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Impact of fiber rotation on suspension rheology – a first try

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Man-made and natural particle suspensions

Particulate pollution from factories

Sand storm in a desert

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Rotation (spin) of inertial particles in viscous fluids - I

- **Fibers and other non-spherical particles:** Spin of fiber-like particles is an essential feature of the particle dynamics. The particle spin is strongly coupled to the particle orientation.

  P.H. Mortensen et al., Phys. Fluids 2008
  C. Marchioli et al., Phys. Fluids 2010

- **Spherical particles:** Nobody cares about the orientation of a spherical particle. But the rotation might matter.

Rotation (spin) of inertial particles in viscous fluids - II

- **Spherical particles:** An optical probe invented by Frish & Webb (1981) measures the fluid vorticity. Assumption: solid spheres rotate at the same rate as the local fluid.
- Inertial spheres may rotate slower than the local fluid. Shown in direct numerical simulations (DNS) by Mortensen et al. (2007) and Zhao & Andersson (2011) and in measurements by Bellani et al. (2012).

M.B. Frish & W.W. Webb, J. Fluid Mech. 1981
P.H. Mortensen et al., Phys. Fluids 2007
L. Zhao & H.I. Andersson, Phys. Fluids 2011
G. Bellani et al., J. Fluid Mech. 2012

Microfluids and Micropolar fluids

- **Microfluids:** A fluid medium whose properties and behaviour are affected by the local motion of material particles; Eringen (1964).
- **Micropolar fluids:** A subclass of microfluids that exhibit microrotational effects and microrotational inertia; Eringen (1966).
- A micropolar fluid represents the rheology of fluid suspensions containing bar-like elements, e.g.blod cells.

A.C. Eringen, Int. J. Eng. Sci. 1964
A.C. Eringen, J. Math. Mech. 1966
G. Lukaszewicz: *Micropolar Fluids – Theory and Applications*, 1999

Continuum mechanical modeling - I

Conservation of *linear momentum* (Newton's 2nd law) can be expressed in Cartesian tensor notation as:

$$\rho \frac{D u_i}{D t} = \frac{\partial T_{ji}}{\partial x_j} + \rho f_i$$

This is Cauchy's equation of motion. The Navier-Stokes equation is obtained with Stokes' symmetric stress tensor T_{ij} for an incompressible viscous fluid:

$$T_{ij} = -p\delta_{ij} + \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

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Continuum mechanical modeling - II

A.C. Eringen (1966) showed that the stress tensor for an incompressible microfluid simplified to:

$$T_{ij} = -p\delta_{ij} + \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) + \mu_r\left(\frac{\partial u_j}{\partial x_i} - \frac{\partial u_i}{\partial x_j}\right) - 2\mu_r \varepsilon_{mij}\omega_m$$

The second parenthesis can be written:

$$\left(\frac{\partial u_j}{\partial x_i} - \frac{\partial u_i}{\partial x_j}\right) = 2\Omega_m \varepsilon_{mij}$$

where the angular velocity is: $\vec{\Omega} = 1/2\nabla \times \vec{u}$ $\quad \Omega_m = 1/2\varepsilon_{mji}\partial u_i / \partial x_j$

We have used the identity: $\varepsilon_{ijk}\varepsilon_{rsk} = \delta_{ir}\delta_{js} - \delta_{is}\delta_{jr}$

Continuum mechanical modeling - III

Eringen's expression can be rewritten as:

$$T_{ij} = -p\delta_{ij} + \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) + \underbrace{2\mu_r \varepsilon_{mij}\left(\Omega_m - \omega_m\right)}_{T_{ij}^P}$$

Micropolar rheology is associated with the last term:

$$T_{ij}^P = 2\mu_r \varepsilon_{mij}\left(\Omega_m - \omega_m\right)$$

The *micro-rotation* **ω** should be obtained from a separate transport equation for the angular momentum.

Lagrangian modeling of inertial spheres - I

Rotational motion of a spherical particle (Euler's equation) :

$$I\frac{d\vec{\omega}}{dt}=\vec{N}$$

The torque vector ***N*** acting on the particle from the surrounding Newtonian fluid can be expressed as:

$$\vec{N}=8\pi\mu a^{3}\left(\vec{\Omega}-\vec{\omega}\right)$$

$\vec{\Omega}=1/2\nabla\times\vec{u}$ is the angular velocity of the fluid.

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Lagrangian modeling of inertial spheres - II

Euler's equation for the rotational particle motion:

$$\frac{d\vec{\omega}}{dt} = \frac{1}{\tau_r}\left(\vec{\Omega} - \vec{\omega}\right)$$

Rotational response time:

$$\tau_r = \frac{Da^2\rho}{15\mu}$$

<u>Note:</u> This rotational response time is exactly 3/10 of the relaxation time for translational motion; Mortensen et al. (2007).

Lagrangian modeling of inertial spheres - III

The torque from a single particle on the fluid is $-\vec{N}$. This is equivalent with Newton's principle *action equals reaction.*

$$-\vec{N} = -8\pi\mu\, a^3 \left(\vec{\Omega} - \vec{\omega}\right)$$

To any vector $-N_m$ corresponds an anti-symmetric tensor of 2^{nd} - order that contains the same information as the vector.

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Lagrangian modeling of inertial spheres - IV

The torque vector $-N_m$ can therefore be obtained from a *particle stress tensor* T^P according to:

$$-N_m = -\frac{1}{2}\varepsilon_{mij}T_{ij}^{P}\Delta \qquad T_{ij}^{P} = \frac{1}{\Delta}\varepsilon_{mij}N_m$$

This equivalence holds if the particle stress tensor is *anti-symmetric*: $T_{ij}^{P} = -T_{ji}^{P}$.
Δ is the volume of a small fluid element surrounding the particle, for instance a grid cell volume $\Delta x \Delta y \Delta z$.

