

DNS of Particles and Fibers Transport and Deposition in Duct Flows

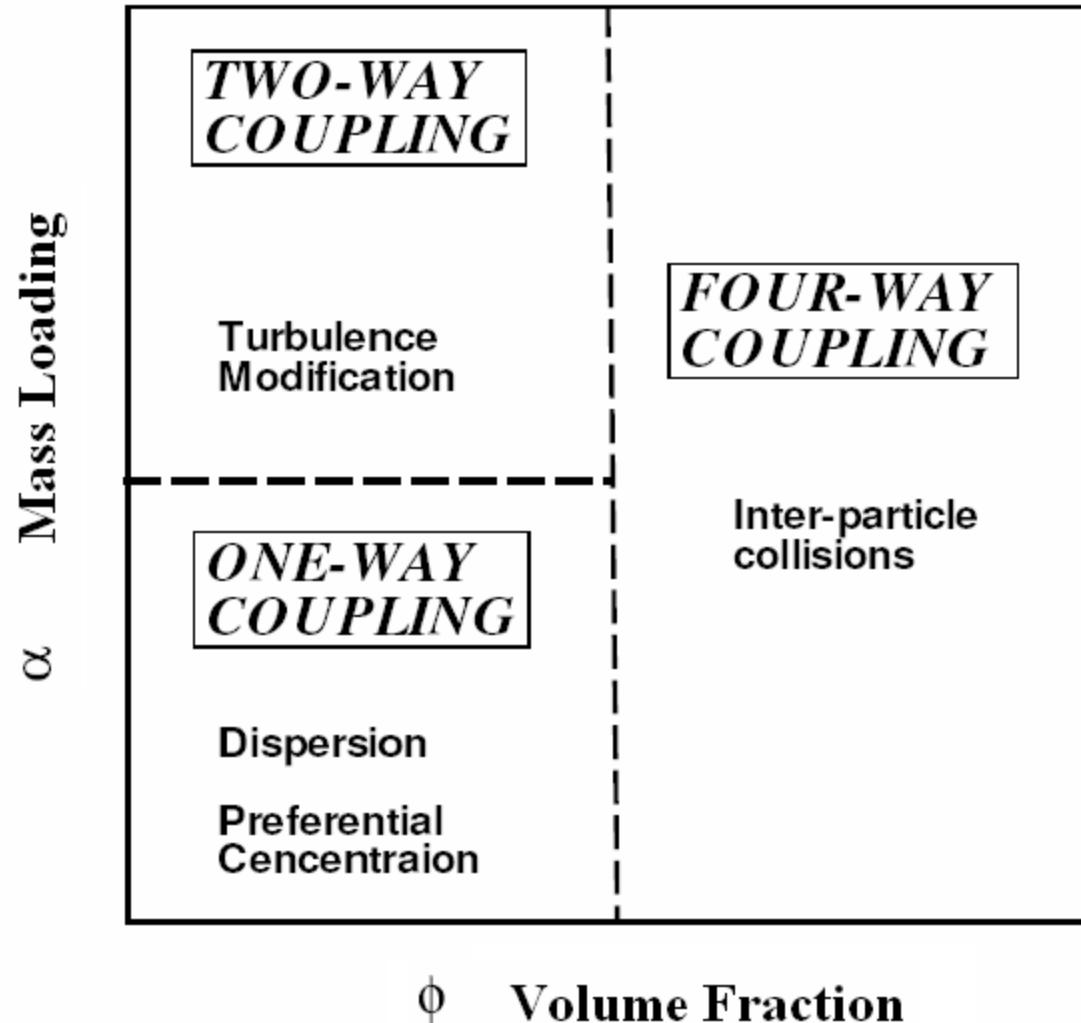
Goodaz Ahmadi

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Outline

- Literature Review
- Particles
 - Governing Equations
 - Results
- Fibers
 - Governing Equations
 - Results
- Concluding Remarks

Flow Regimes



Direct Numerical Simulations

Wall Units

$$u_i^+ = \frac{u_i}{u^*}$$

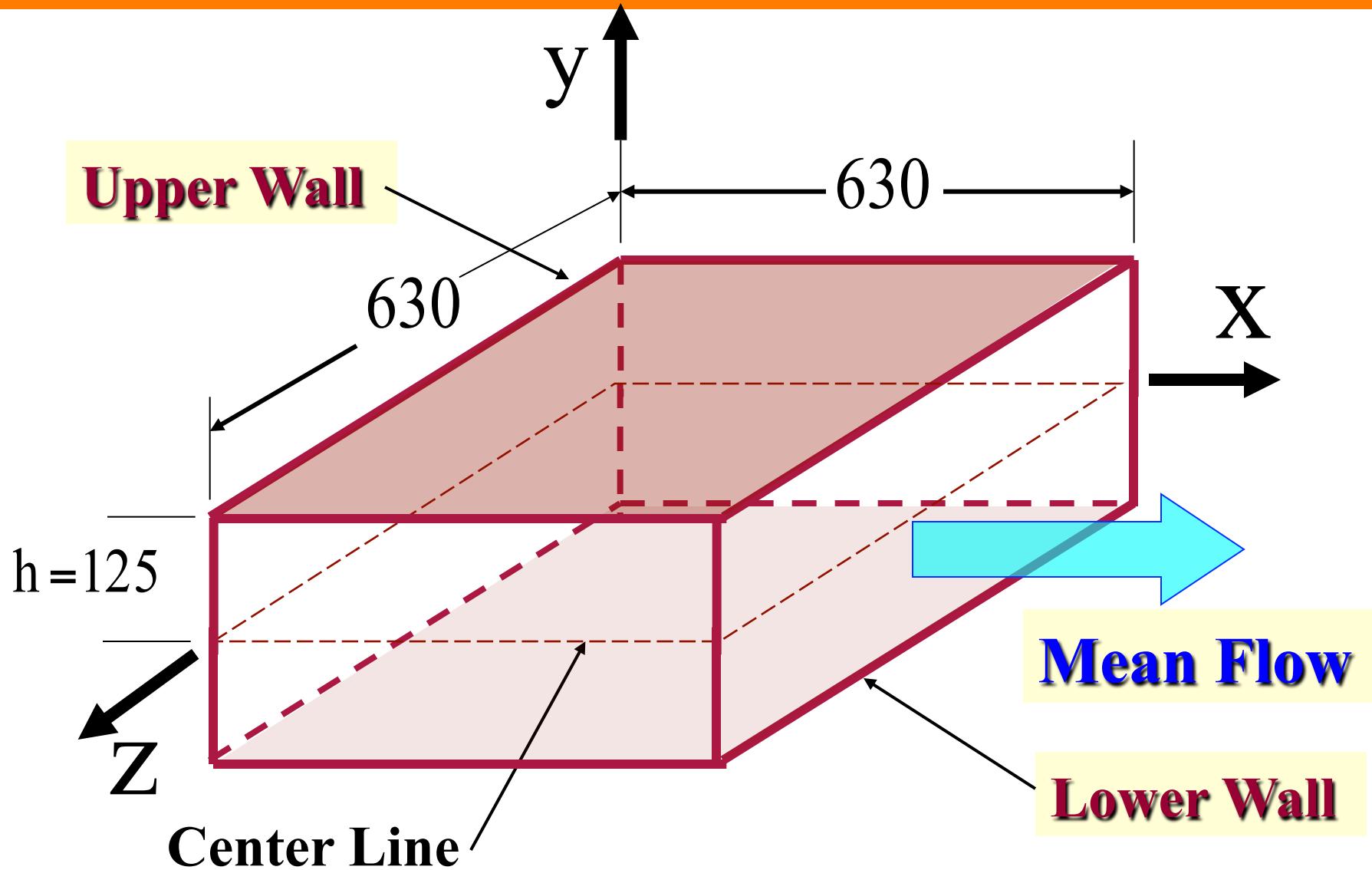
$$x_i^+ = \frac{u^* x_i}{\nu}$$

Navier-Stokes

$$\nabla^+ \cdot \mathbf{u}^+ = 0$$

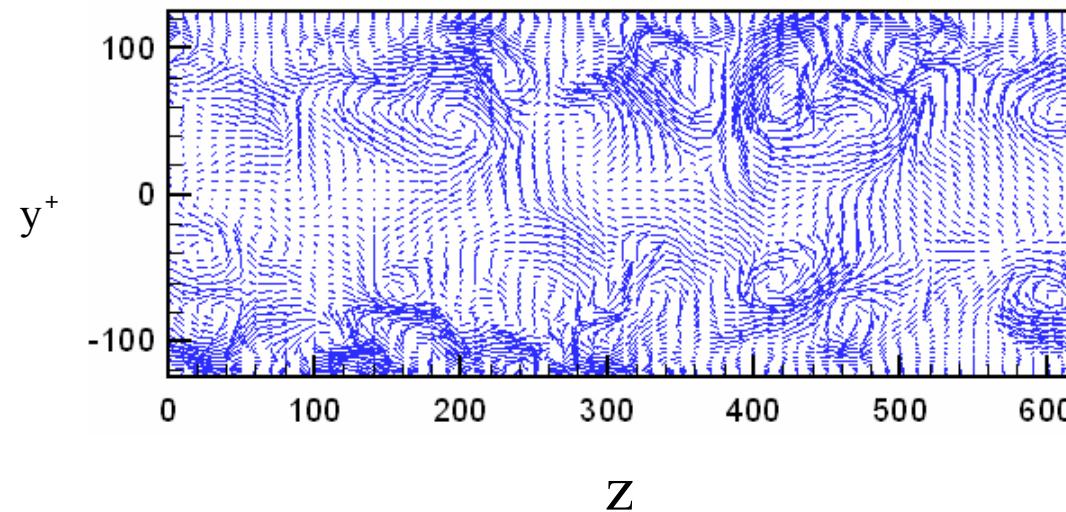
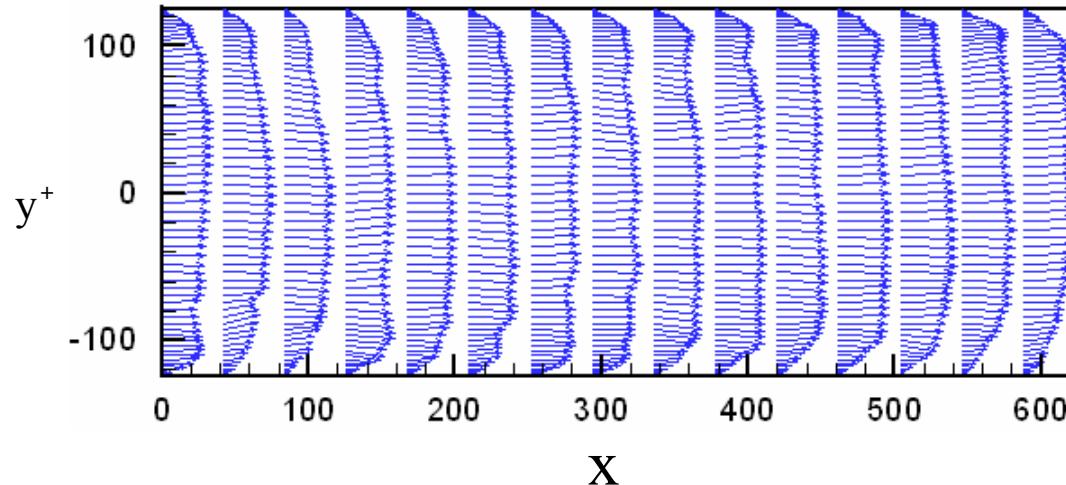
$$\frac{\partial \mathbf{u}^+}{\partial t^+} + \mathbf{u}^+ \cdot \nabla^+ \mathbf{u}^+ = \nabla^{+2} \mathbf{u}^+ - \nabla^+ P^+ + S_u^p$$

