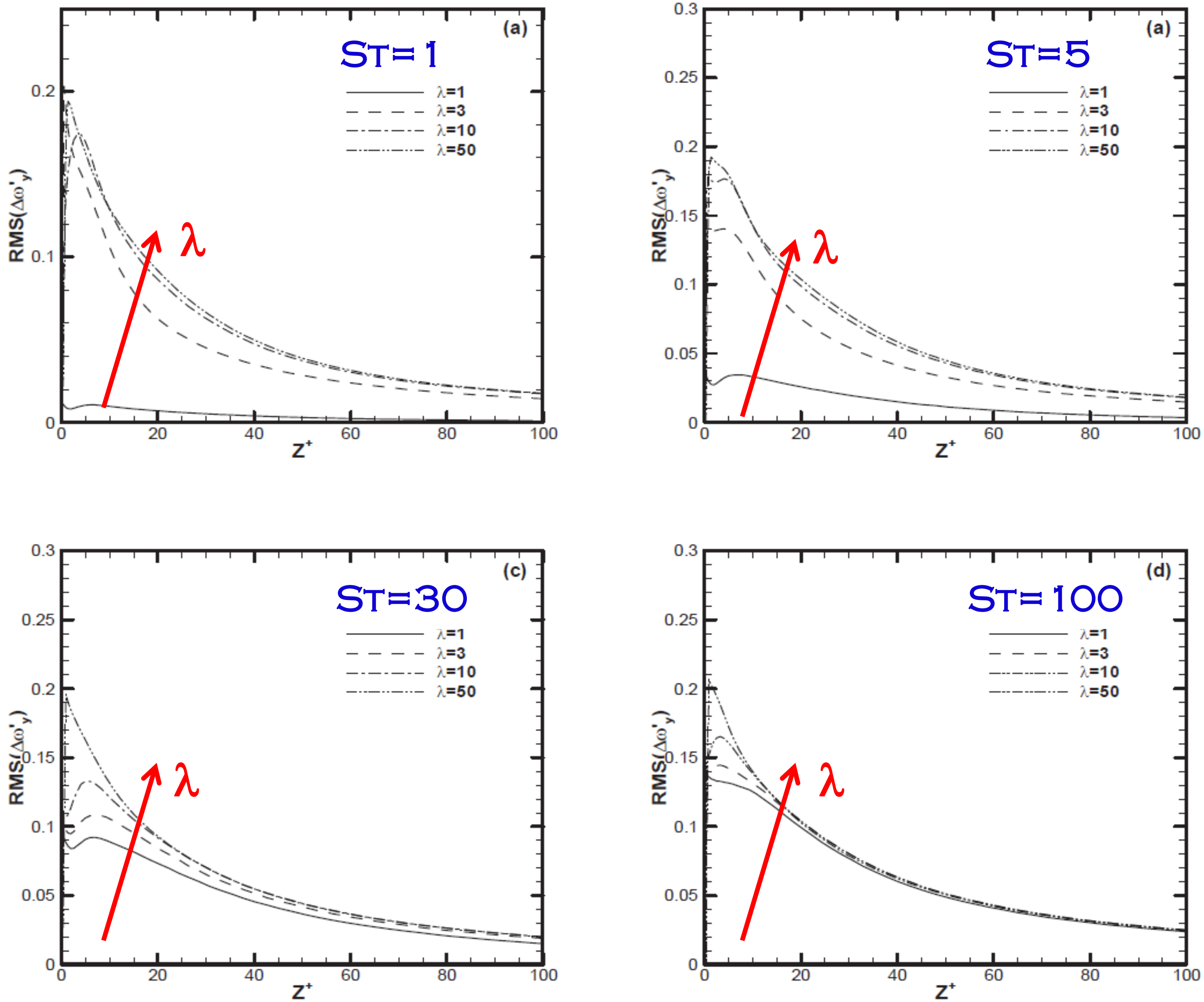


Results: Spanwise slip spin - Correlation w wall-normal vel.



spanwise slip spin $\Delta\omega_y$ versus fiber wall-normal velocity W_p conditionally sampled at the position of the St=100 fibers in the region $10 < z+ < 30$

Results: Spanwise slip spin - RMS values



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Results: Spanwise slip spin vs streamwise slip velocity

