# Particles settling in a cellular flow field at low Stokes number

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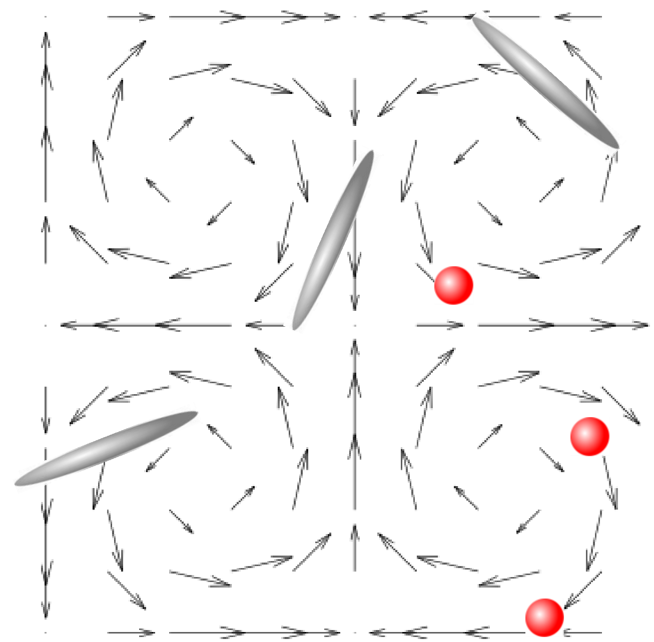
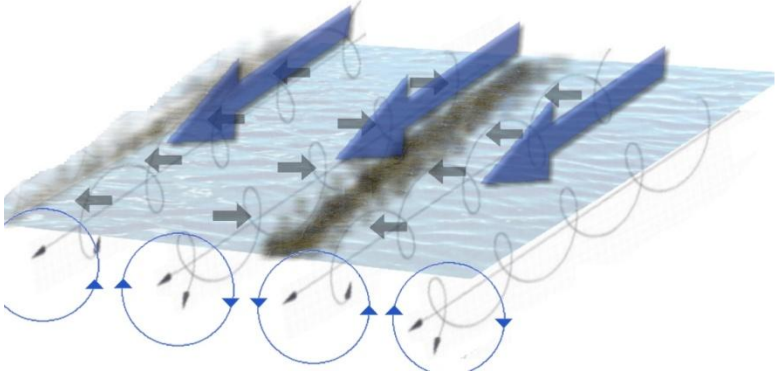
*Caen, October 22, 2014*



# Transport and sedimentation



# Flow turbulence effect on settling particles

## Preferential sweeping and typical trajectories

*Wang & Maxey JFM 1993*

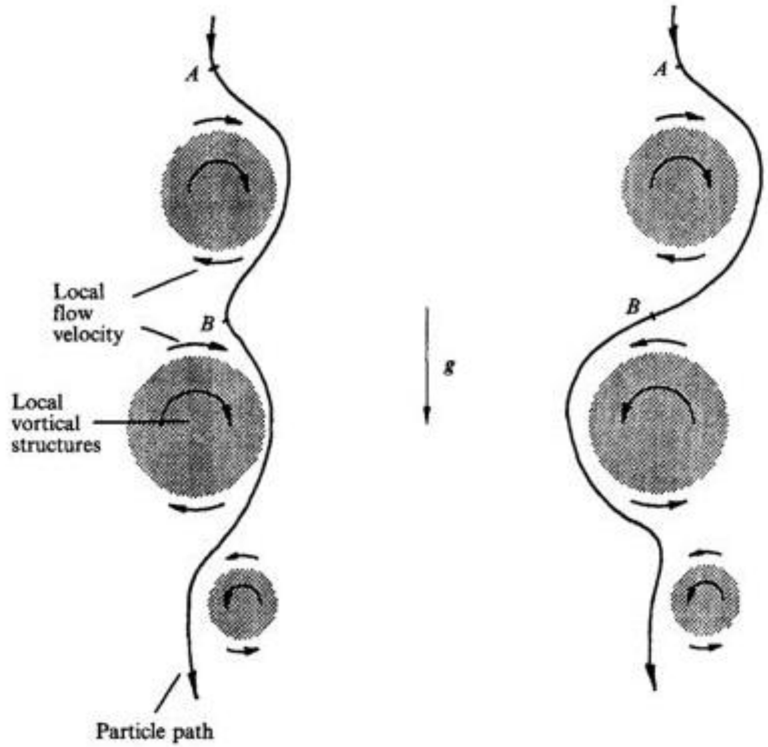



FIGURE 13. Sketch showing the preferential sweeping mechanism for a heavy particle interacting with local flow vortical structures under its inertia and body force.

*Maxey PoF 1987, Aliseda et al. JFM 2002, Yang & Shy PoF 2003, Climent & Maxey IJMF 2003, Bosse et al. PoF 2006, Marchioli et al. PoF 2007 . . .*