Flow Between Two Parallel Plates



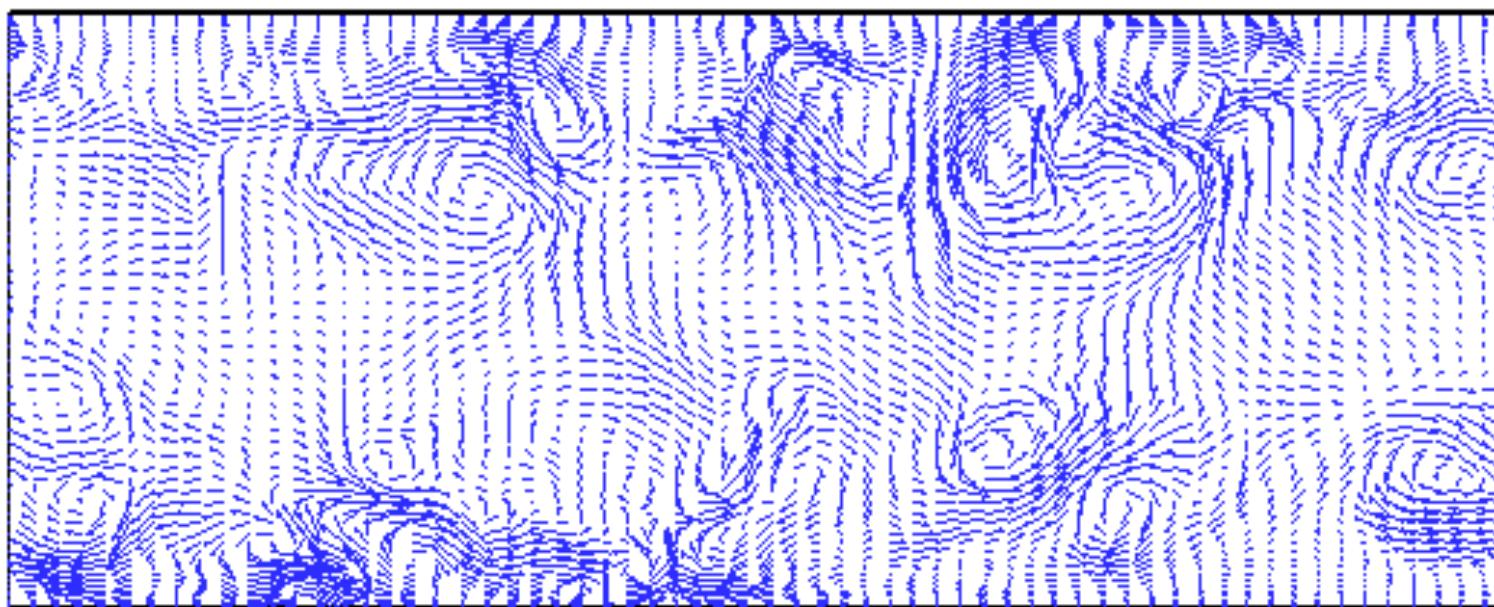
Instantaneous Velocity Field

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Flow Field Visualization

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Governing Equations

Spherical Particle Equation of Motion

$$\frac{dv^+}{dt^+} = g^+ + F_d^+ + F_L^+ + n^+(t^+)$$

Gravity

Drag

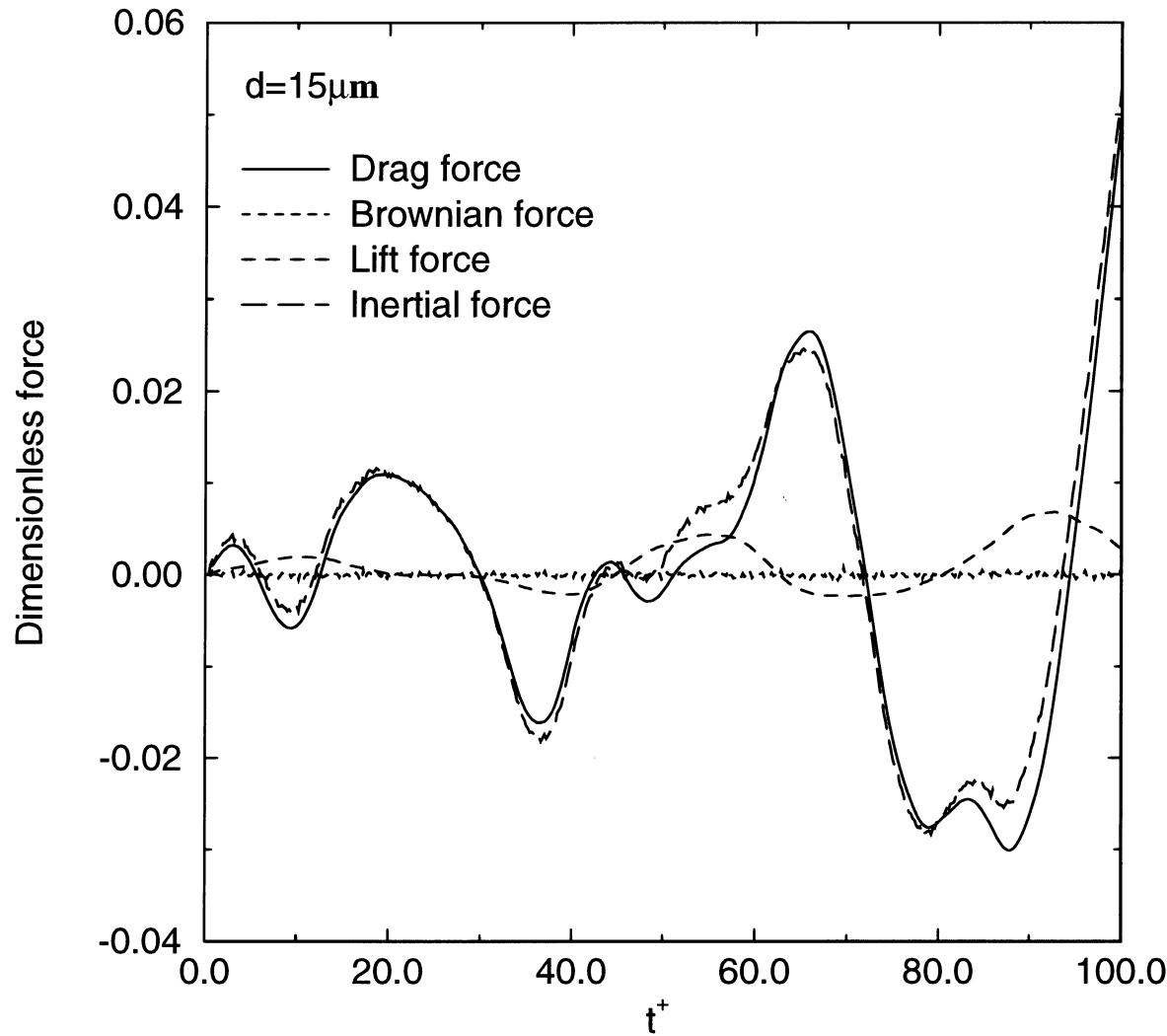
Lift

Brownian

$$\frac{dx^+}{dt} = v^+$$

Comparison of Force Magnitudes

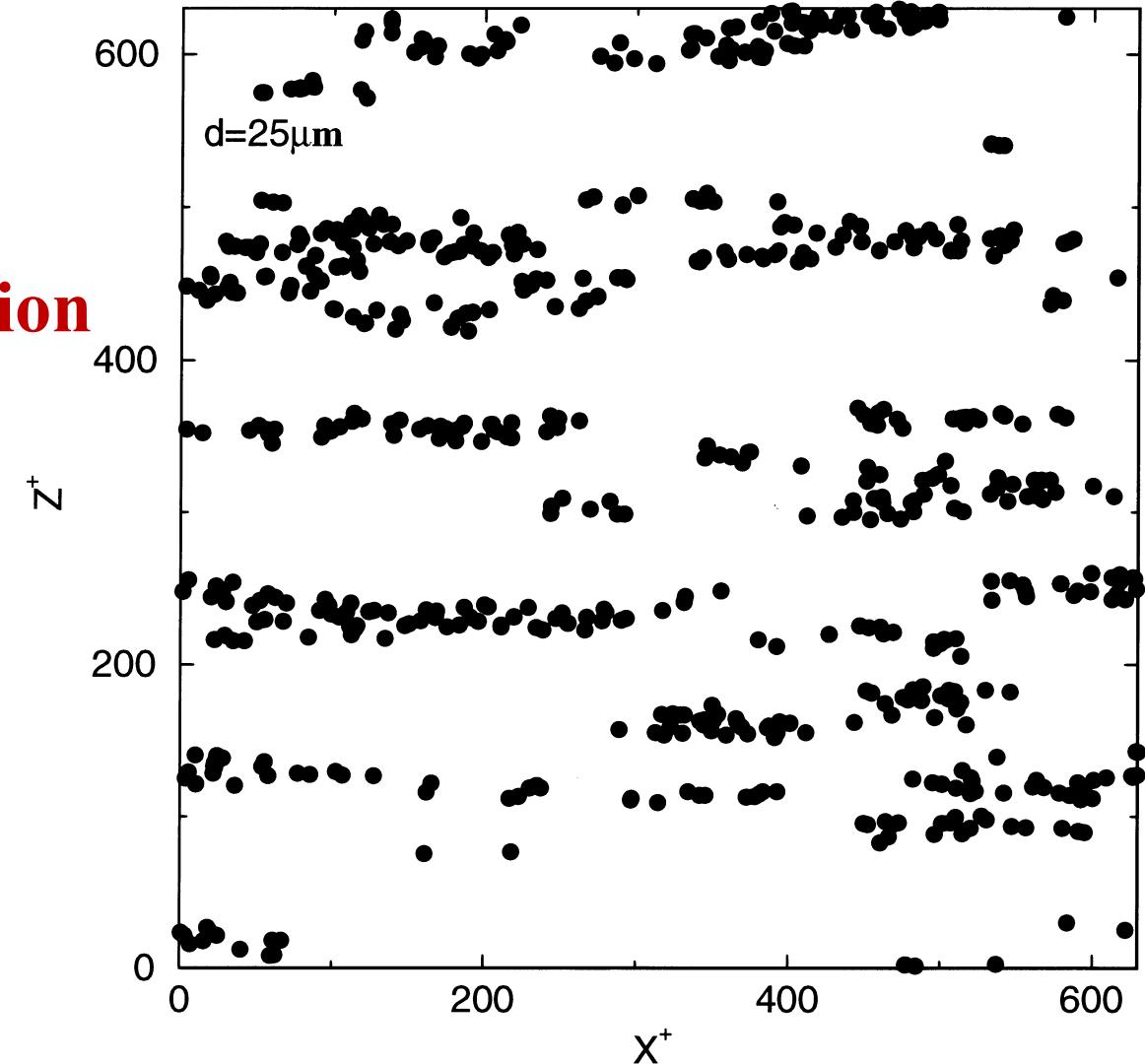
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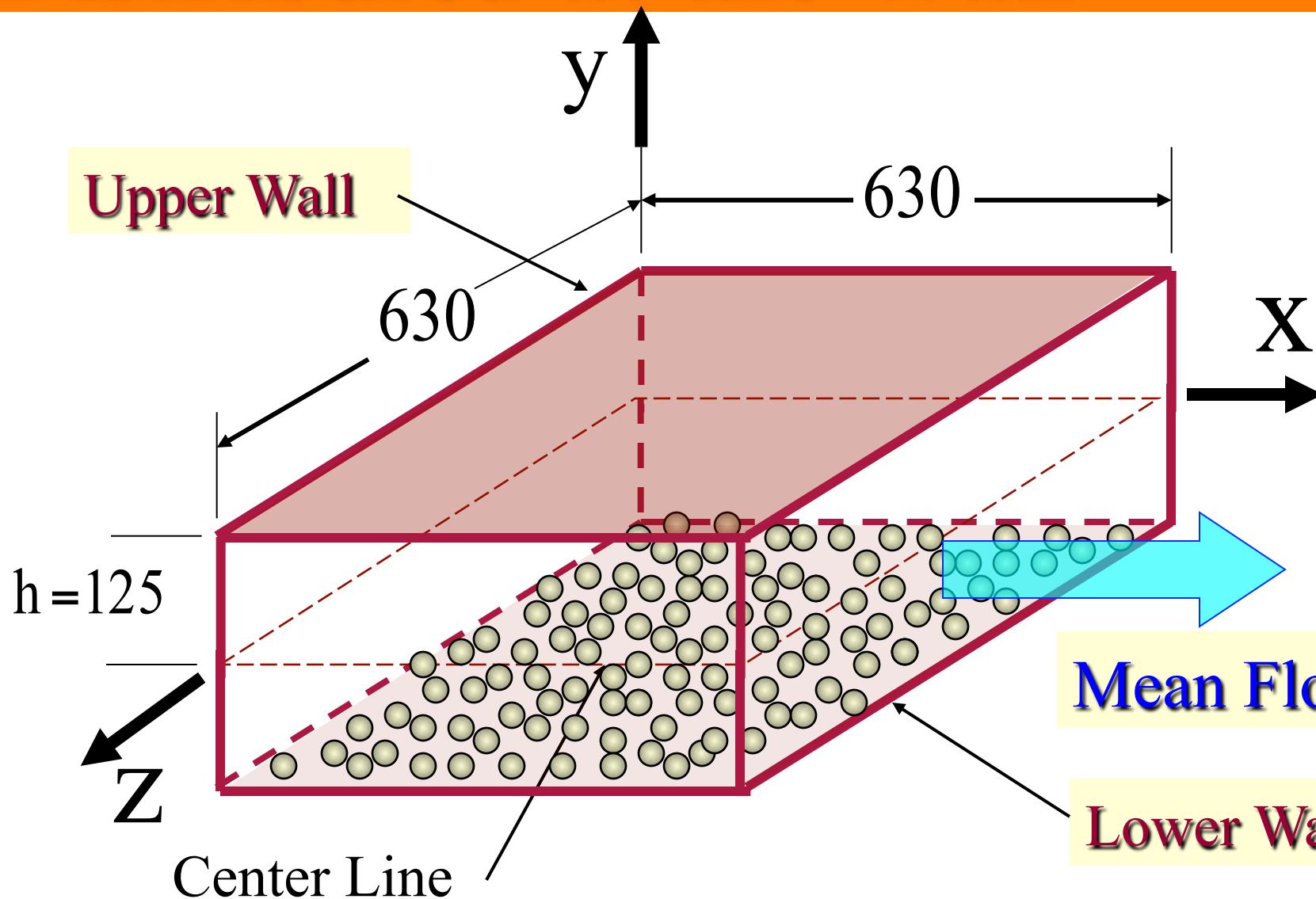
Deposition Pattern

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Mean Flow Direction



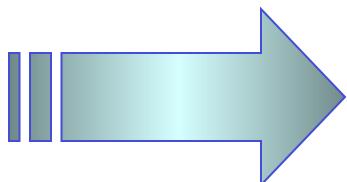
Particles Are Randomly Distributed at the Wall



Particle Removal Pattern

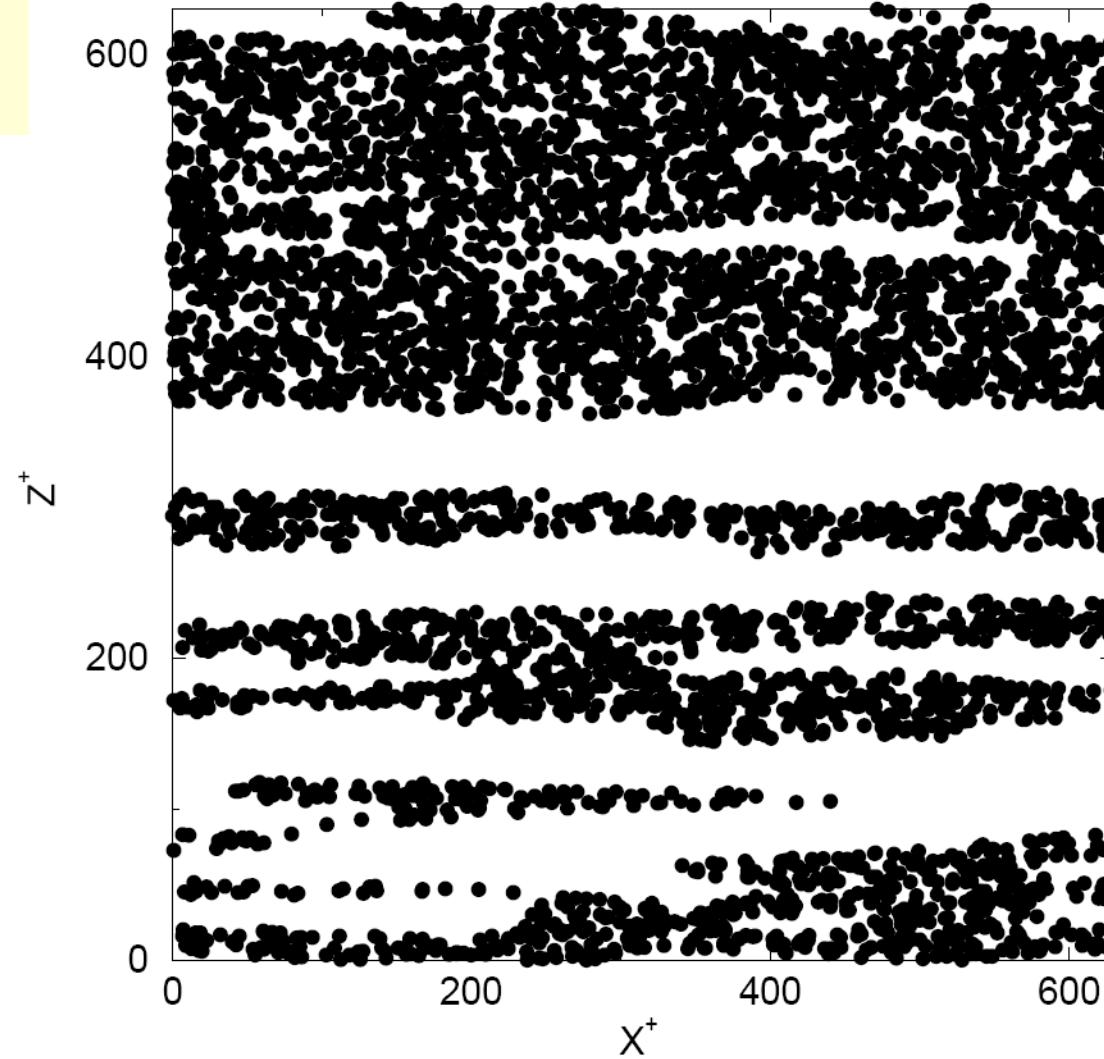
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Surface
Concentration