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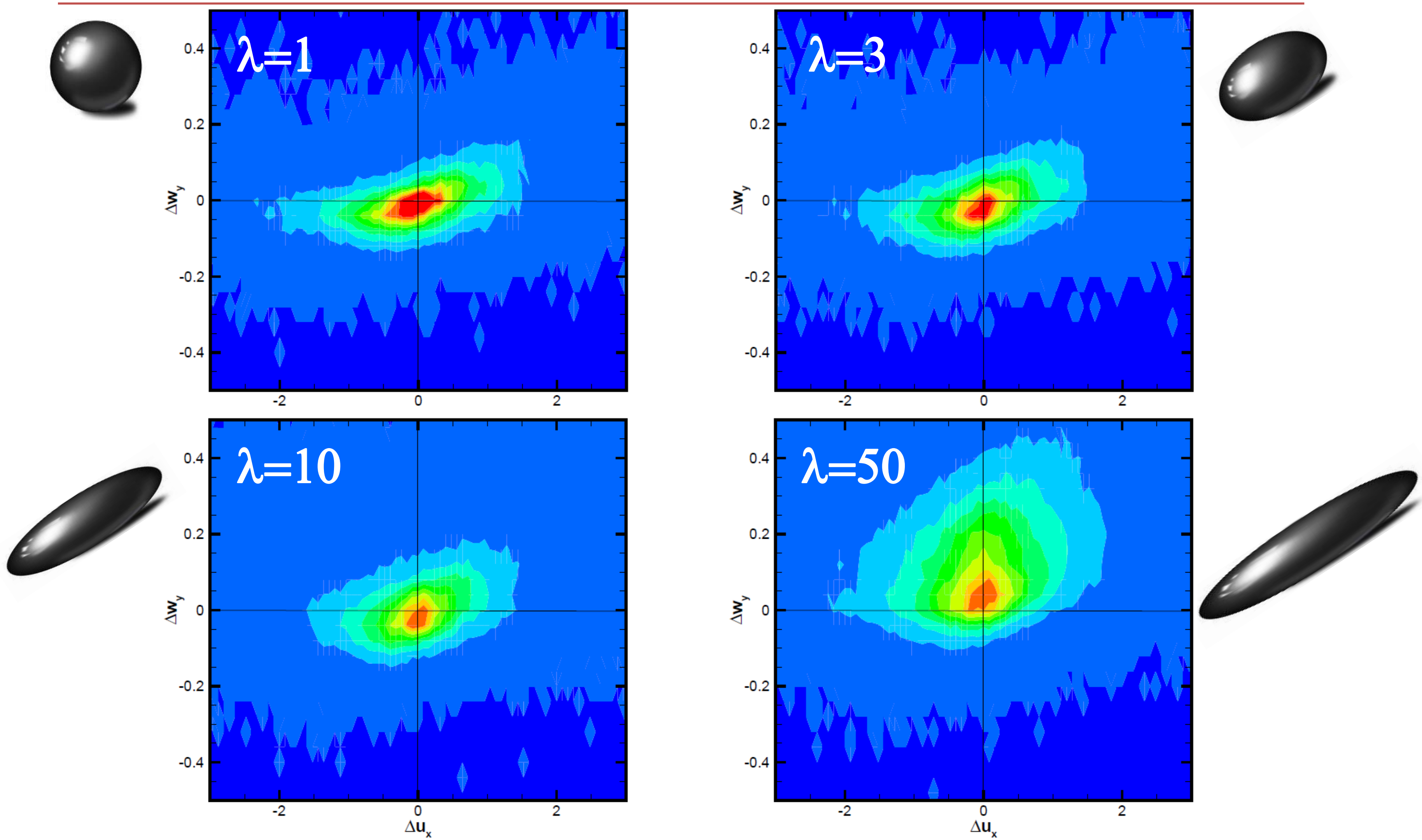

Spanwise slip spin $\Delta\omega_y$ versus streamwise slip velocity Δu_x conditionally sampled at the position of the St=30 fibers **in the viscous region** $3 < z+ < 7$.

Concluding remarks

Slip velocity and spin are useful measures of fibers-turbulence interaction in wall-bounded flows: their statistics provide useful indications for modeling turbulent fiber dispersion

Slip velocity statistics depend both on fiber elongation (quantitatively) and fiber inertia (also qualitatively!)

RMS exceeds the corresponding mean value by roughly 3 to 5 times: the instantaneous slip velocity may thus frequently change sign

Slip spin is significantly influenced by fiber elongation ("more" than the slip velocity) but inertia has a relatively weak effect on it ("less" than the slip velocity)

The two quantities seem correlated only for small inertia (both translational and rotational)