I. Experiments

II. Spheres

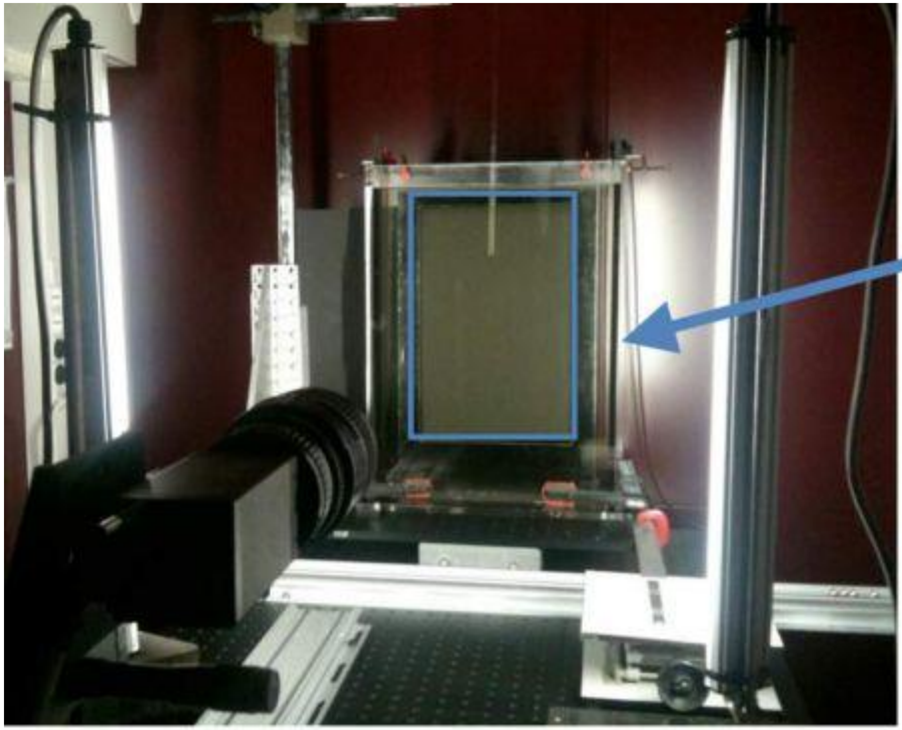
1. Model
2. Particle trajectories and results as inertia increases

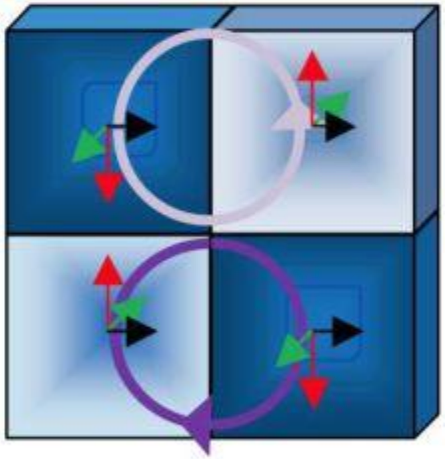
III. Fibres

1. Model
2. Particle trajectories and preliminary results

# Experimental setup

## Flow field generated by electroconvection





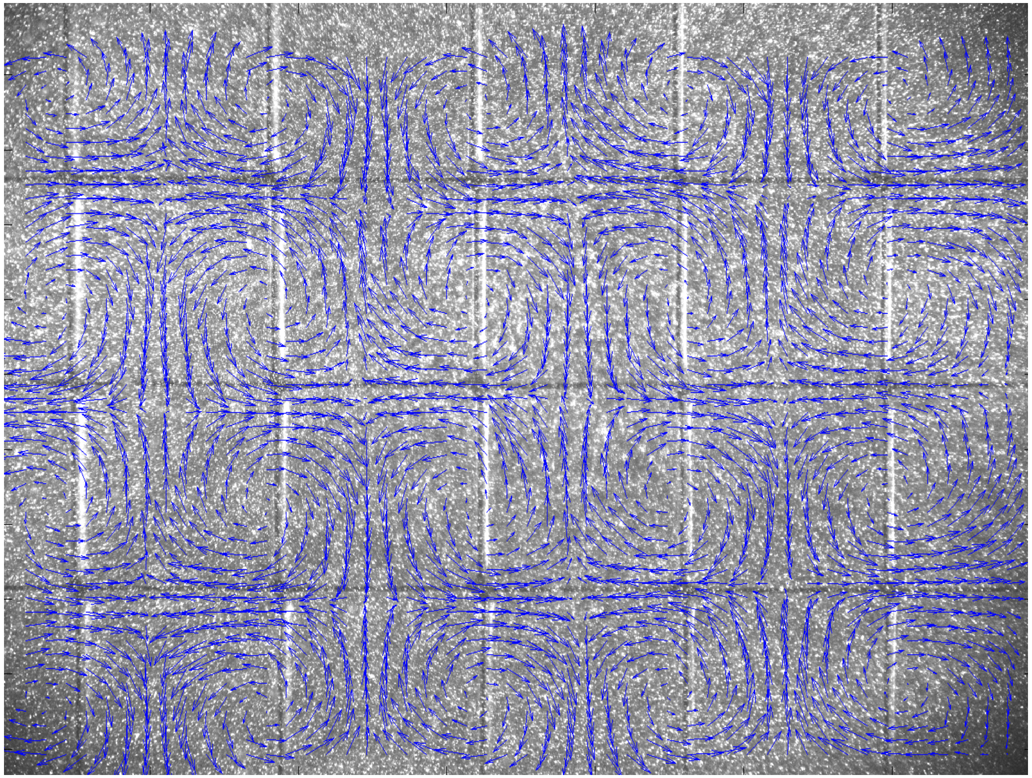
Magnetic filed (permanent magnets)