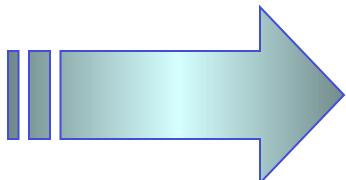
$t^+ = 40$

30 μm



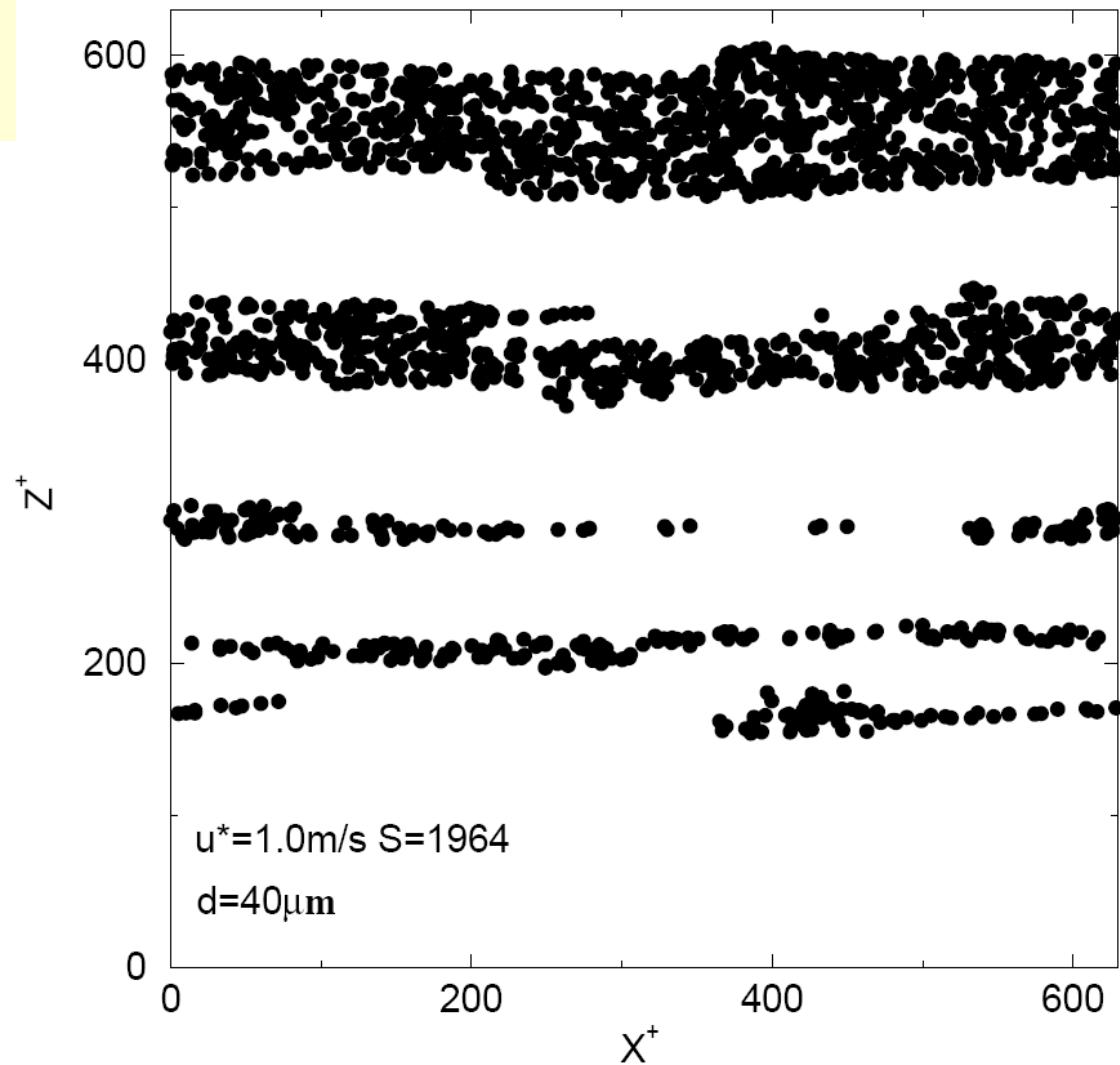
Particle Removal Pattern

Surface
Concentration



$t^+ = 40$

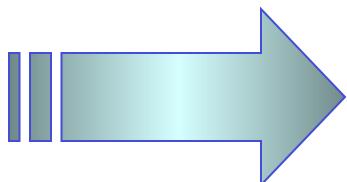
40 μm



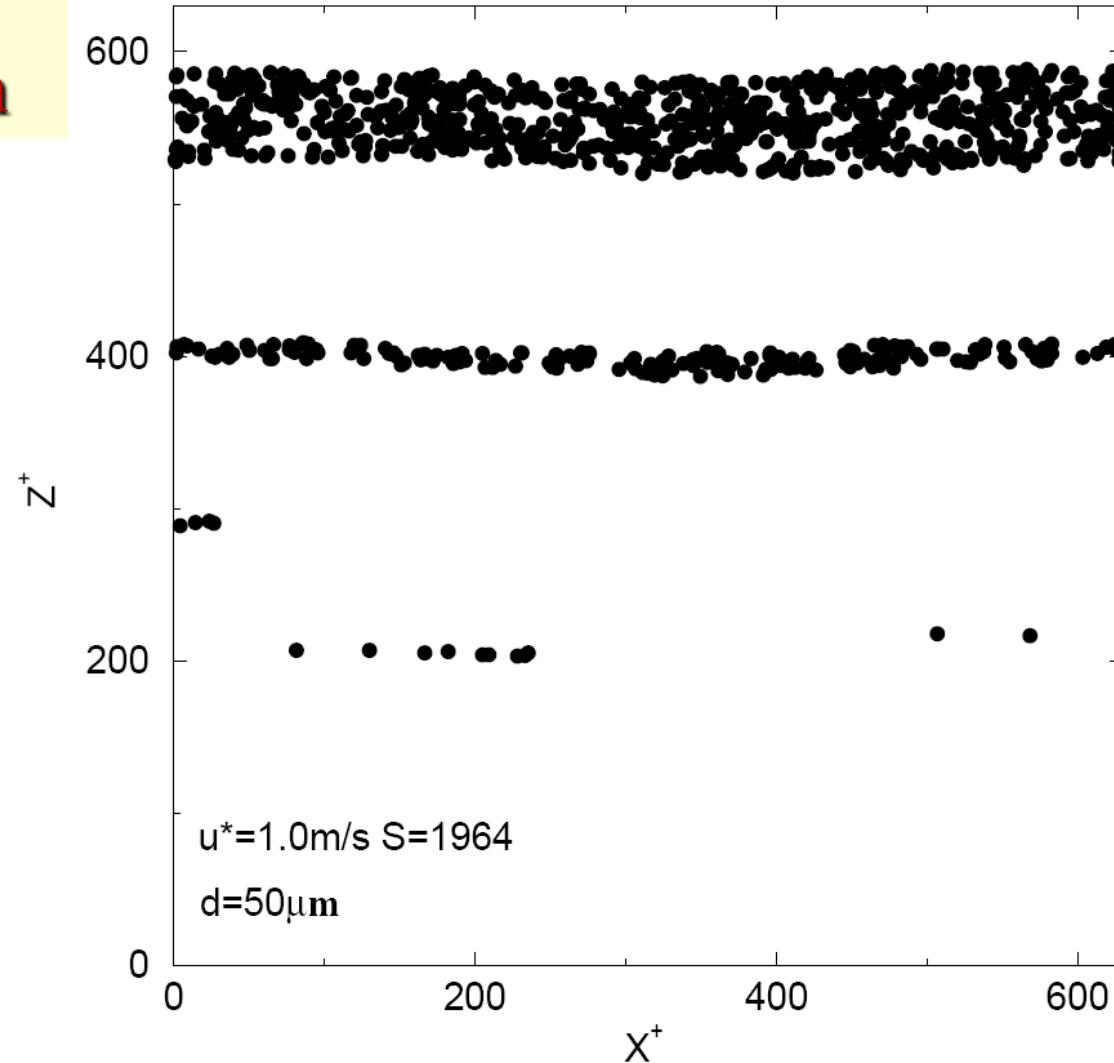
Particle Removal Pattern

50 μm

Surface
Concentration

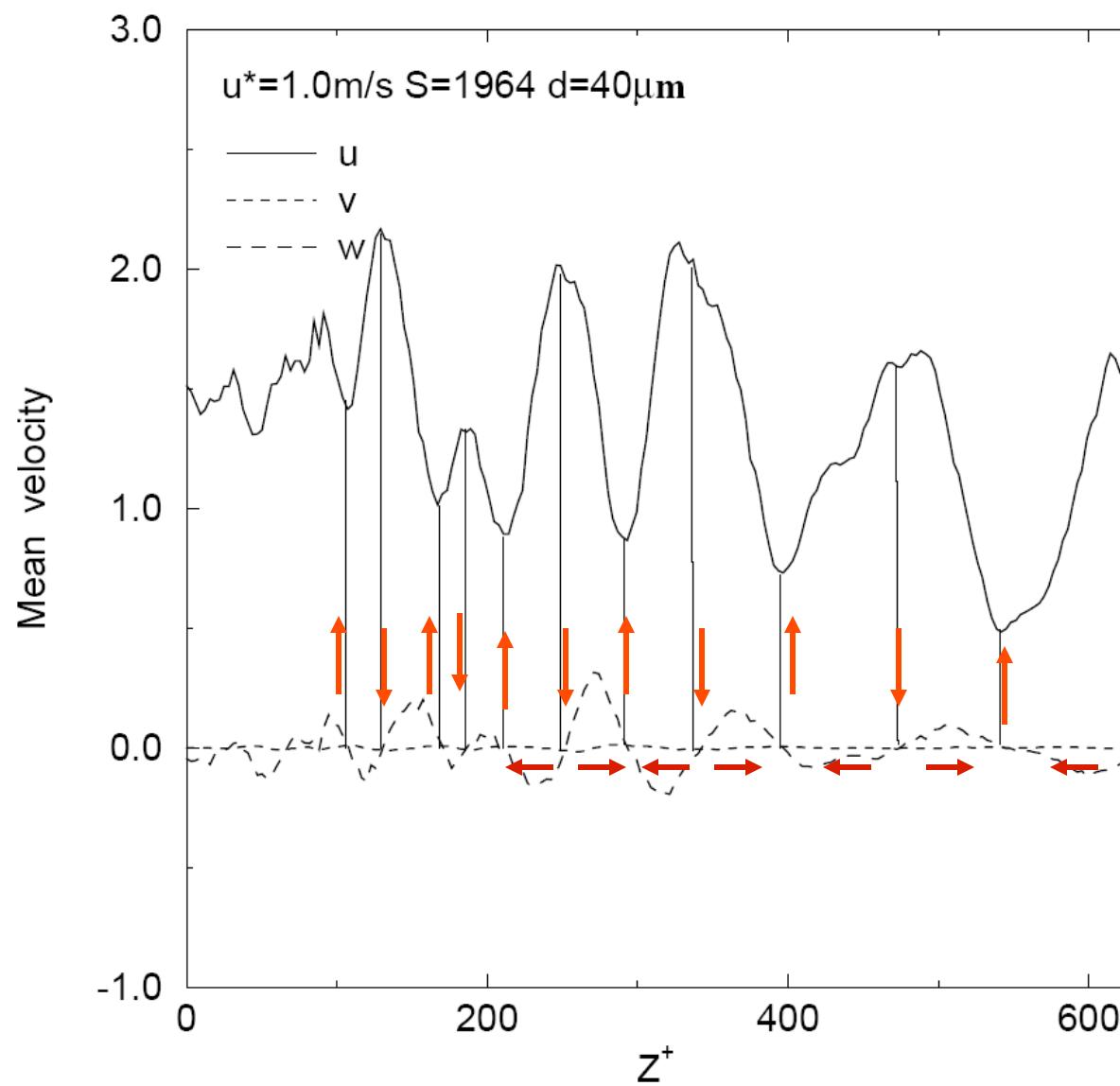


$t^+ = 40$



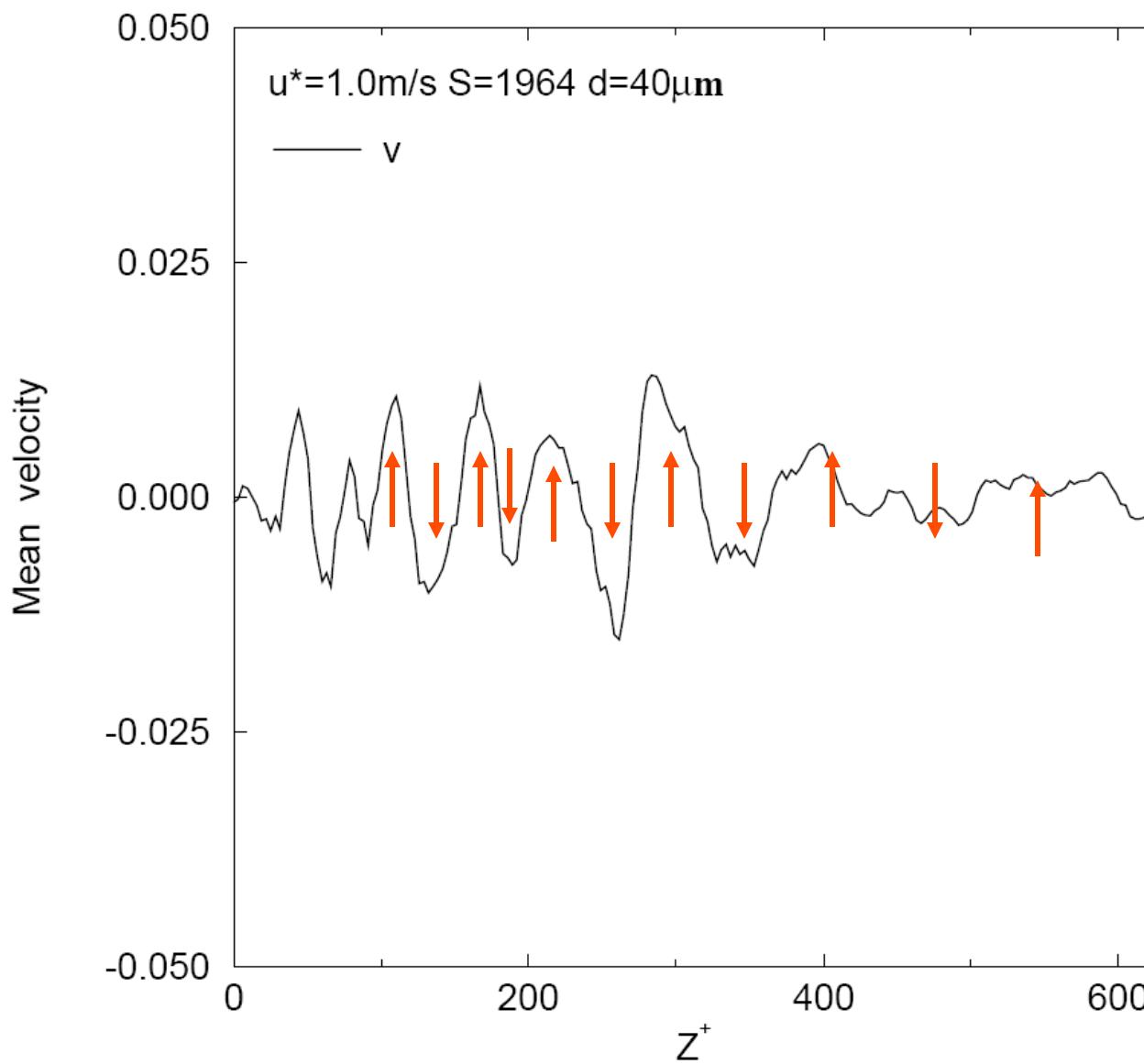
Variations of Averaged Velocities

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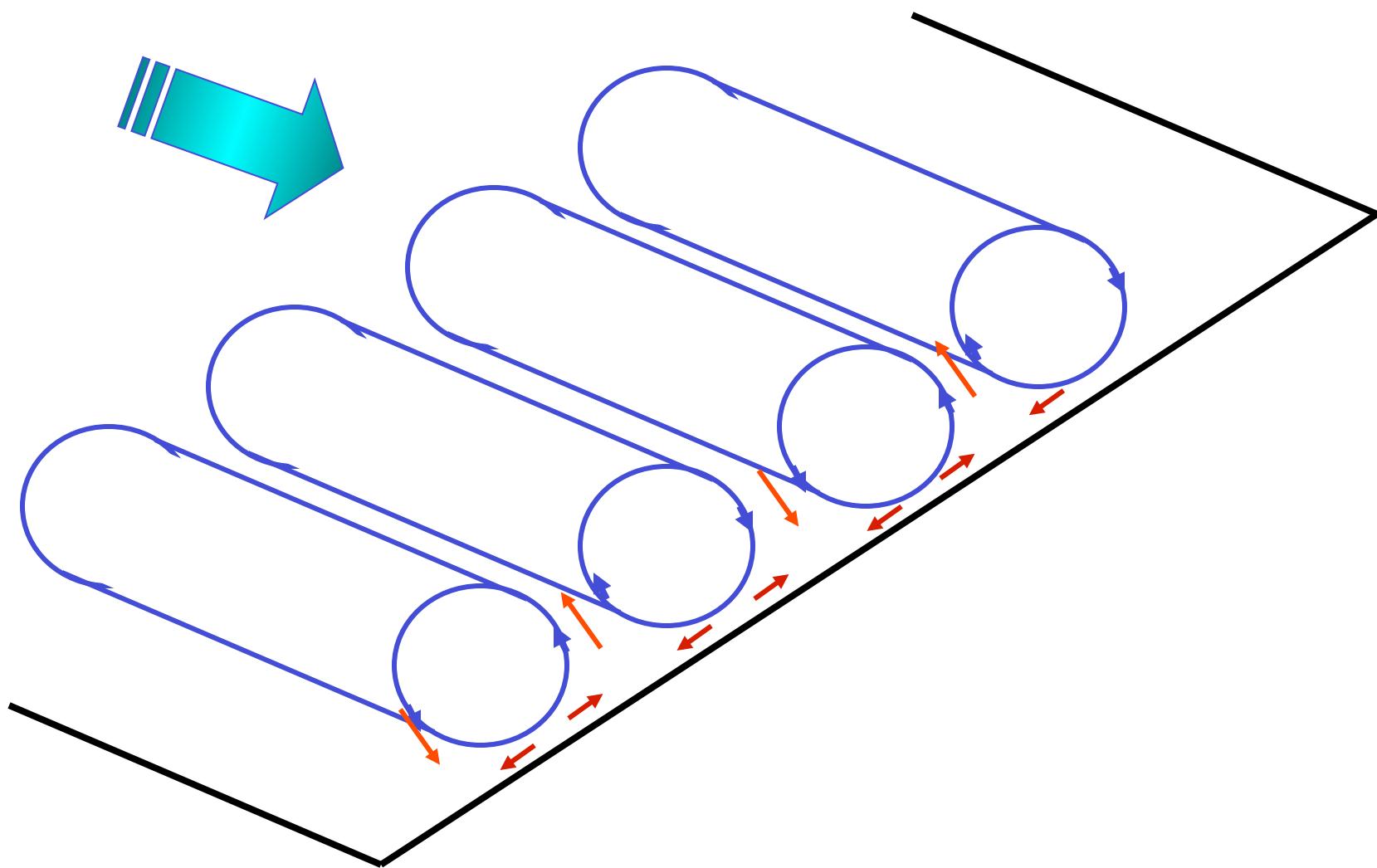
Variation of Averaged V-Velocities

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Coherent Wall Vortices

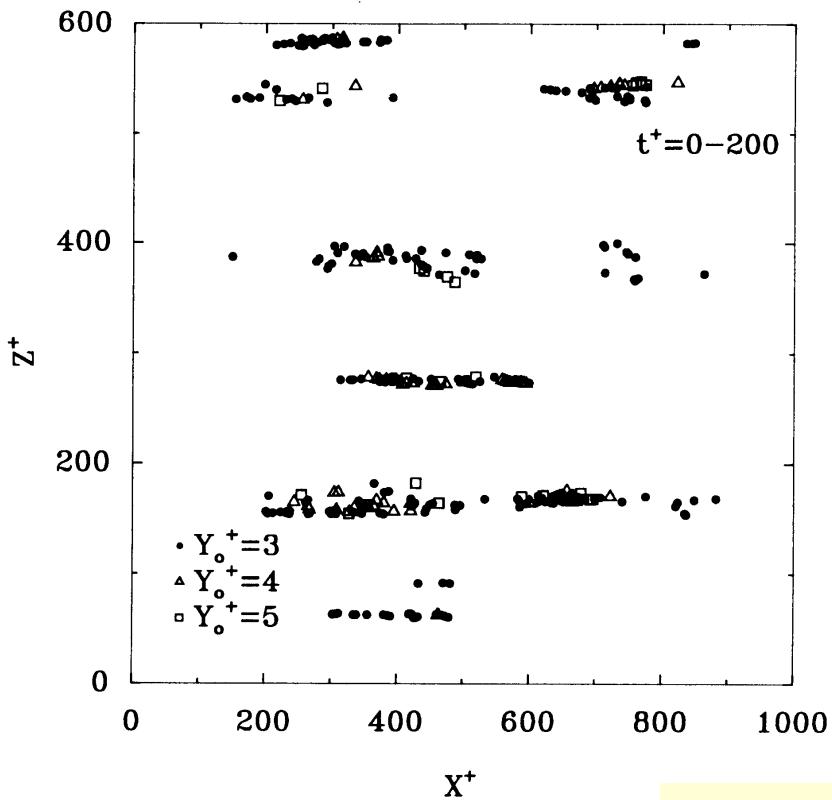
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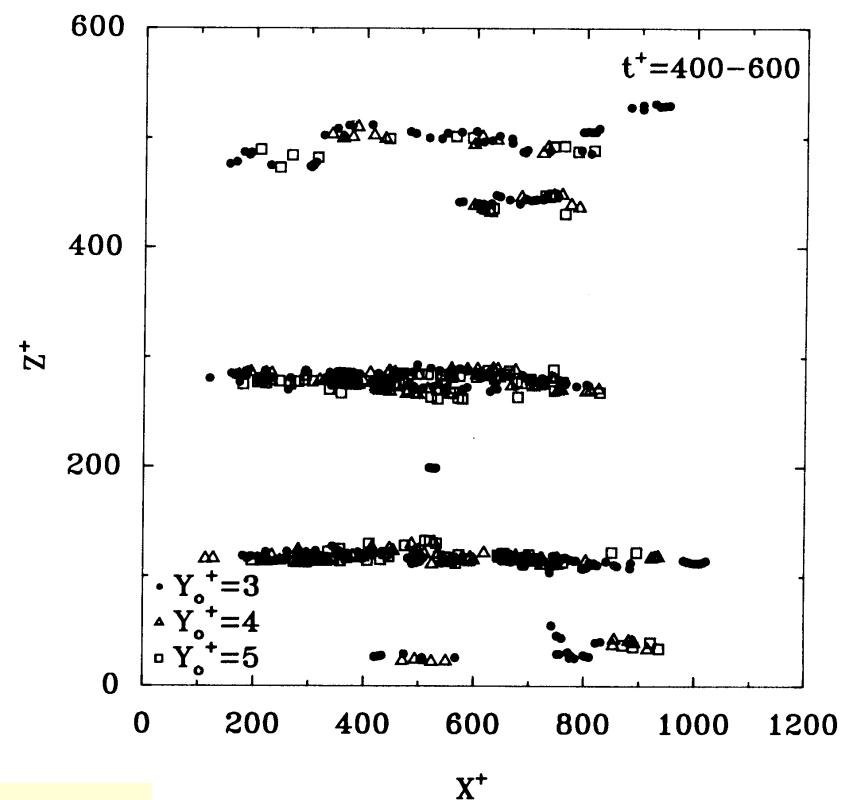
Particle Transport Pattern