Lagrangian modeling of inertial spheres - V

More than one particle may reside in a given fluid element.
Summation should be carried out over the n_p-particles in Δ:

$$T_{ij}^{P} = \frac{1}{\Delta}\sum_{\ell=1}^{n_p} \varepsilon_{mij} N_m = \frac{8\pi\mu a^3}{\Delta}\sum_{\ell=1}^{n_p} \varepsilon_{mij}\left(\Omega_m - \omega_m\right)$$

$$T_{ij}^{P} = \frac{1}{\Delta}\sum_{\ell=1}^{n_p} \begin{vmatrix} 0 & +N_3 & -N_2 \\ -N_3 & 0 & +N_1 \\ +N_2 & -N_1 & 0 \end{vmatrix} = \frac{8\pi\mu a^3}{\Delta}\sum_{\ell=1}^{n_p} \begin{vmatrix} 0 & +(\Omega_3 - \omega_3) & -(\Omega_2 - \omega_2) \\ -(\Omega_3 - \omega_3) & 0 & +(\Omega_1 - \omega_1) \\ +(\Omega_2 - \omega_2) & -(\Omega_1 - \omega_1) & 0 \end{vmatrix}$$

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Continuum versus point-particle formulation - I

Point-particle formulation:

$$T_{ij}^{P} = \frac{1}{\Delta}\sum_{\ell=1}^{n_p} \varepsilon_{mij} N_m = \frac{8\pi\mu\, a^3}{\Delta}\sum_{\ell=1}^{n_p} \varepsilon_{mij}\left(\Omega_m - \omega_m\right)$$

Continuum mechanical formulation (Eringen):

$$T_{ij}^{P} = 2\mu_r \varepsilon_{mij}\left(\Omega_m - \omega_m\right)$$

Assume that all particles inside Δ have the same relative angular velocity. This gives the microrotation viscosity:

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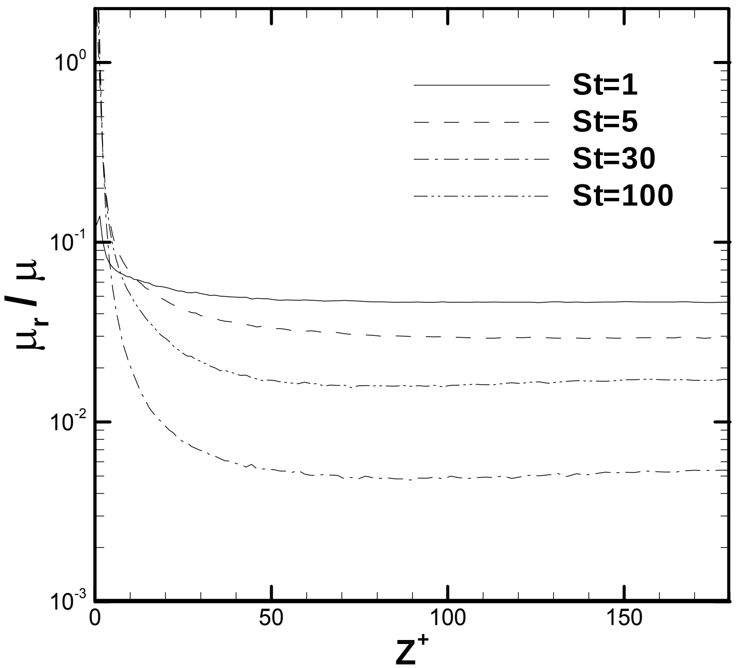
Continuum versus point-particle formulation - II

Microrotation viscosity μ_r :

$$\mu_r = 4\pi\mu a^3 n_p / \Delta = 3\mu V_p n_p / \Delta$$

- depends on μ
- depends on volumetric particle loading $V_p n_p / \Delta$
- no explicit dependence on mass

Deduced from DNS data:



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Conluding remarks and outlook - I

The present analysis has been based on a number of simplifying assumptions:

- Eringen’s stress tensor for micropolar fluids
- Lagrangian dynamics of spherical particles
- Dilute suspension of point-particles
- Stokes-flow approximation for the torque from the fluid

Published paper:

H.I. Andersson and L. Zhao: “Bridging the gap between continuum mechanical microrotation viscosity and Lagrangian point-particles”, *ASME Journal of Fluids Engineering*, Vol. 135, 124502, December 2013.

Conluding remarks and outlook - II

The present reasoning is valid also for non-spherical particles, e.g. fibers. However, the mathematical modeling of the particle rotation will then be more complicated:

- The moment of inertia I is a tensor rather than a scalar.
- The Jeffery torque components include not only the relative rotation but also the shear rate of the flow.
- Intuitively, the microrotation viscosity should be a tensor and not a scalar as in the present study for spheres.

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- The end
- FIN

Simulation specifications

Simulations by Zhao & Andersson, Comm. Comp. Phys. 2012.

- Frictional Reynolds number Re_τ=180 (Kim et al. JFM 1987)
- Mesh
  - 192^3
  - $\Delta x^+=11.6$, $\Delta y^+=5.3$, $\Delta z^+=0.9$--2.8
- Domain size
  - 12h*6h*2h (x*y*z)
- Particle St and radius
  - St = 1, 5, 30, 100, $a^+=0.36$
- Particle number
  - $n_p = 10^5$