Electric current

Water + citric acid + Ucon Oil® : possibility to control density and viscosity

Electromagnetic forcing $\propto \mathbf{j} \times \mathbf{B}$

# Experimental flow



PIV measurements
velocities from 0.5 to 10 mm/s

Stationary flow *Tabeling et al., EPL 1987*

Quasi-2D flow modeled by Taylor-Green Vortices

$$\psi = U_0 \frac{L}{\pi} \sin(\pi x/L) \sin(\pi y/L)$$

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# Model

## Advection of a small particle in an unstationary flow: Dimensionless Basset-Boussinesq-Oseen equation

*Gatignol JMTA 1983*
*Maxey & Riley PoF 1983*

Using size *L* and velocity $U_0$ of the vortices

$$\begin{aligned}\frac{d\mathbf{v}}{dt} &= \frac{1}{St}(\mathbf{W}+\mathbf{u}-\mathbf{v}+\frac{P^2}{6}\nabla^2\mathbf{u}) \\ &+ R(\frac{3}{2}\mathbf{u}\cdot\nabla\mathbf{u}+\frac{P^2}{20}\mathbf{u}\cdot\nabla\,\nabla^2\mathbf{u}) \\ &+ \frac{3}{\sqrt{2}}\sqrt{\frac{R}{St}}\int_0^t \frac{-d\mathbf{v}/d\tau+\mathbf{v}\cdot\nabla\mathbf{u}+\frac{P^2}{6}\mathbf{v}\cdot\nabla\nabla^2\mathbf{u}}{\sqrt{\pi(t-\tau)}}d\tau\end{aligned}$$

$$St = \frac{2}{9}\frac{(\rho_p+\rho_f/2)a^2U_0}{\mu L} = \frac{\text{Particle response time}}{\text{Flow time scale}}$$

$$P = a/L \ll 1$$

$$\mathbf{W} = \frac{\mathbf{W_s}}{U_0}$$

$$R = \frac{\rho_f}{\rho_p+\rho_f/2}$$

# Model

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$$
\begin{aligned}
\frac{d\mathbf{v}}{dt} &= \frac{1}{St}\left(\mathbf{W} + \mathbf{u} - \mathbf{v} + \frac{P^2}{6}\nabla^2\mathbf{u}\right) \\
&+ R\left(\frac{3}{2}\mathbf{u}\cdot\nabla\mathbf{u} + \frac{P^2}{20}\mathbf{u}\cdot\nabla\,\nabla^2\mathbf{u}\right) \\
&+ \frac{3}{\sqrt{2}}\sqrt{\frac{R}{St}}\int_0^t \frac{-d\mathbf{v}/d\tau + \mathbf{v}\cdot\nabla\mathbf{u} + \frac{P^2}{6}\mathbf{v}\cdot\nabla\nabla^2\mathbf{u}}{\sqrt{\pi(t-\tau)}}d\tau
\end{aligned}
$$

$$St = \frac{2}{9}\frac{(\rho_p+\rho_f/2)a^2U_0}{\mu L} = \frac{\text{Particle response time}}{\text{Flow time scale}}$$

$$P = a/L \ll 1$$

$$\mathbf{W} = \frac{\mathbf{W_s}}{U_0}$$

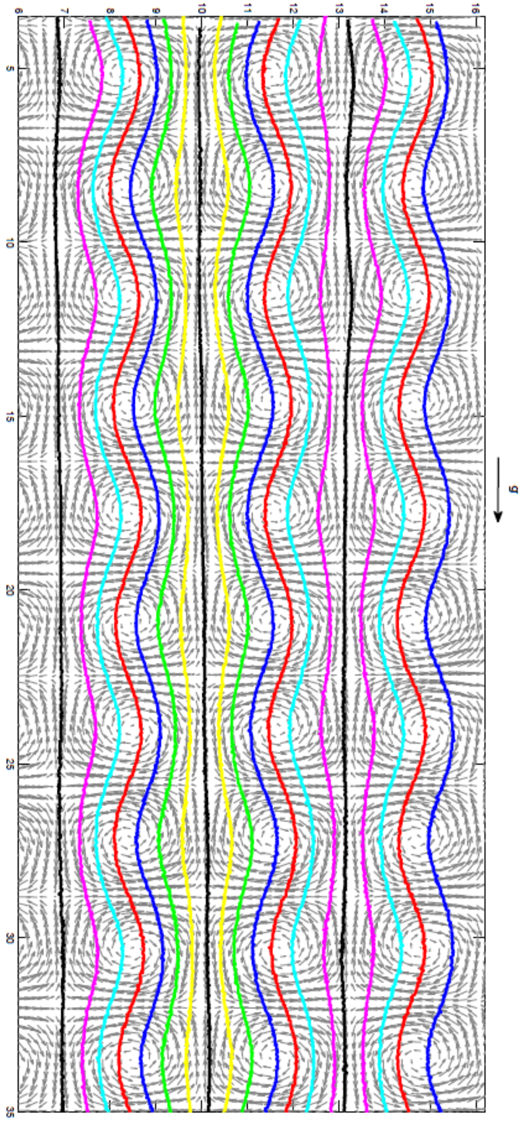
$$R = \frac{\rho_f}{\rho_p+\rho_f/2}$$

Low Stokes number: Drag – Buoyancy only $\quad \mathbf{v} = \mathbf{u} + \mathbf{W}$