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$0 < t^+ < 200$



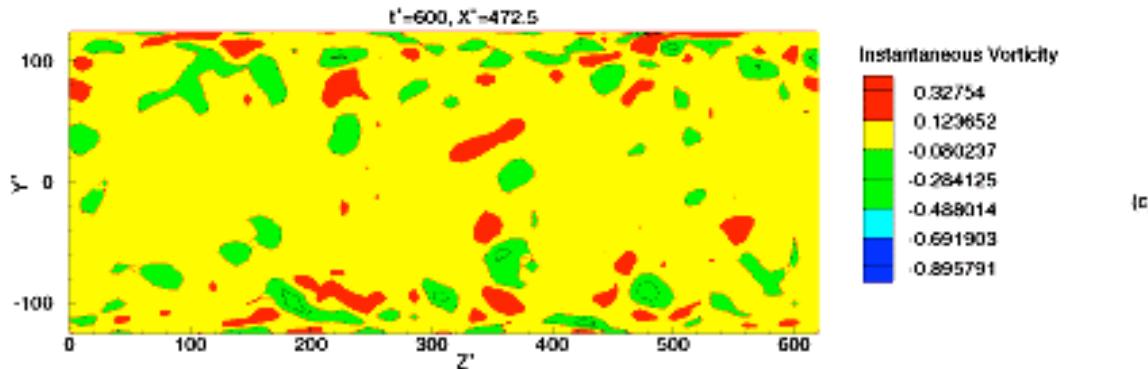
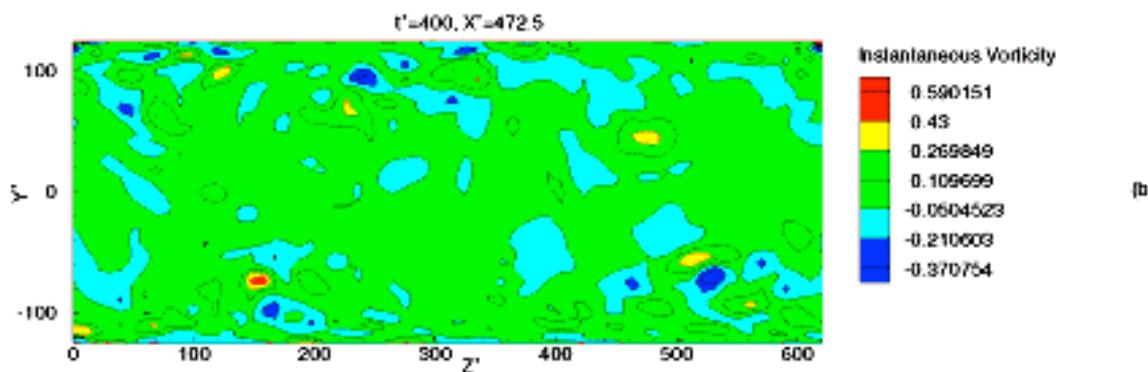
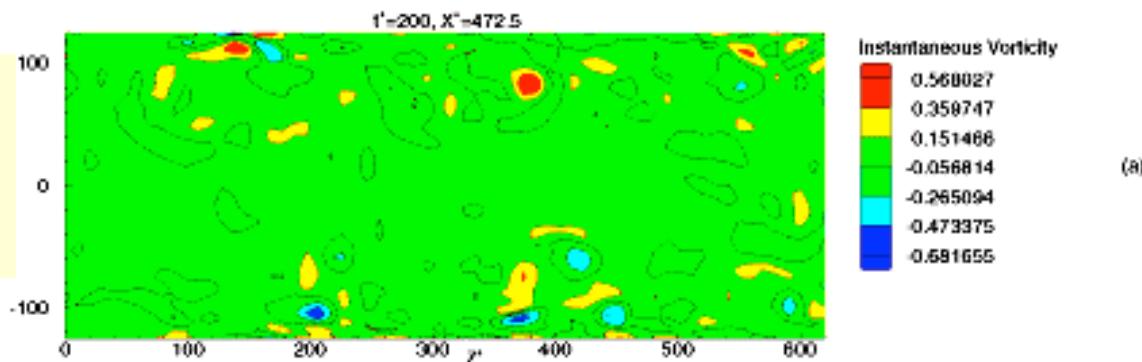
$400 < t^+ < 600$



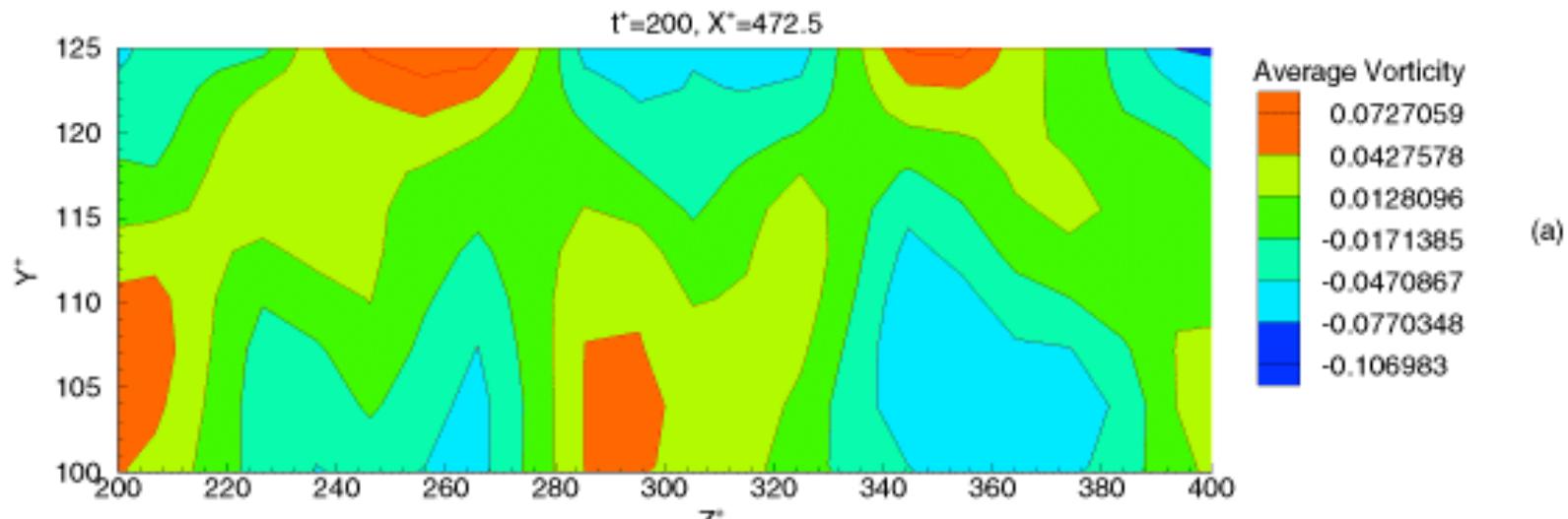
$d=0.5 \mu\text{m}$

Vorticity Variations

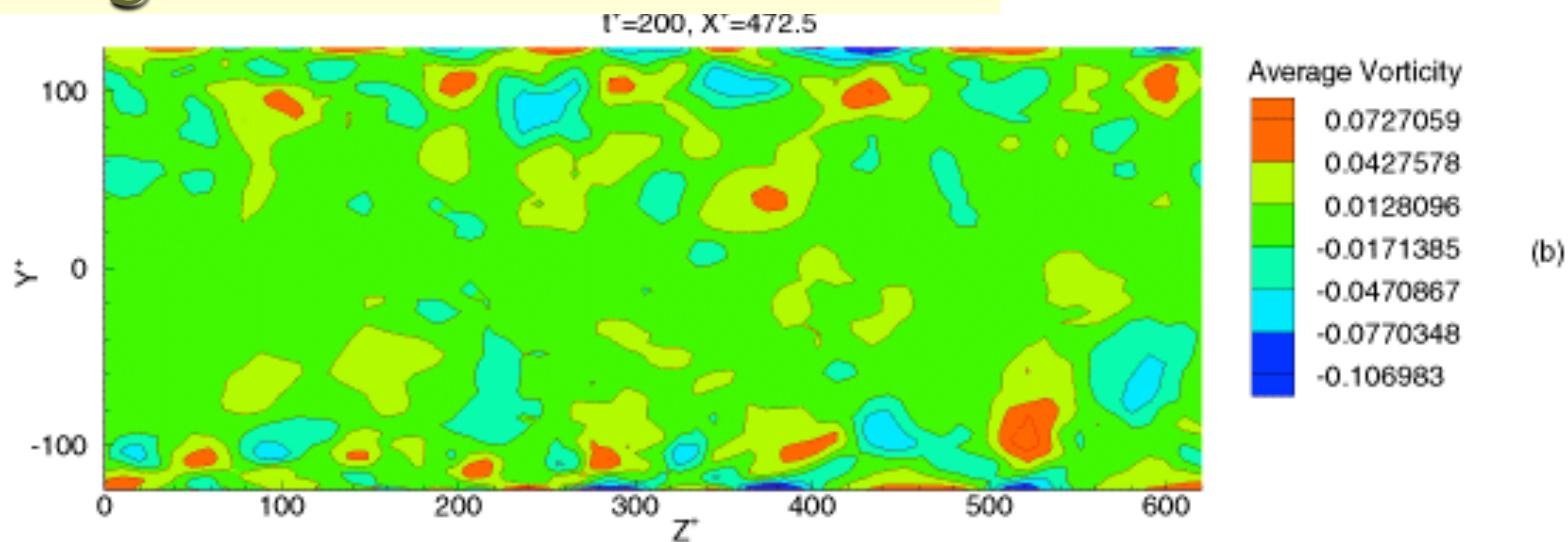
Instantaneous Vorticity



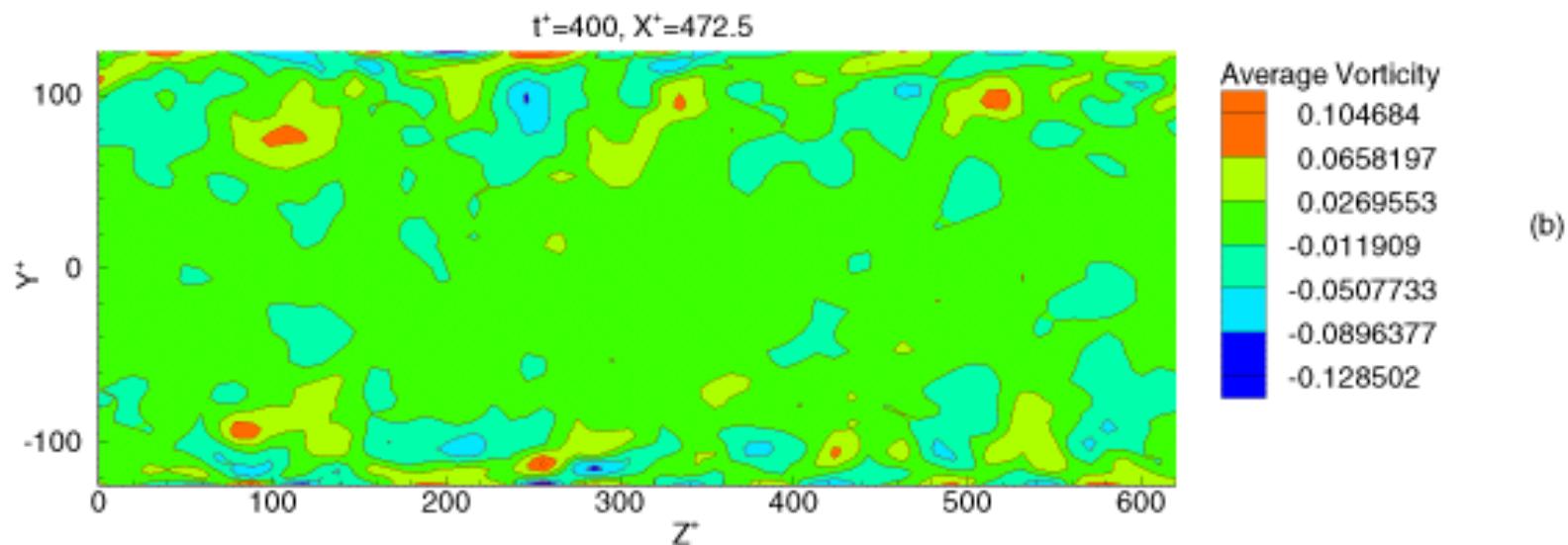
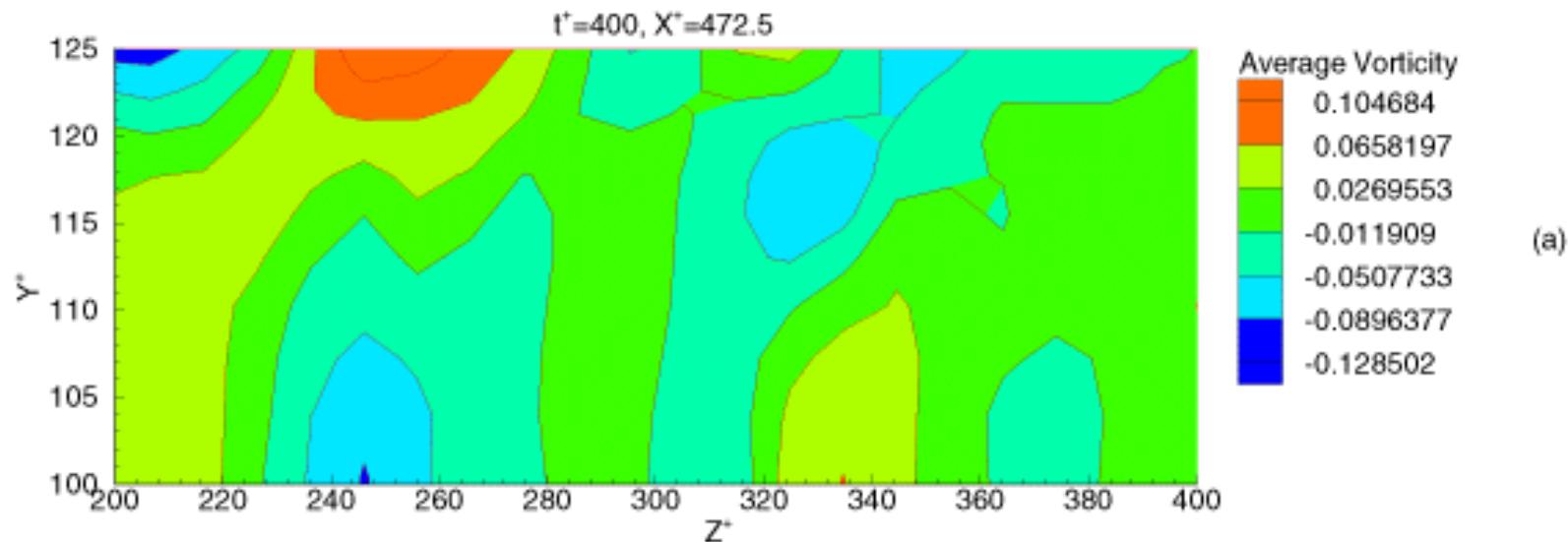
Vorticity Variations



Averaged Over 50 Wall Units

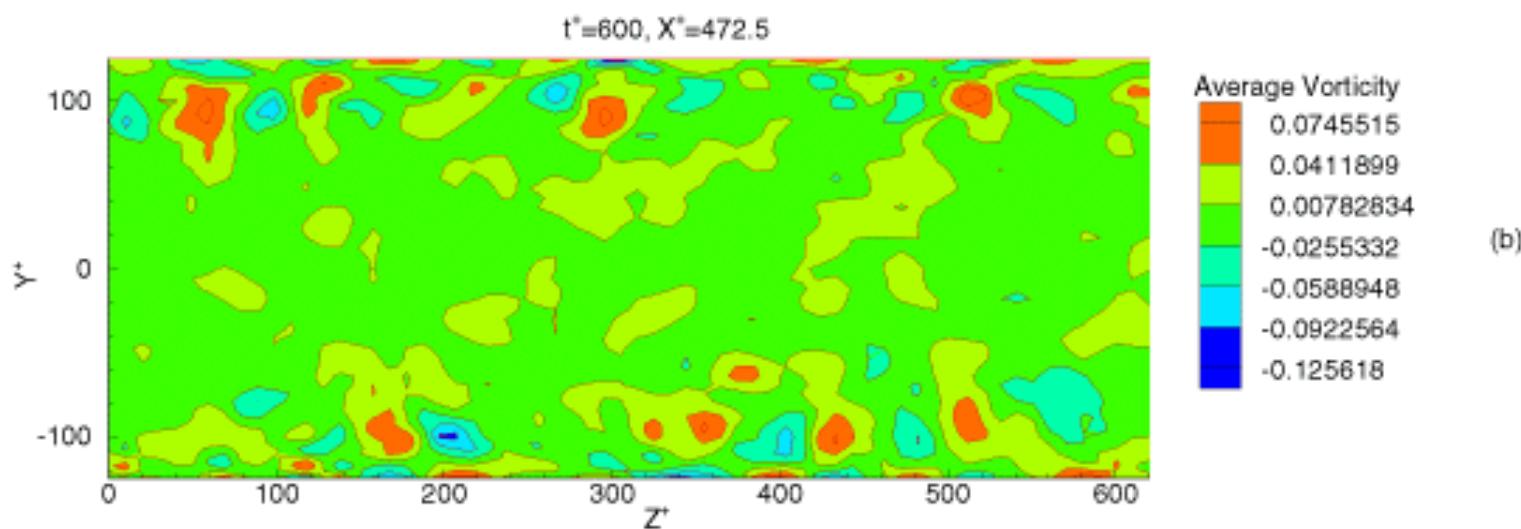
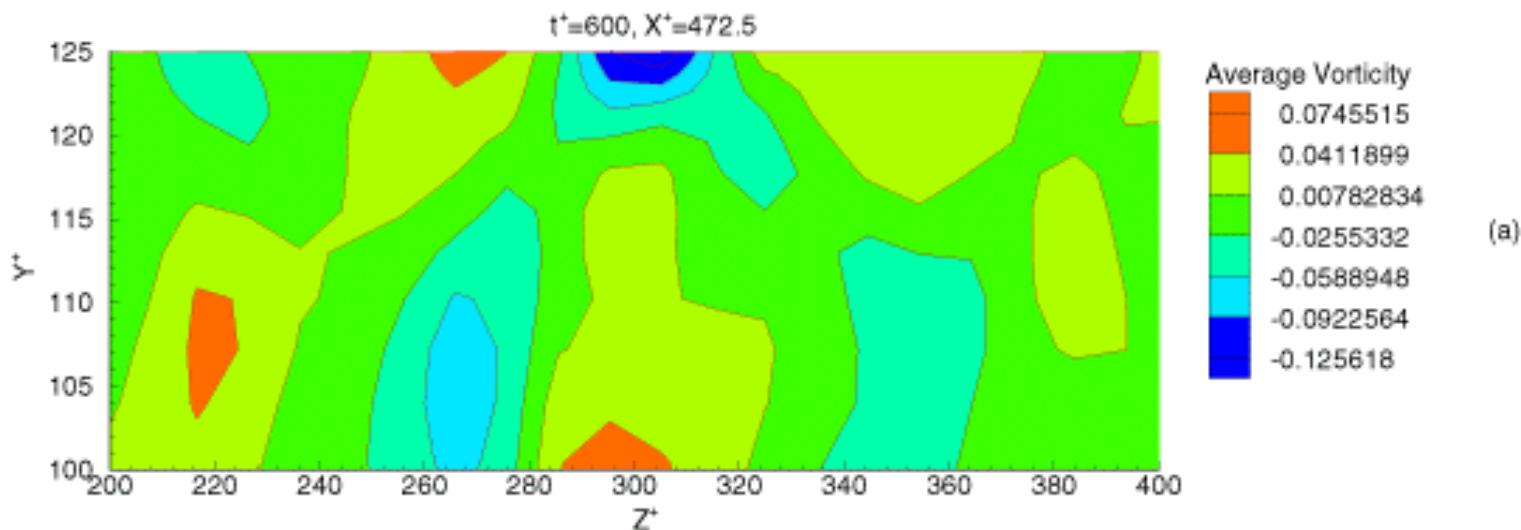