Stommel J Mar Res 1949

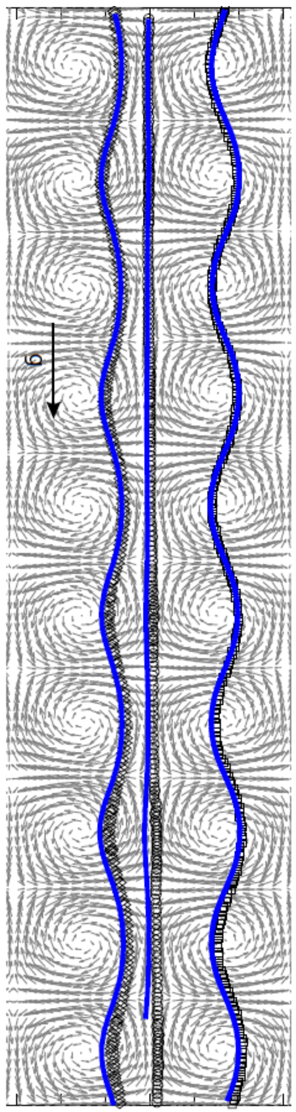
# Experimental results

**St << 1 and Particle Reynolds number $Re_p$ << 1**




Experiment

Periodic motion
No preferential path

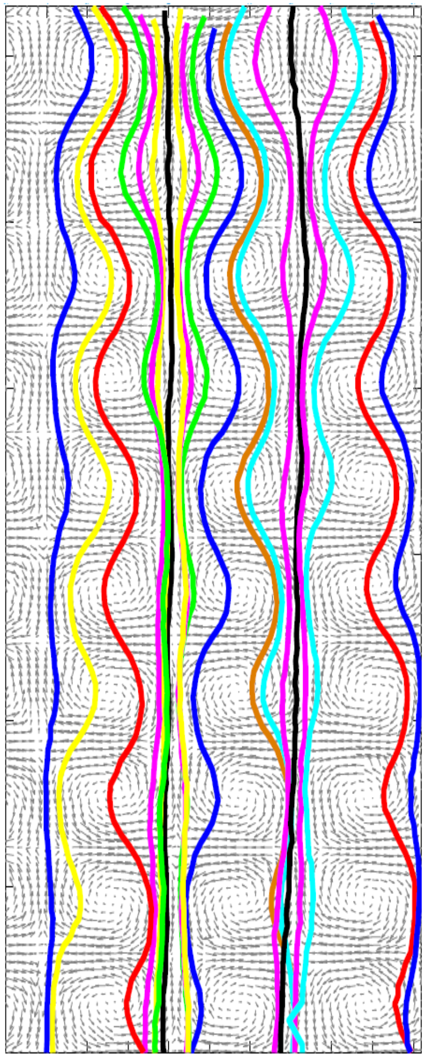


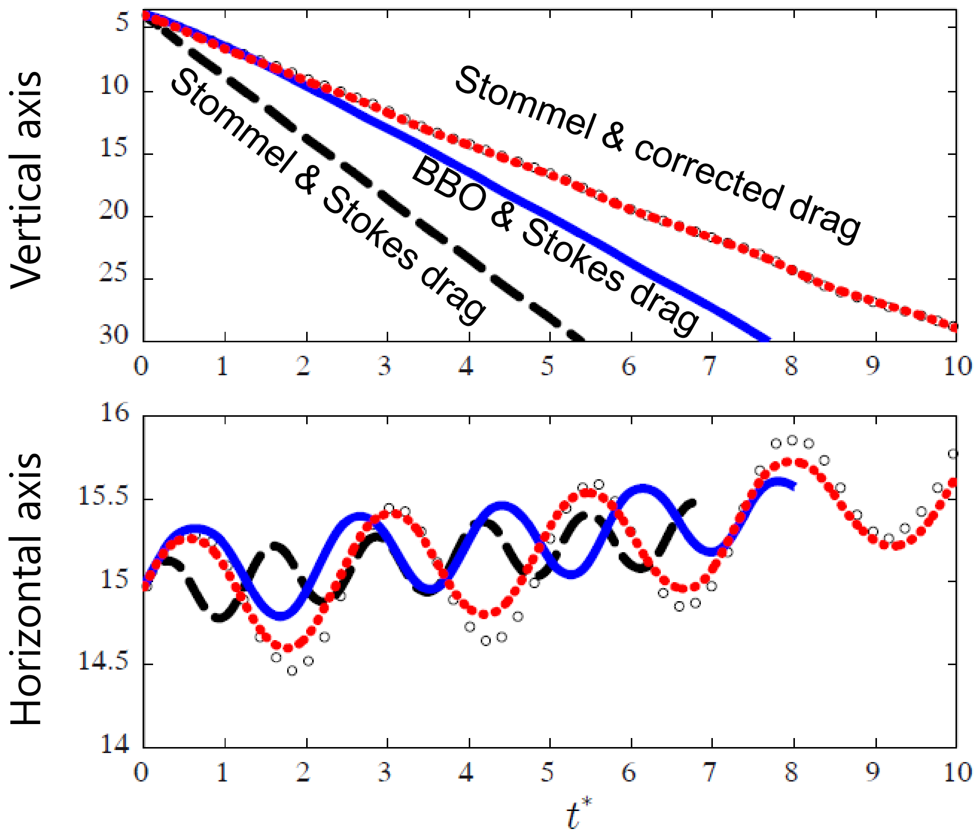

Model (Stommel)

$$\boxed{\mathbf{v} = \mathbf{u} + \mathbf{W}}$$

# Experimental results

**St ~ 0.02 and $Re_p$ ~ 5**






$$\boxed{\mathbf{v} = \mathbf{u} + \mathbf{W}_{\mathrm{SN}}(Re_p)}$$

Dominant effect of $Re_p$: modified drag (Schiller-Naumann)

*Bergougnoux et al., Phys. Fluids 2014*

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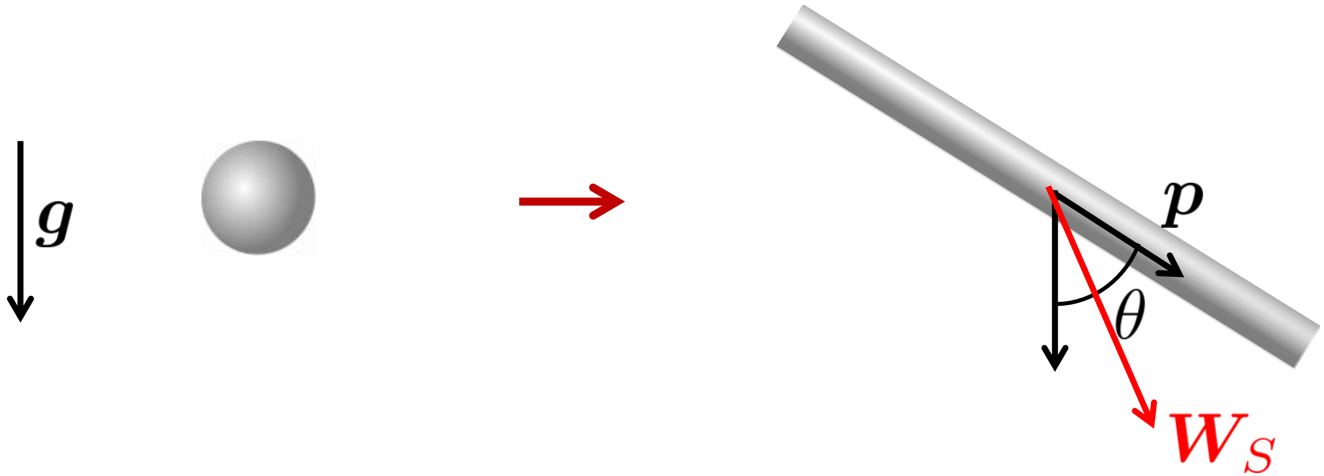
# Effect of anisotropy




Non-zero lift for $\theta \neq 0$ and $\frac{\pi}{2}$

Settling velocity and direction depend on orientation

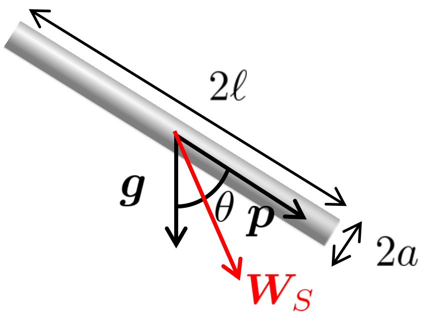
Tumbling dynamics can be chaotic

*Mallier & Maxey PoF 1991*

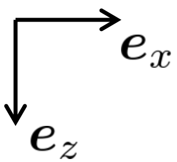
# Low Stokes number regime

*Slender body theory: Jeffery 1922, Batchelor 1970, Cox 1970*

## Velocity: Stommel model




$$\boxed{\mathbf{v} = \mathbf{u} + \mathbf{W}}$$




$$W_z = W_0(1+\cos^2\theta)\left[1+\left(\ln 4 - \frac{1+5\cos^2\theta}{2+2\cos^2\theta}\right)\frac{1}{\ln A}\right] + O\left(\frac{1}{(\ln A)^3}\right)$$

$$W_x = W_0 \sin\theta\cos\theta\left[1+\left(\ln 4 - \tfrac{5}{2}\right)\frac{1}{\ln A}\right] + O\left(\frac{1}{(\ln A)^3}\right)$$

$$A = \frac{\ell}{a}(\gg 1)$$

$$W_0 = \frac{(\rho_p - \rho_f)a^2 g \ln A}{4\mu U_0}$$

## Orientation

$$\frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} = \left[\boldsymbol{\Omega} + \frac{A_e^2 - 1}{A_e^2+1}\,\mathbf{E}\right].\boldsymbol{p}$$

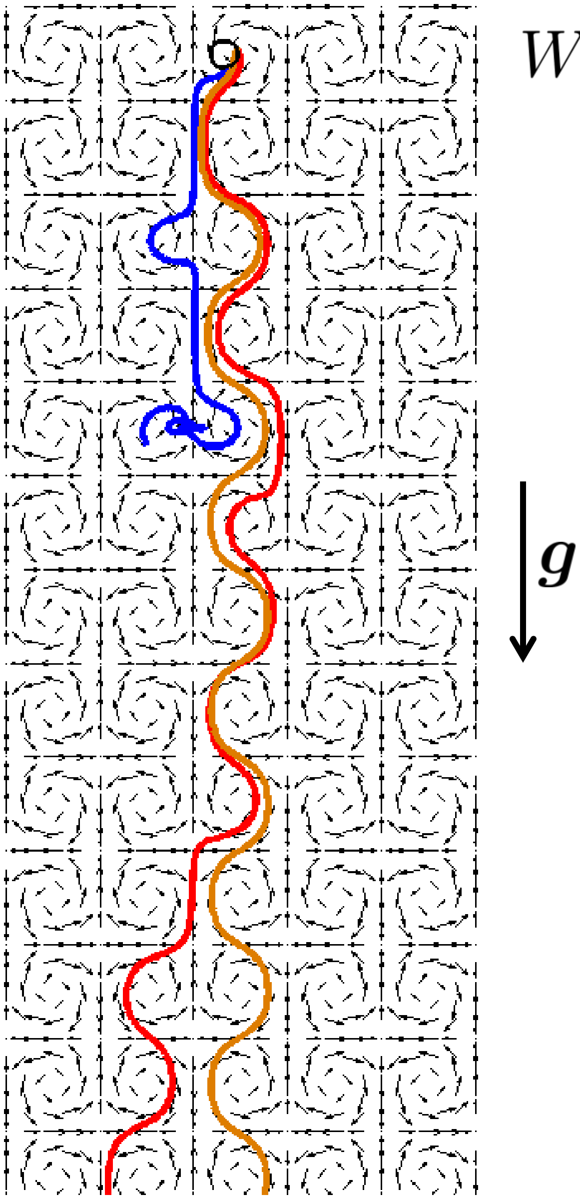
$$\boldsymbol{\Omega} = \frac{1}{2}\left(\nabla \boldsymbol{U} - {}^t\nabla\boldsymbol{U}\right)$$

$$\mathbf{E} = \frac{1}{2}\left(\nabla \boldsymbol{U} + {}^t\nabla\boldsymbol{U}\right)$$