Vorticity Variations



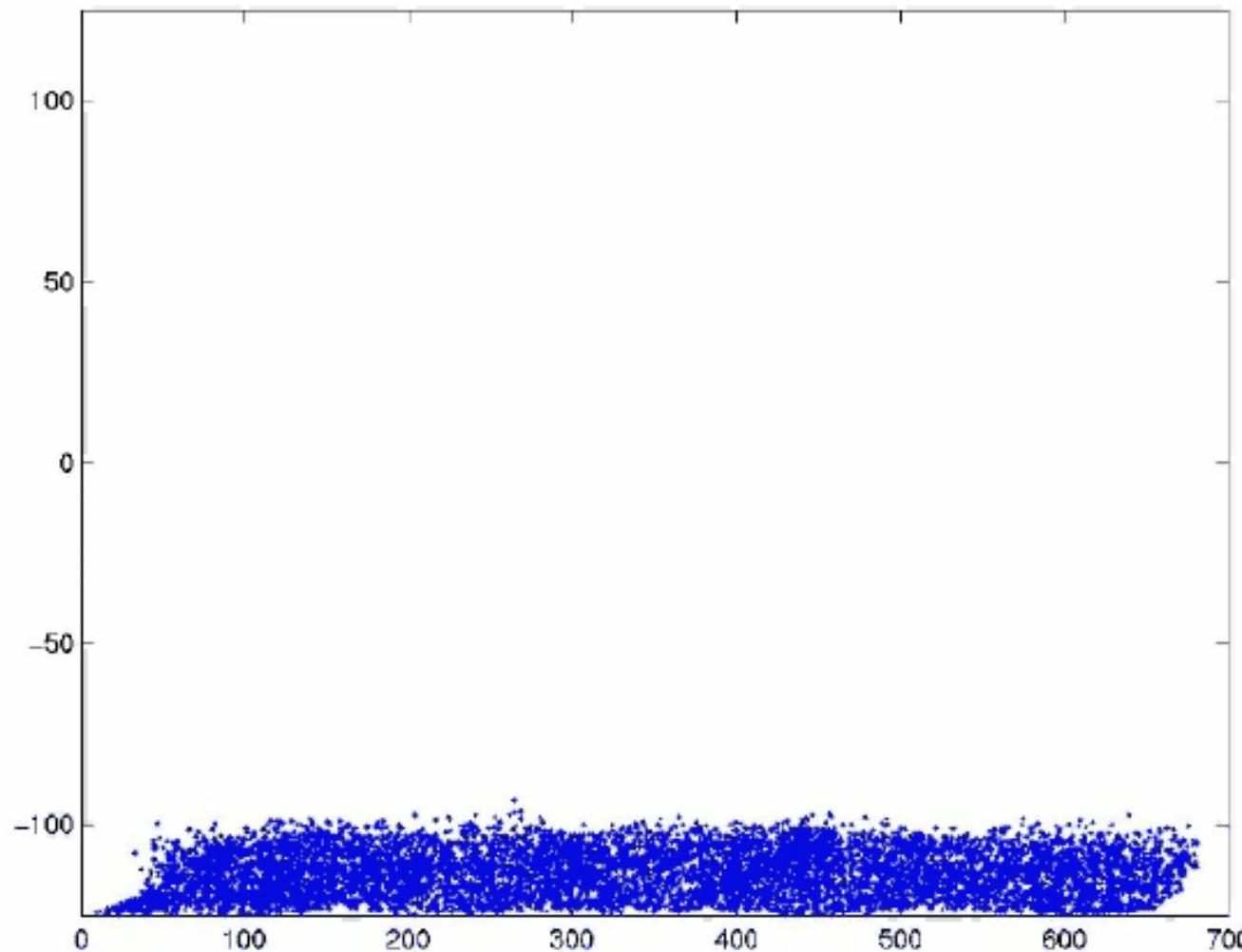
Vorticity Variations

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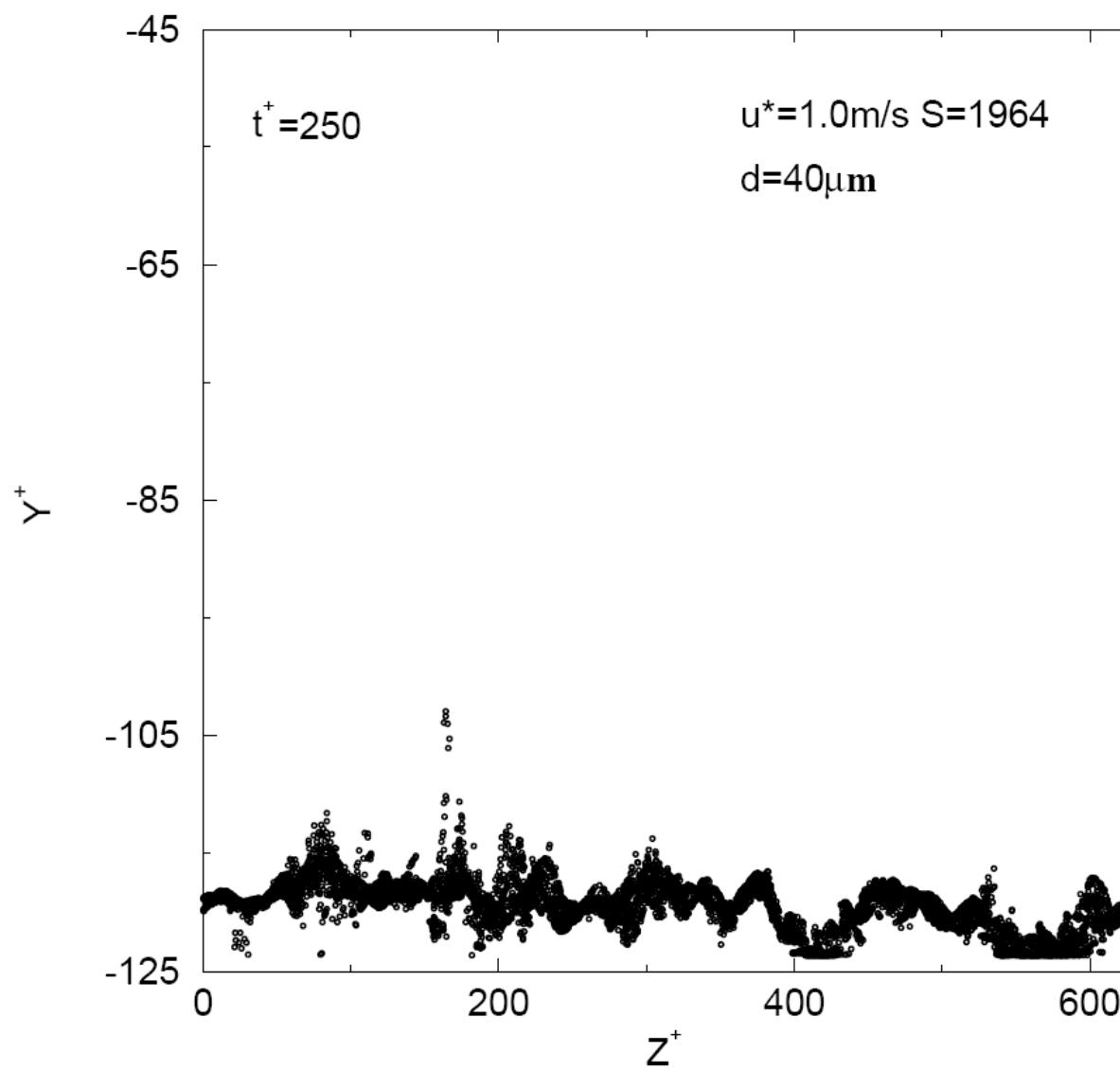
Particle Removal Pattern

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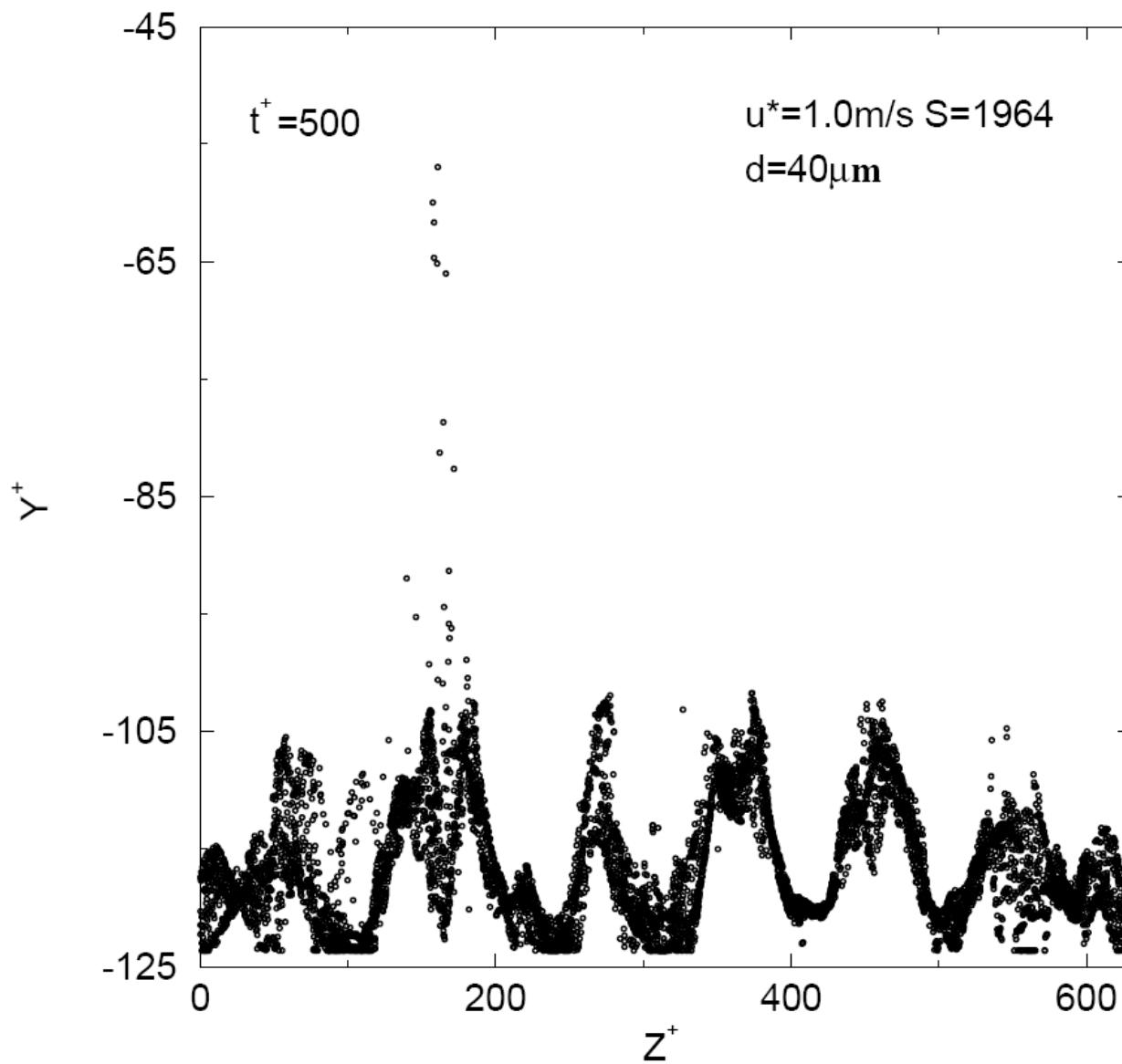
Particle Removal Pattern