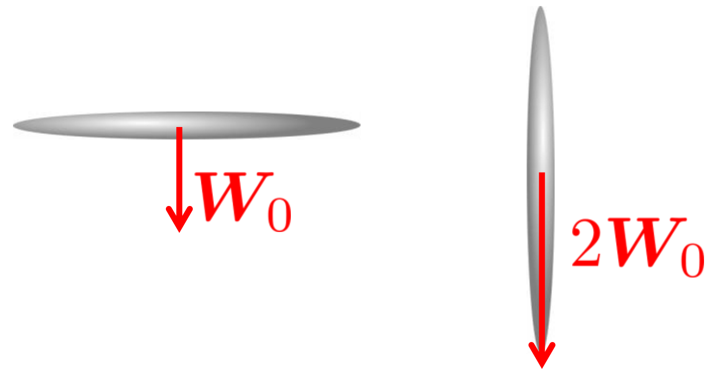
$$A_e \approx 0.7A$$

*Mason PRSA 1956*

# Numerical results (1)



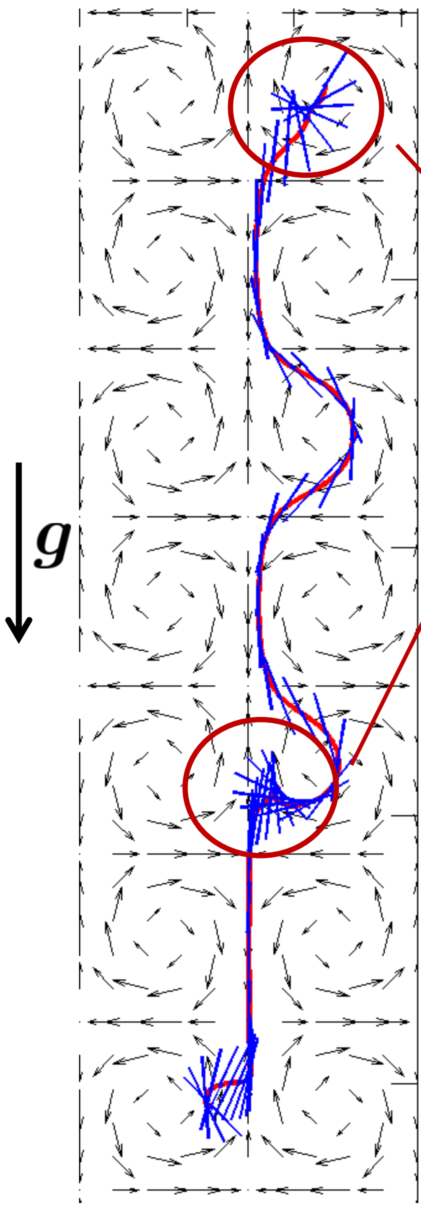

$W_0 = 0.5$ : Non-dimensional settling speed




**Chaotic behavior**

*Mallier & Maxey PoF 1991*

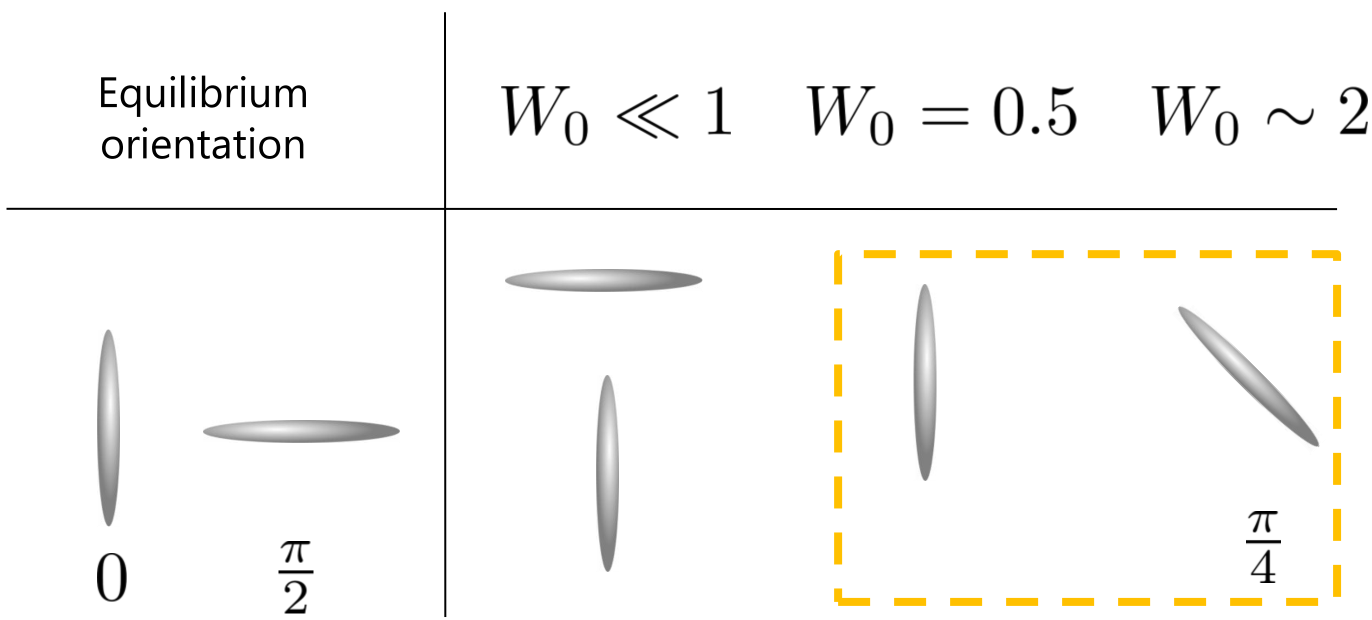
# Numerical results (2)




$W_0 = 0.5$

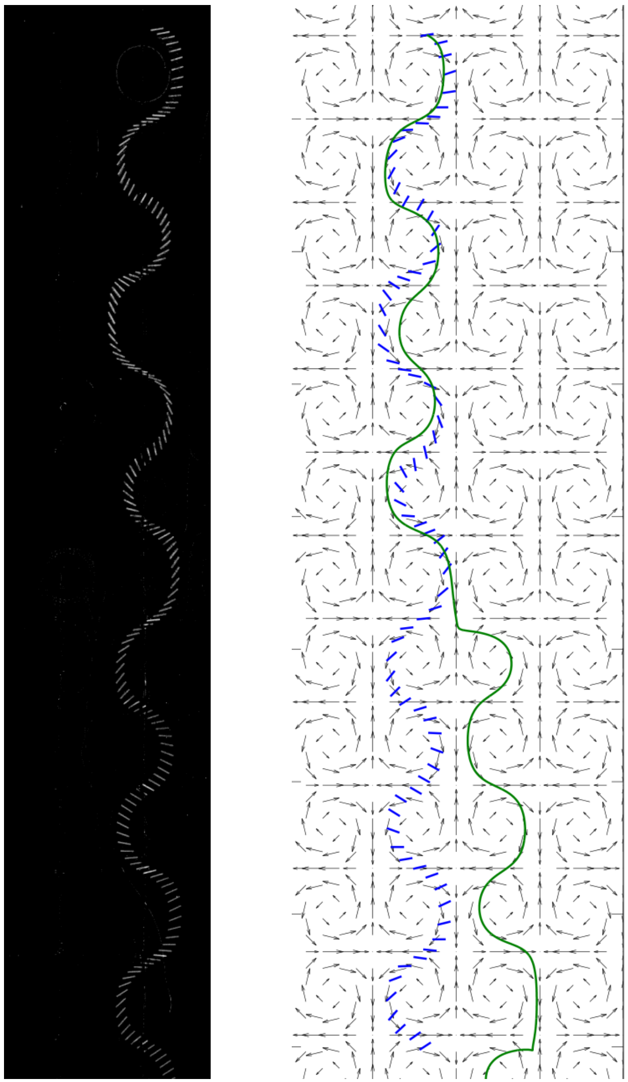
Fast tumbles

Dominant orientations
(in Stokes regime!)