Clarkson
University



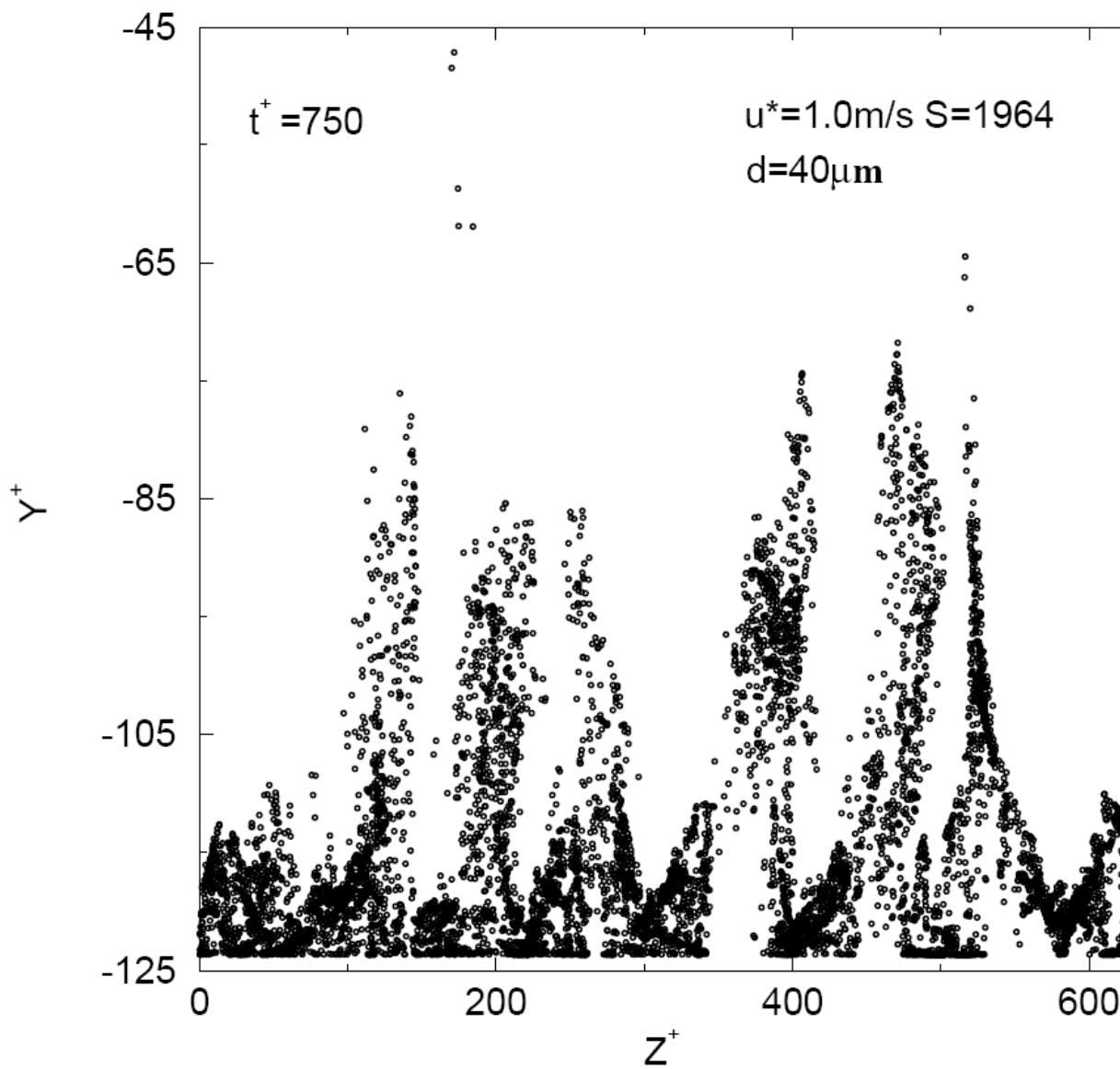
Particle Removal Pattern

Clarkson
University



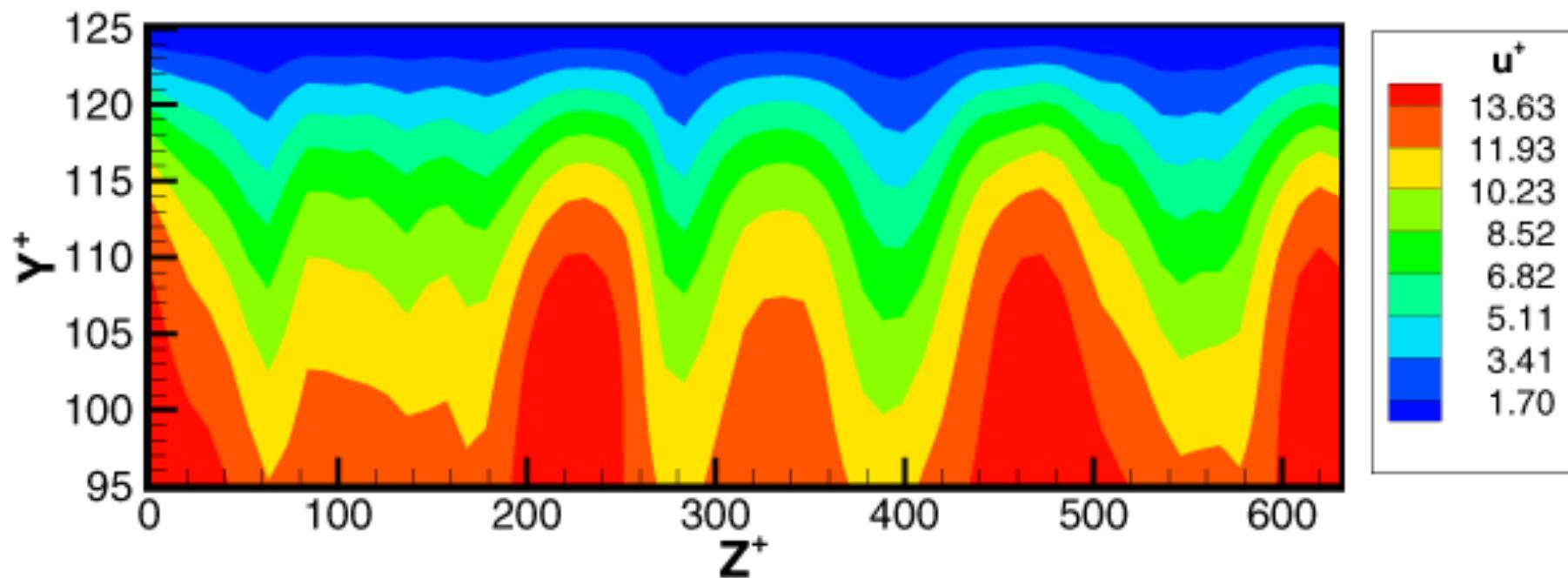
Particle Removal Pattern

Clarkson
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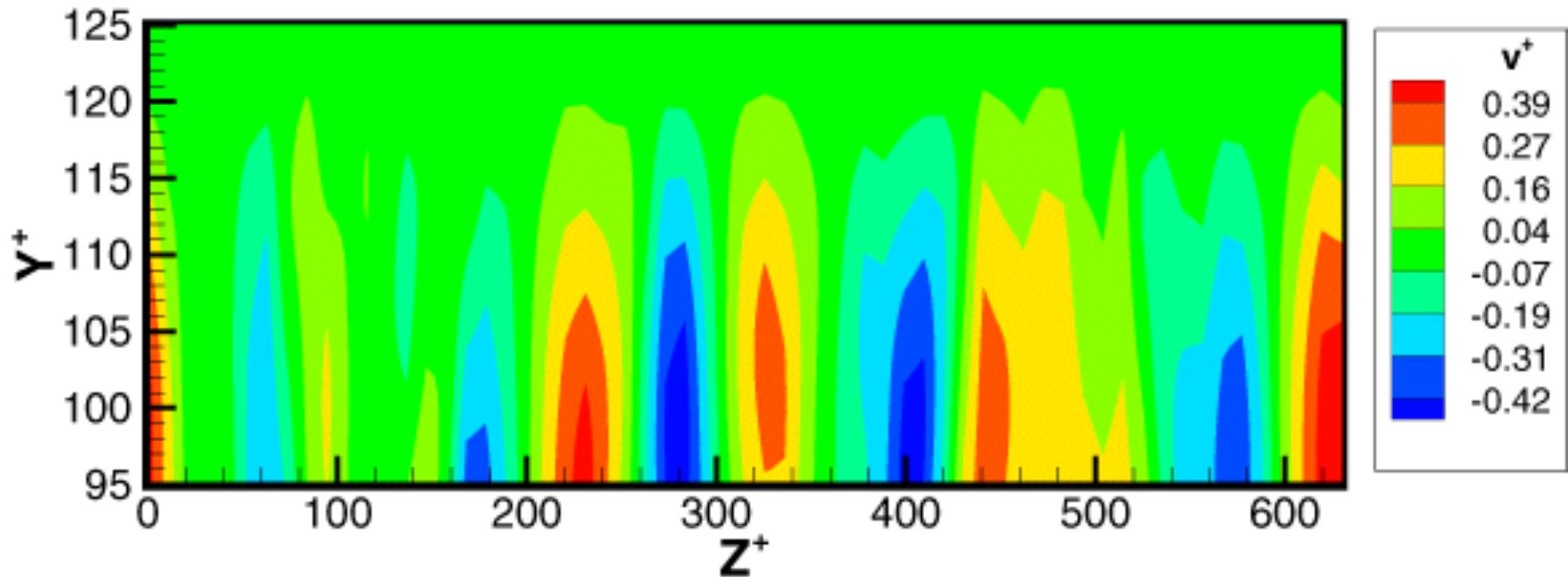
High Speed Streaks

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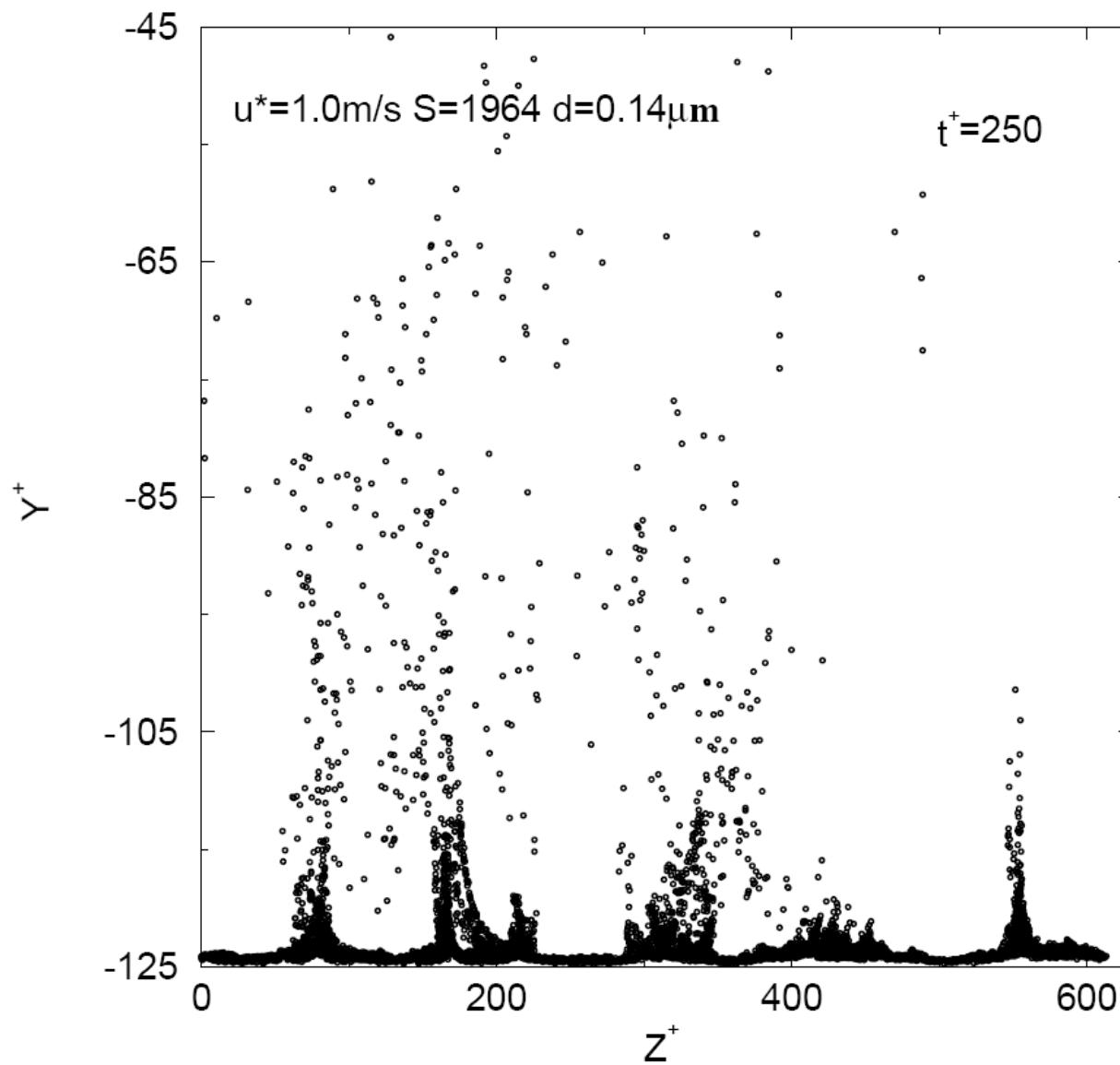
Periodic Vertical Streams

Averaged over 50 time steps



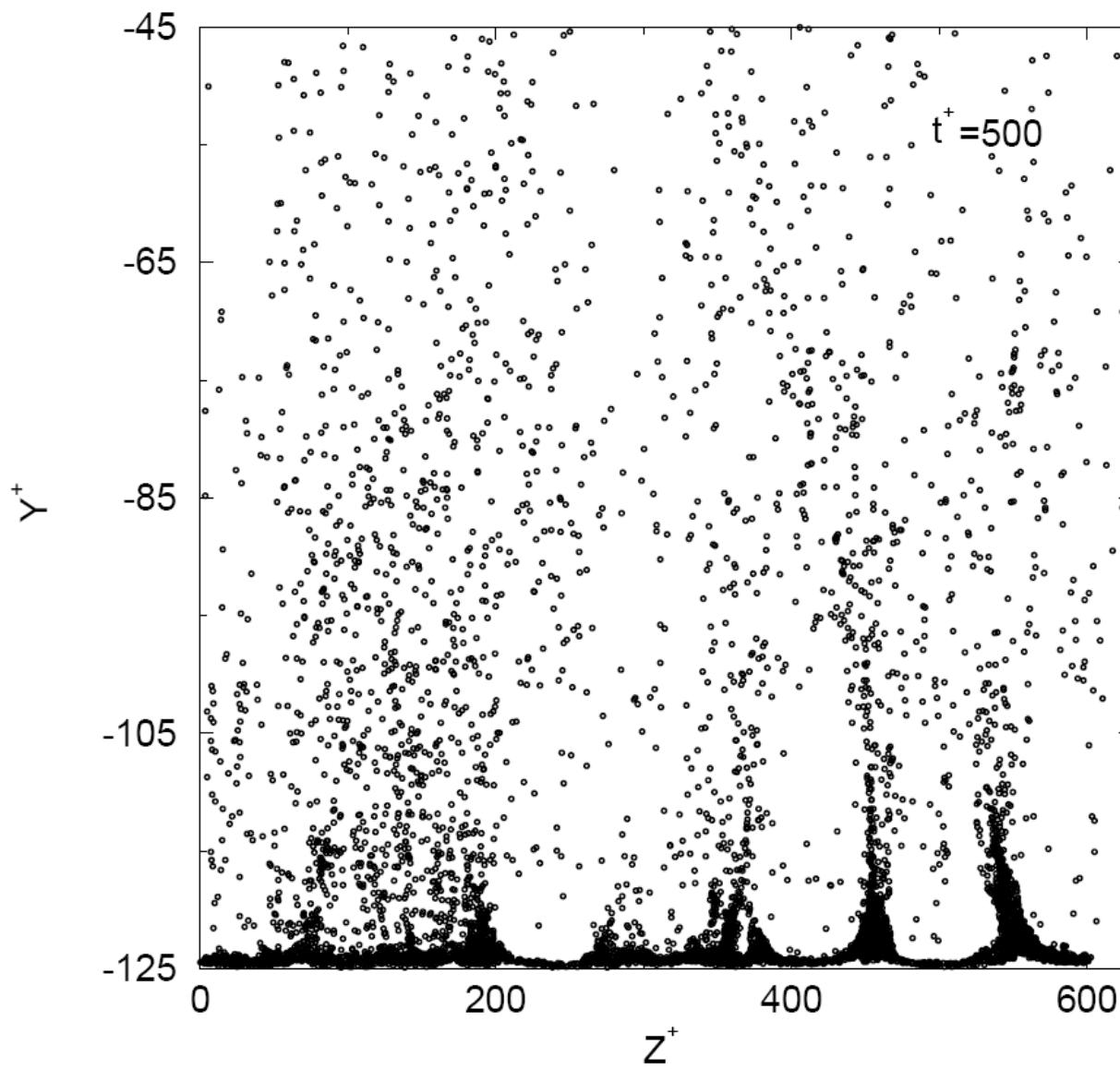
Particle Removal Pattern

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Particle Removal Pattern

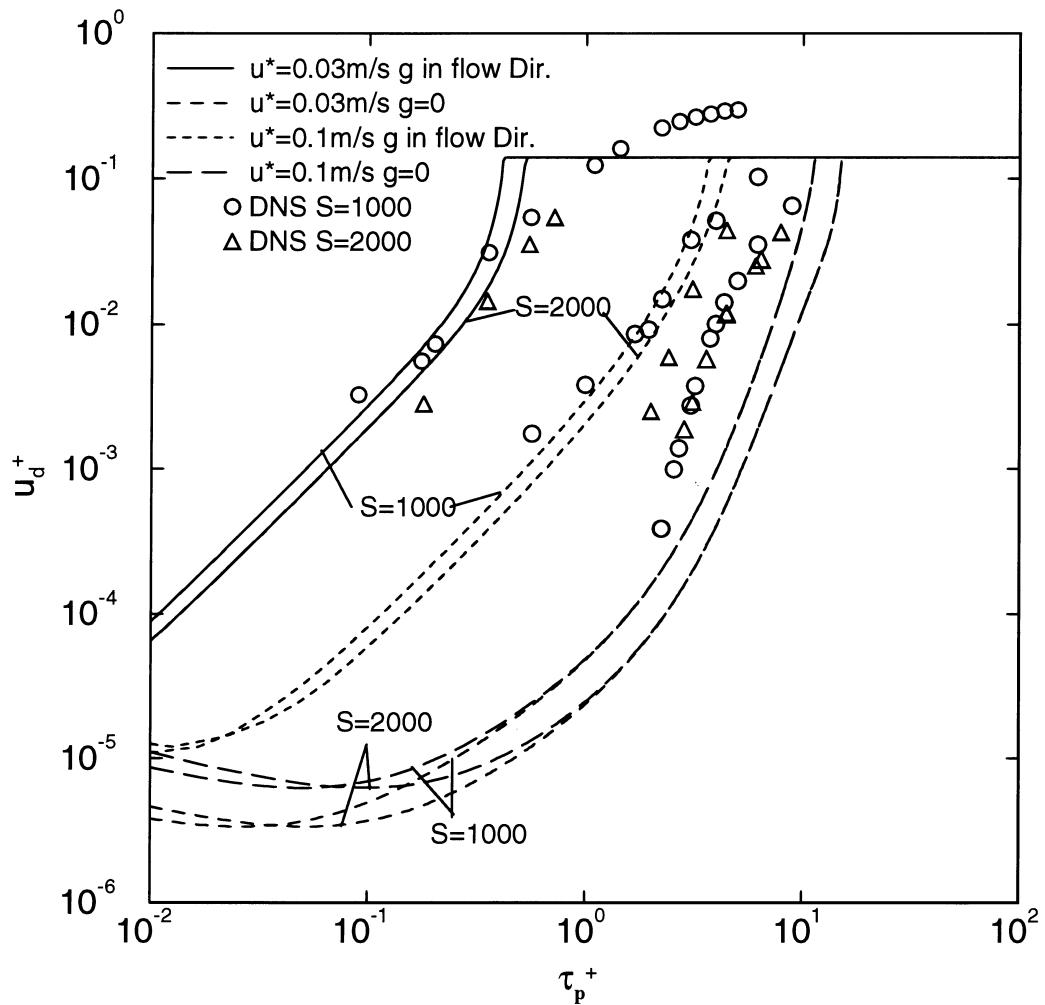
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Deposition Velocity

Vertical Ducts

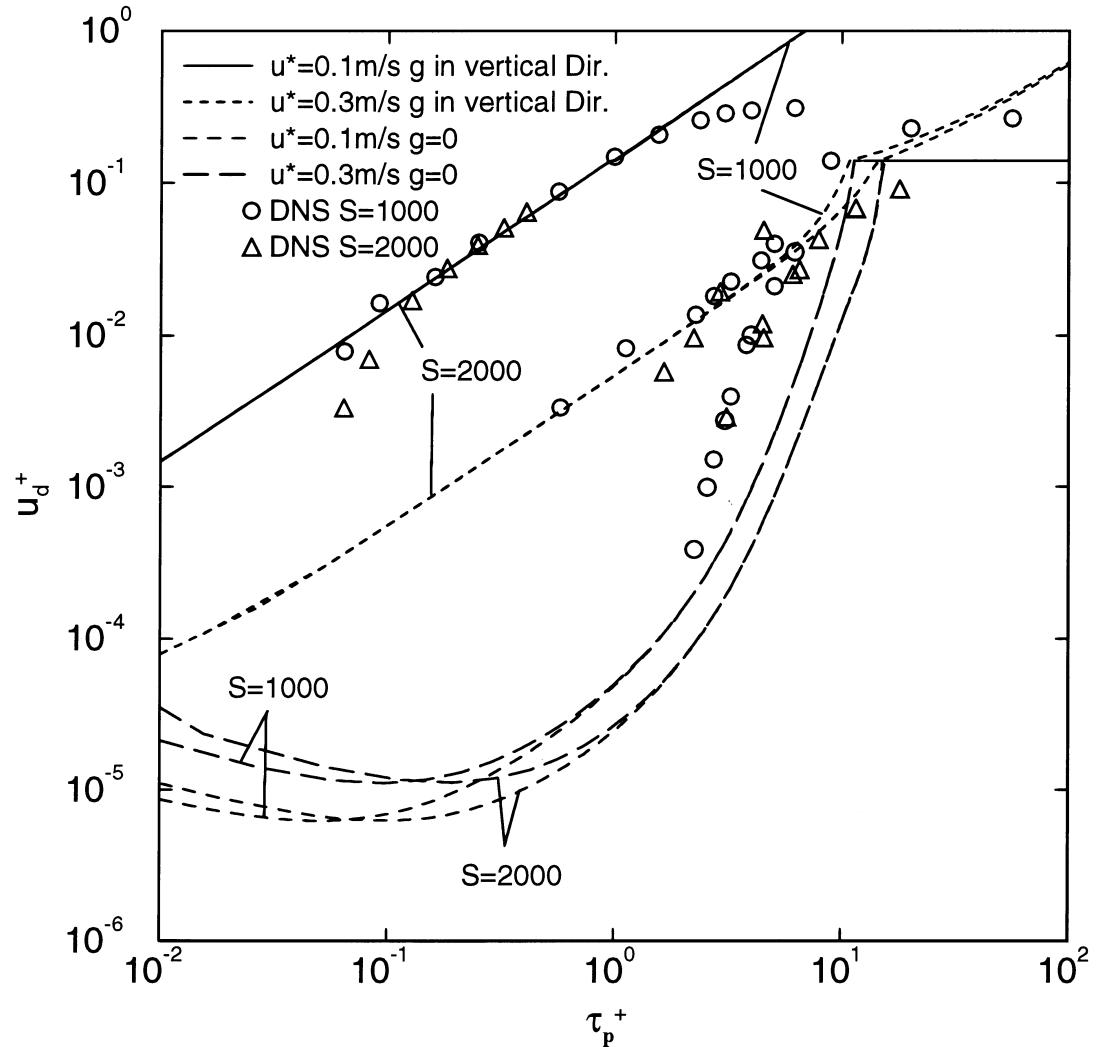
Effects of Shear Velocity and Density Ratio



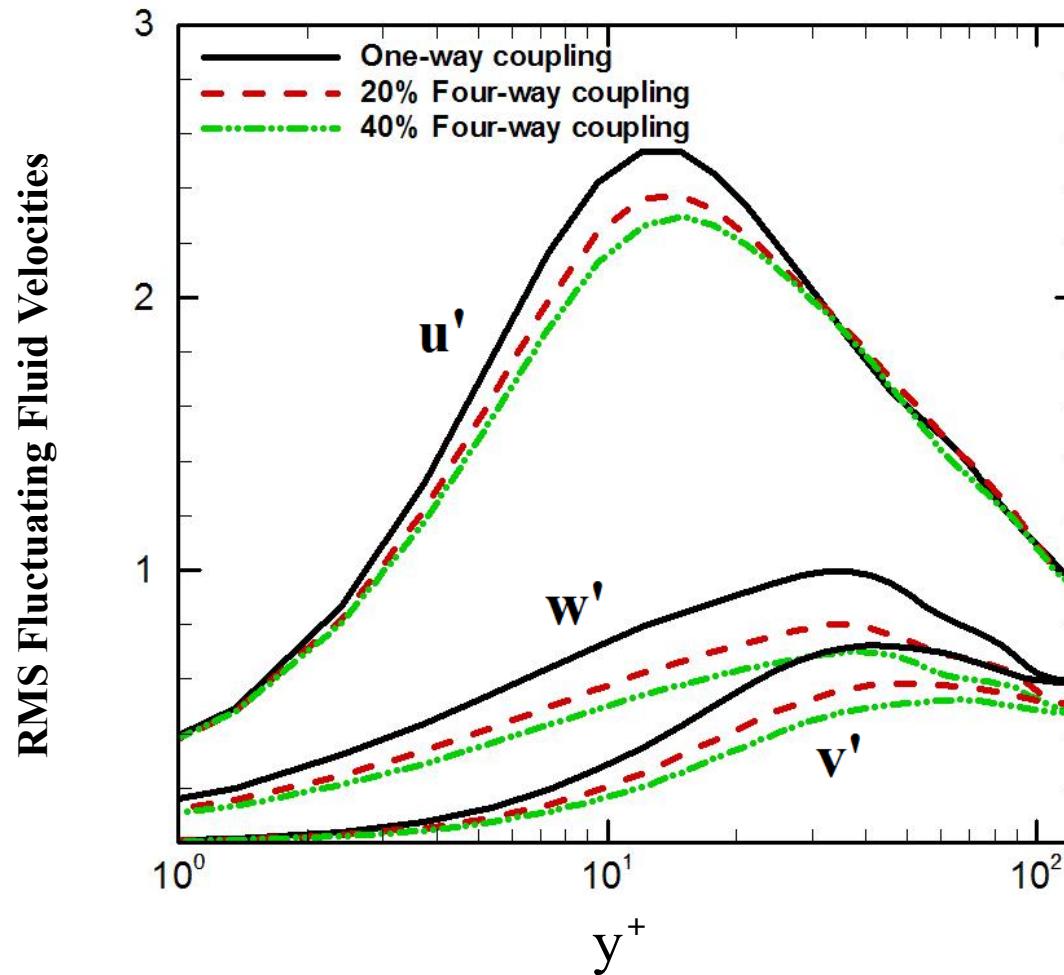
Deposition Velocity

Horizontal Ducts

Effects of Shear
Velocity and
Density Ratio

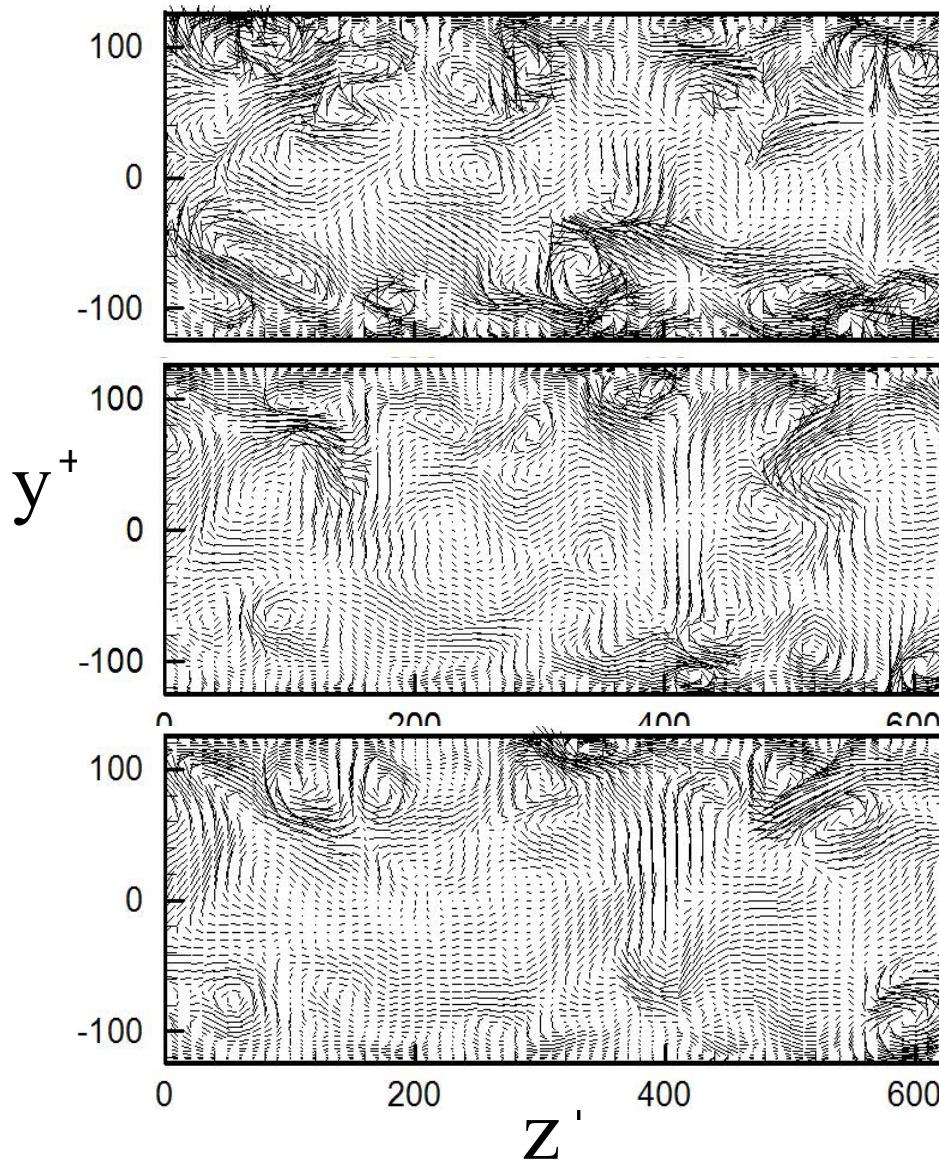


RMS Velocities



Effects of $d=30$ micron particles on the flow fluctuating velocities.

Sample Velocity Vector Plots

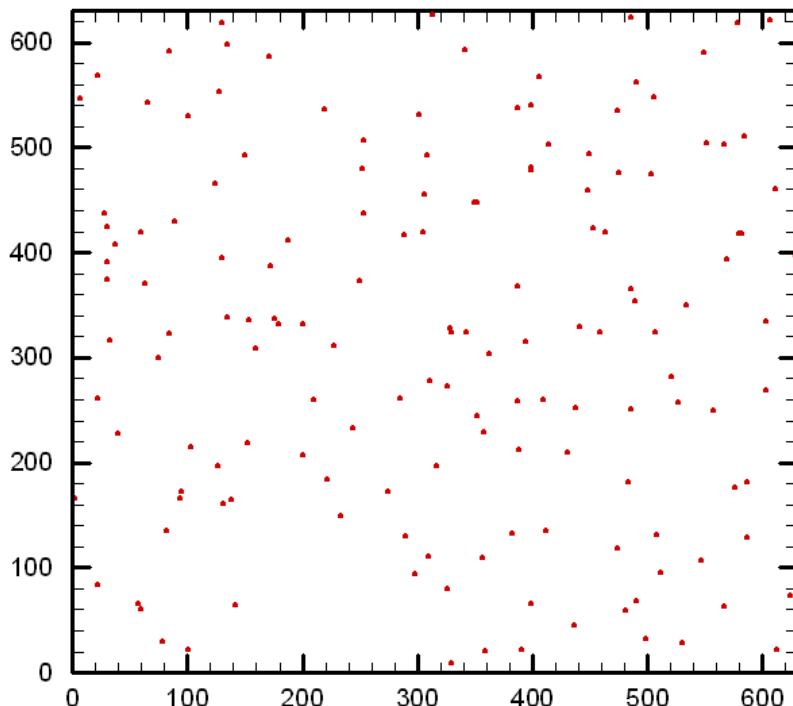


One-way coupling

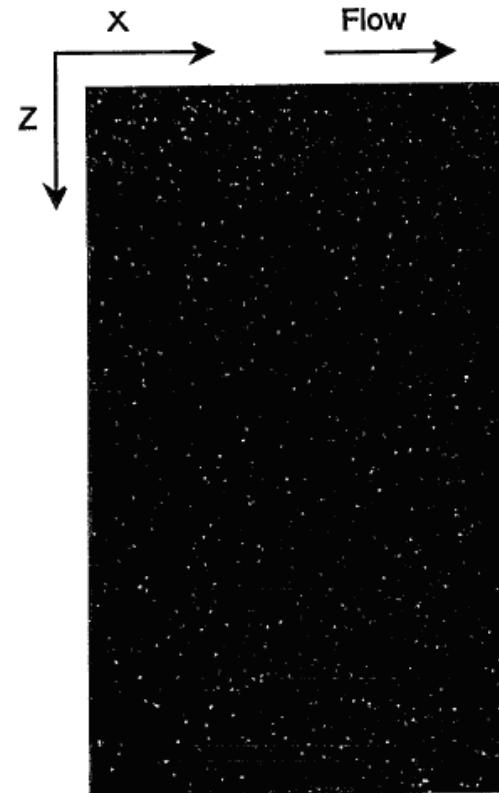
**Four-way coupling,
M.L.=20%**

**Four-way coupling,
M.L.=40%**

Particle Distribution

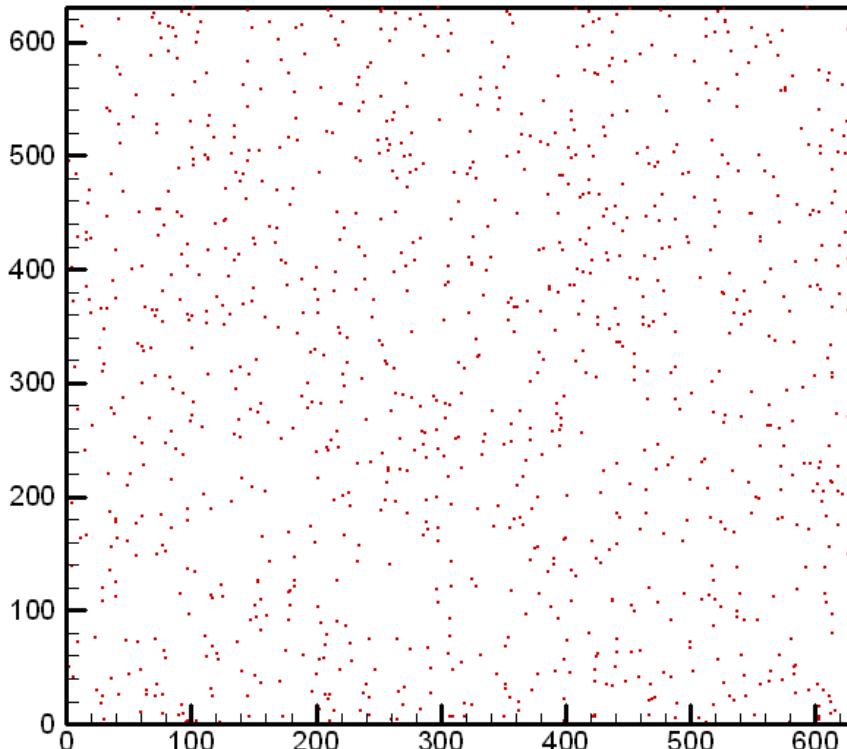


Fessler et al. (1994)

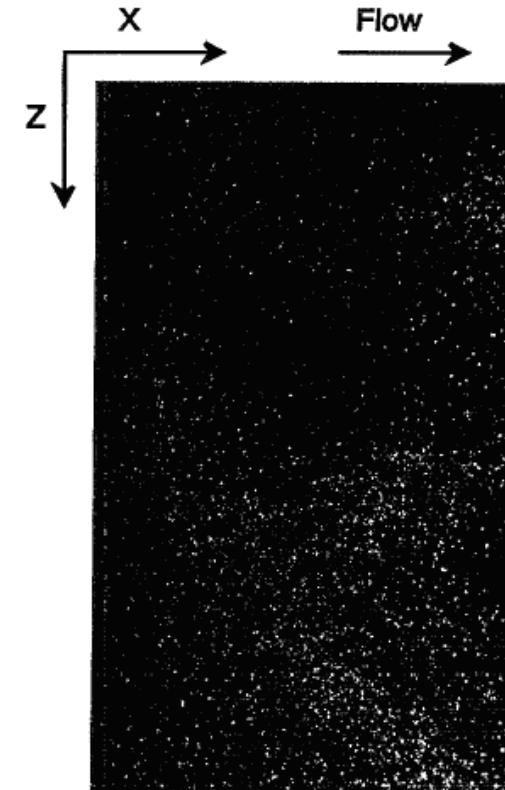


**Distribution of 70 micron copper particles
($S=7333$) at the channel center-plane.**

Particle Distribution

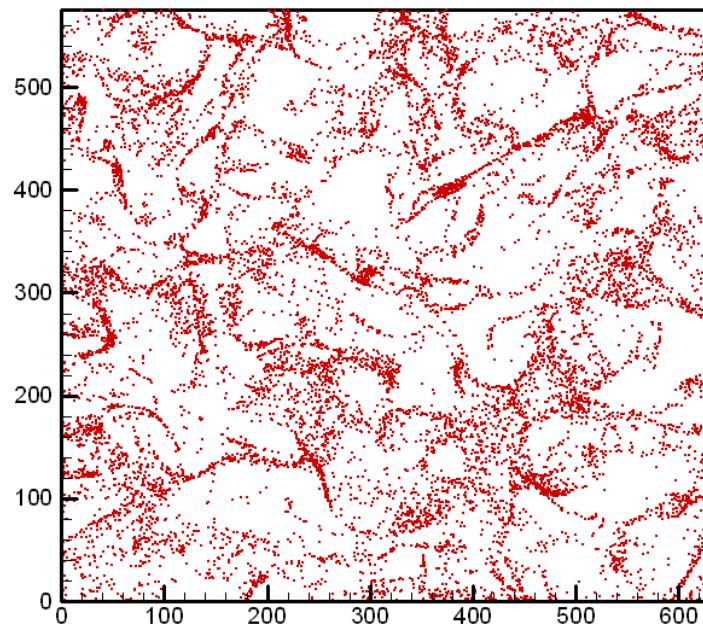


Fessler et al. (1994)

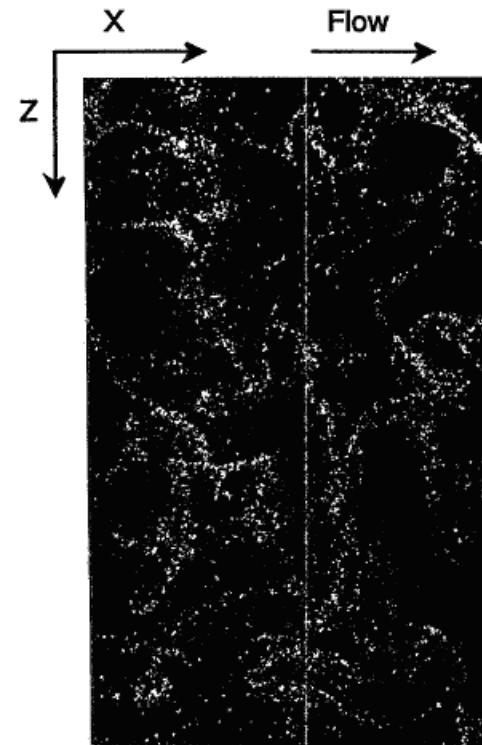


Distribution of 50 micron glass particles ($S=2030$) at the channel center-plane

Particle Distribution



Fessler et al. (1994)



Distribution of 28 micron Lycopodium particles ($S=826$) at the channel center-plane

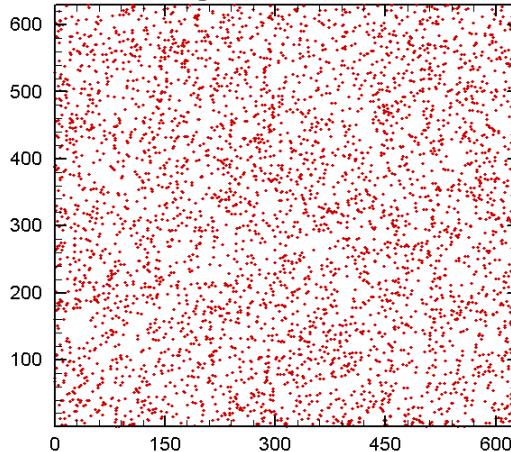
Particle Distribution

The lack of preferential concentration for copper and glass particles could be due to