# Preliminary experimental results

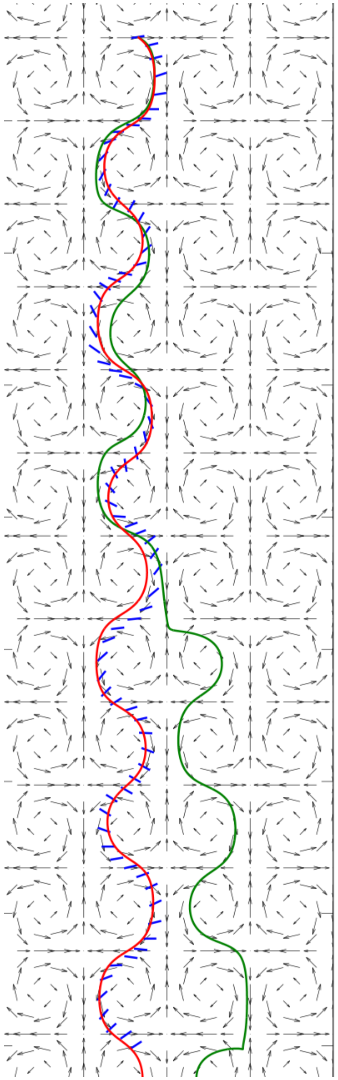
St << 1 and $Re_p$ ~ 0.1



First order drag

# Preliminary experimental results

St << 1 and $Re_p \sim 0.1$

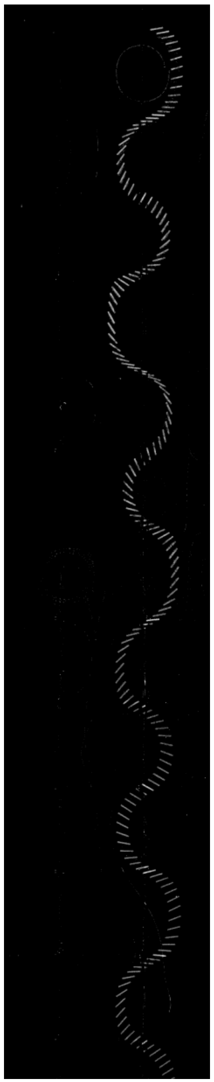


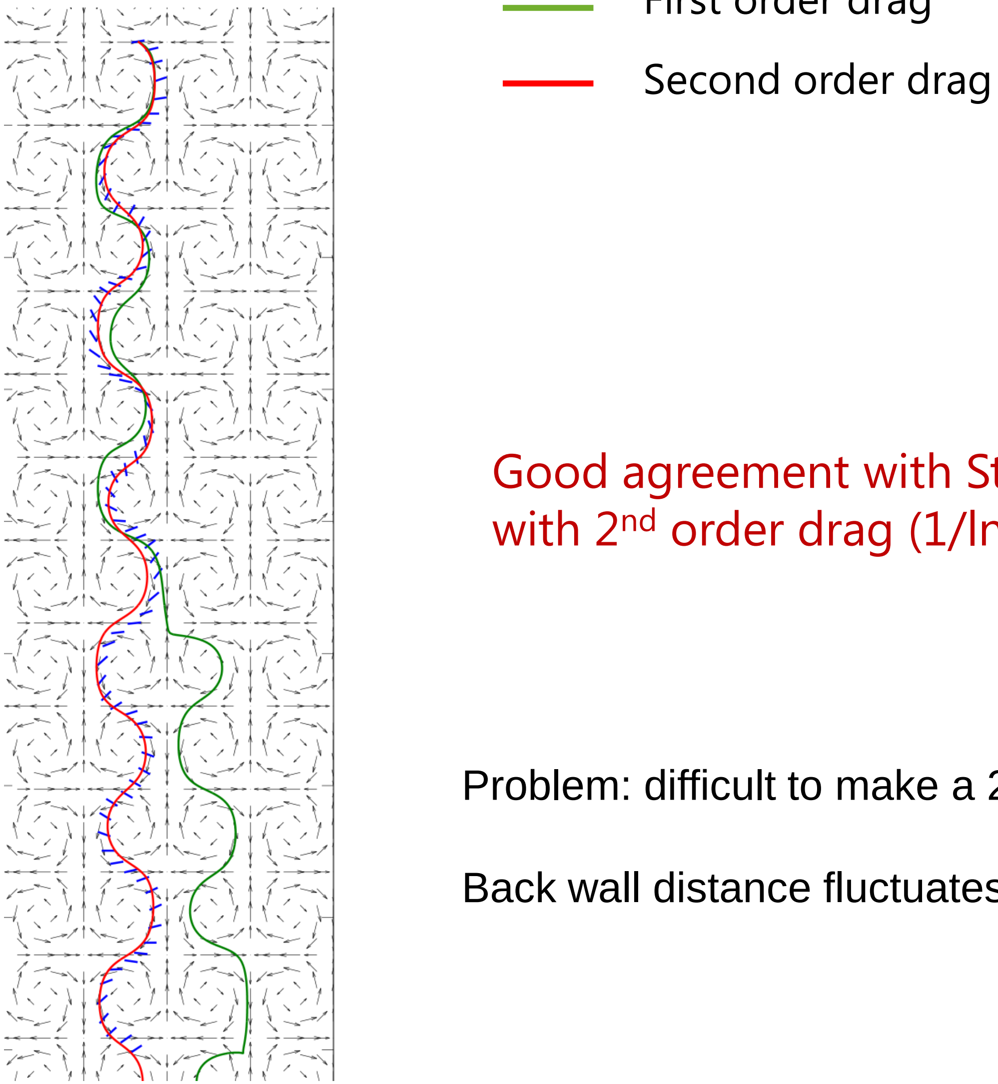
First order drag

Second order drag

# Preliminary experimental results

St << 1 and $Re_p$ ~ 0.1





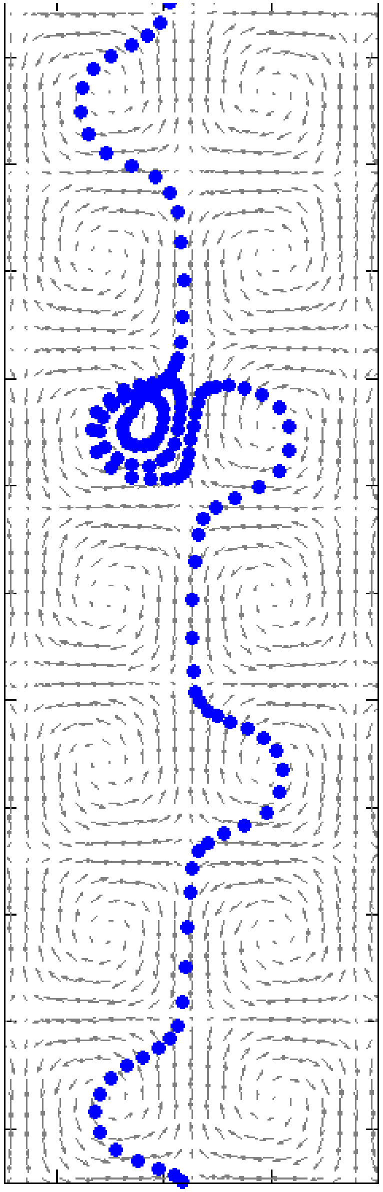


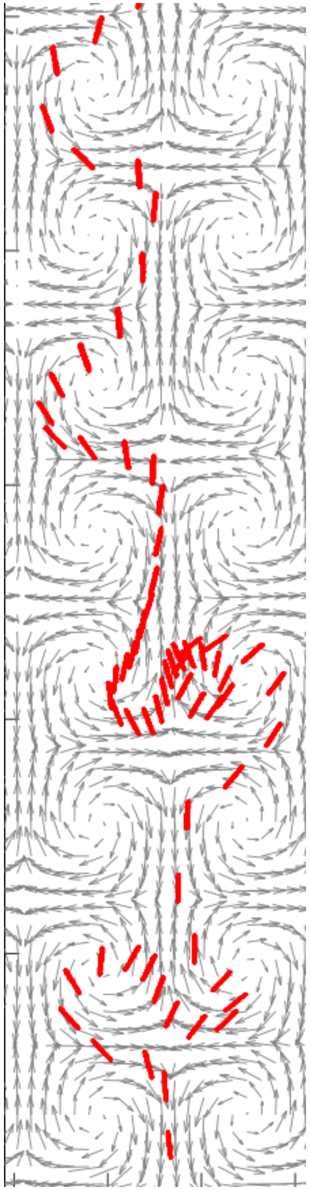
Good agreement with Stommel model with $2^{nd}$ order drag ($1/\ln^2(A)$)

Problem: difficult to make a 2D experiment

Back wall distance fluctuates from $\pm 5a$ to whole tank

# Conclusion





## Settling of spheres in cellular flow

Experimental examination of the settling of individual spherical particles in a 2D cellular flow field

Precise testing of the BBO equation of motion
Dominant effect of inertia: drag correction

*Bergougnoux et al., Phys. Fluids 2014*

## Effect of particle anisotropy

Preliminary results: importance of second order corrections on drag

## Ongoing work

Full 3D analysis in the case of fibres
Inertia effects on fibres (drag and orientation)

Trapping and suspension