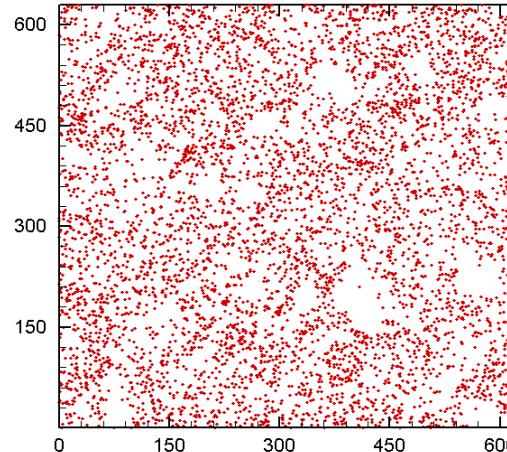
- **Particle Stokes number**
- **Inter-particle collisions**

Particle Distribution at the channel center-plane

$$\tau_p^+ = 10^{-4}$$

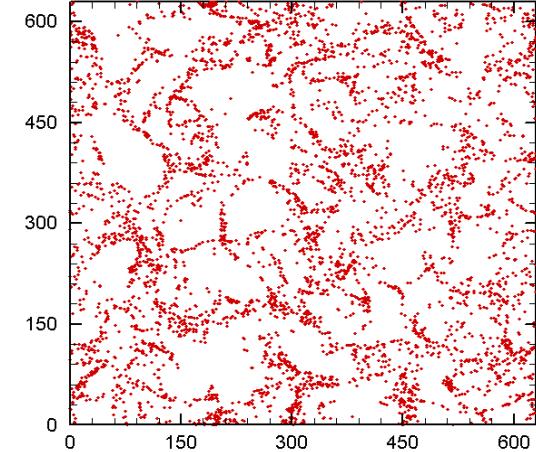


$$\tau_p^+ = 1.0$$

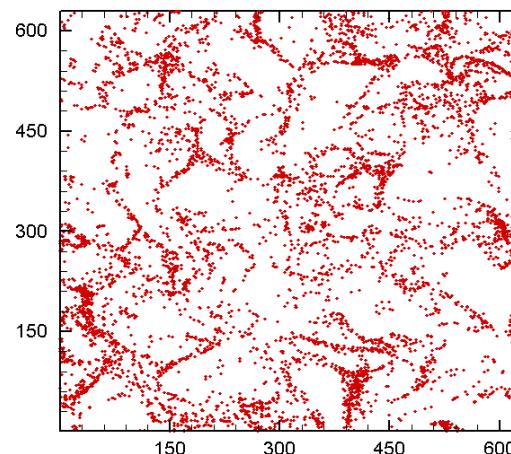


One Way Coupling

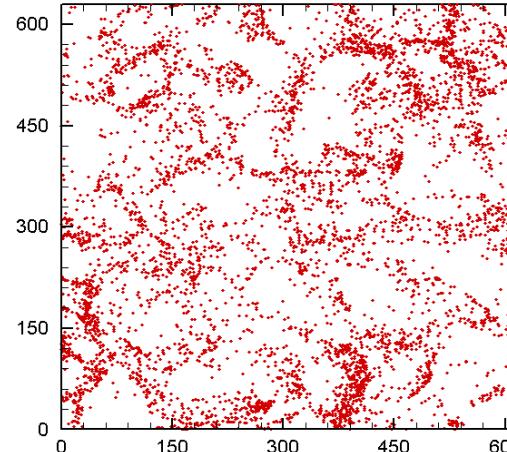
$$\tau_p^+ = 4.4$$



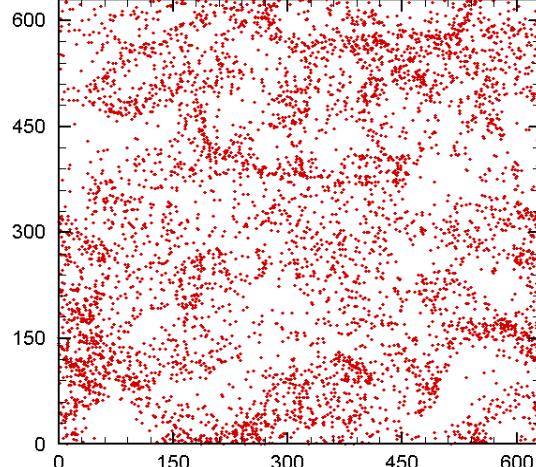
$$\tau_p^+ = 10.0$$



$$\tau_p^+ = 20$$



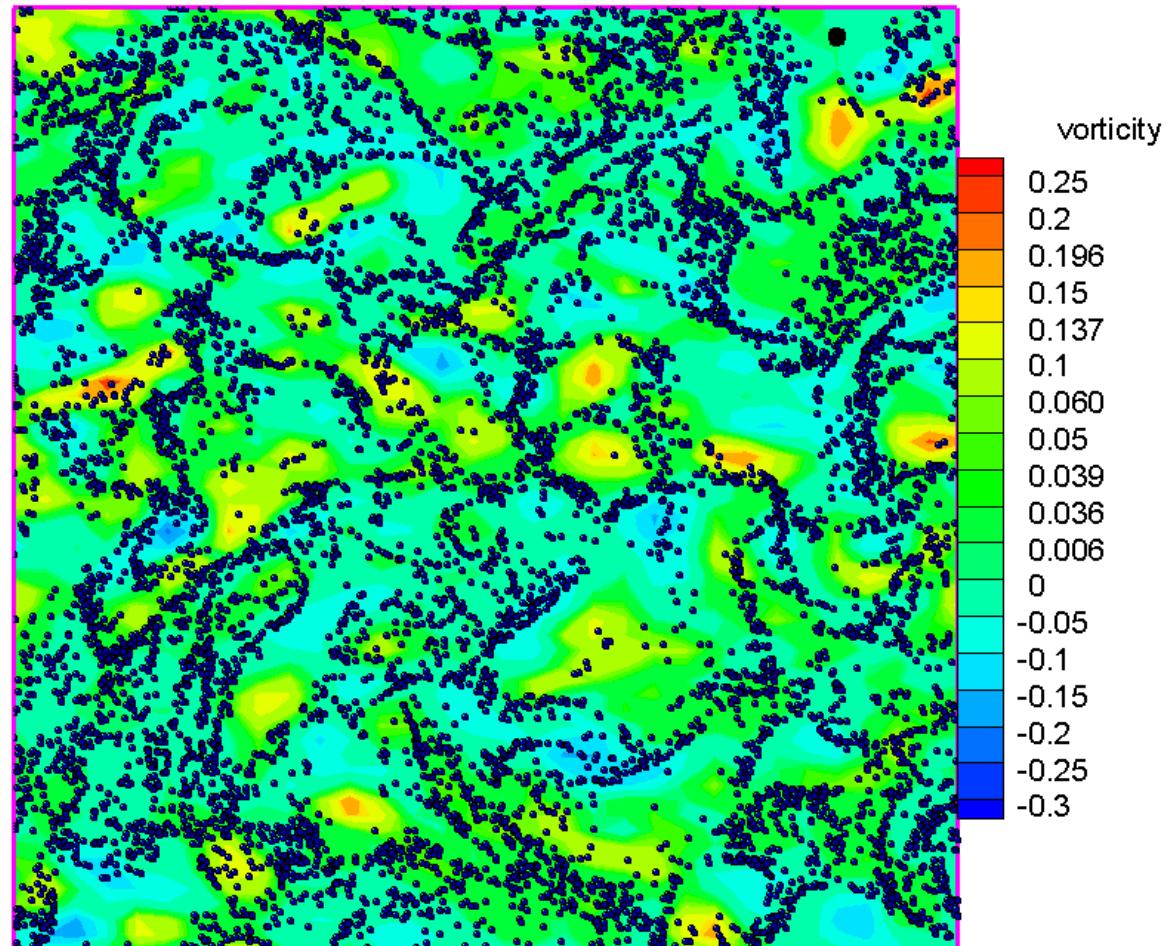
$$\tau_p^+ = 40.0$$



Particle Distribution and Vorticity Contours

**Channel
Center Plane**

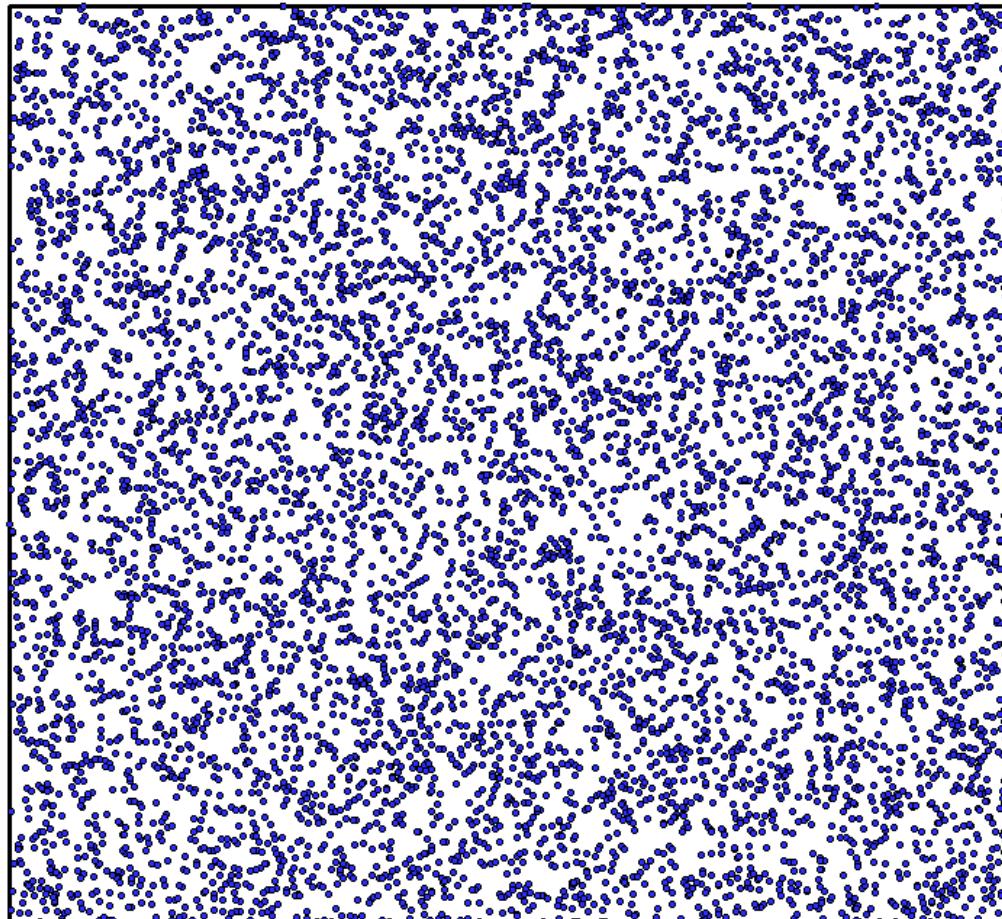
$$\tau_p^+ = 10.0$$



Particle Distribution at Channel Center-Plane

Frame 001 | 06 Dec 2006 |

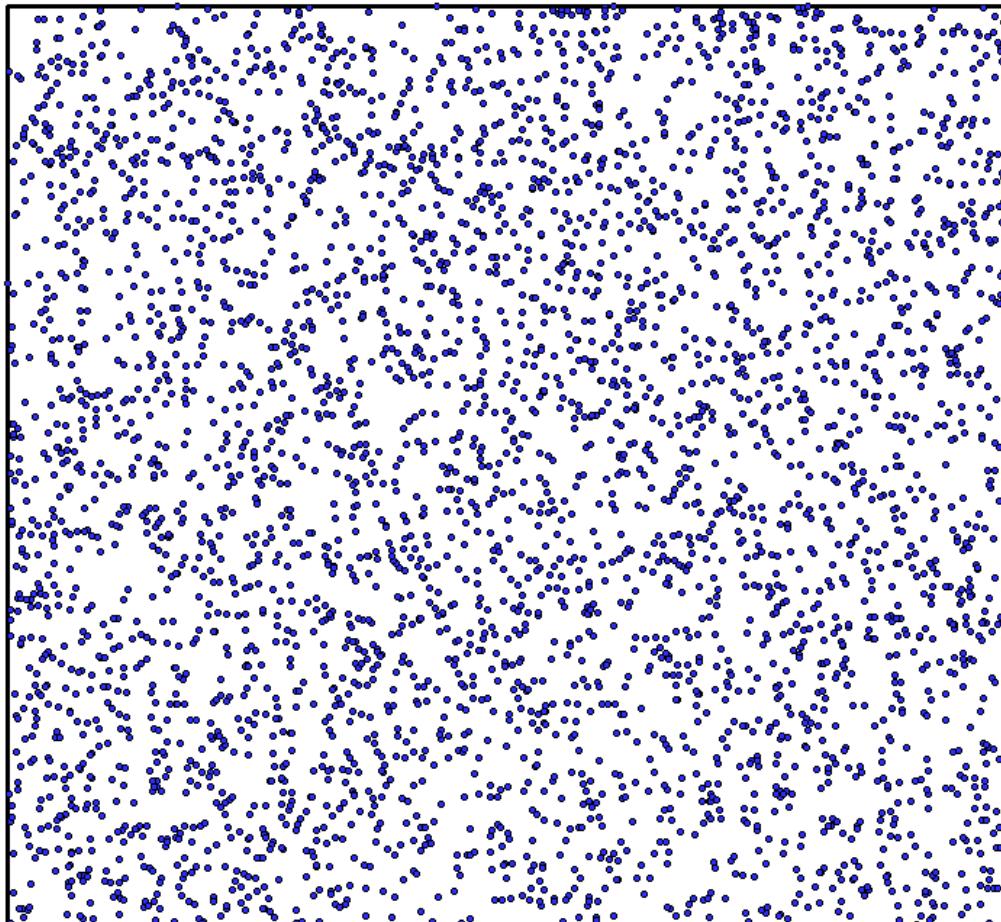
$\tau_p^+ = 10.0$



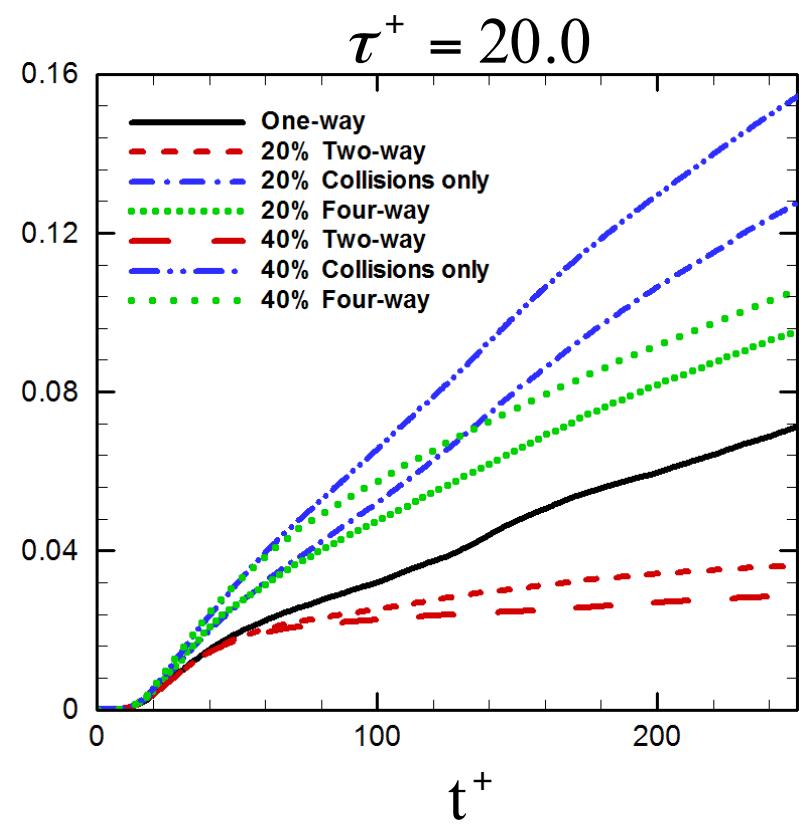
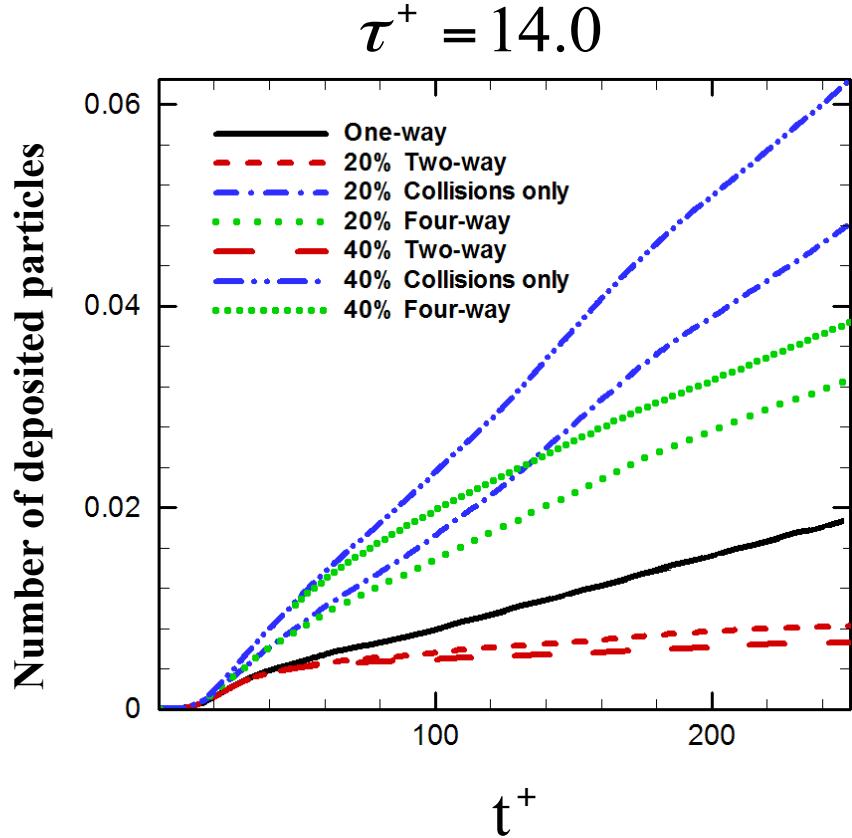
Particle Distribution in the Wall Region

$$\tau_p^+ = 10.0$$

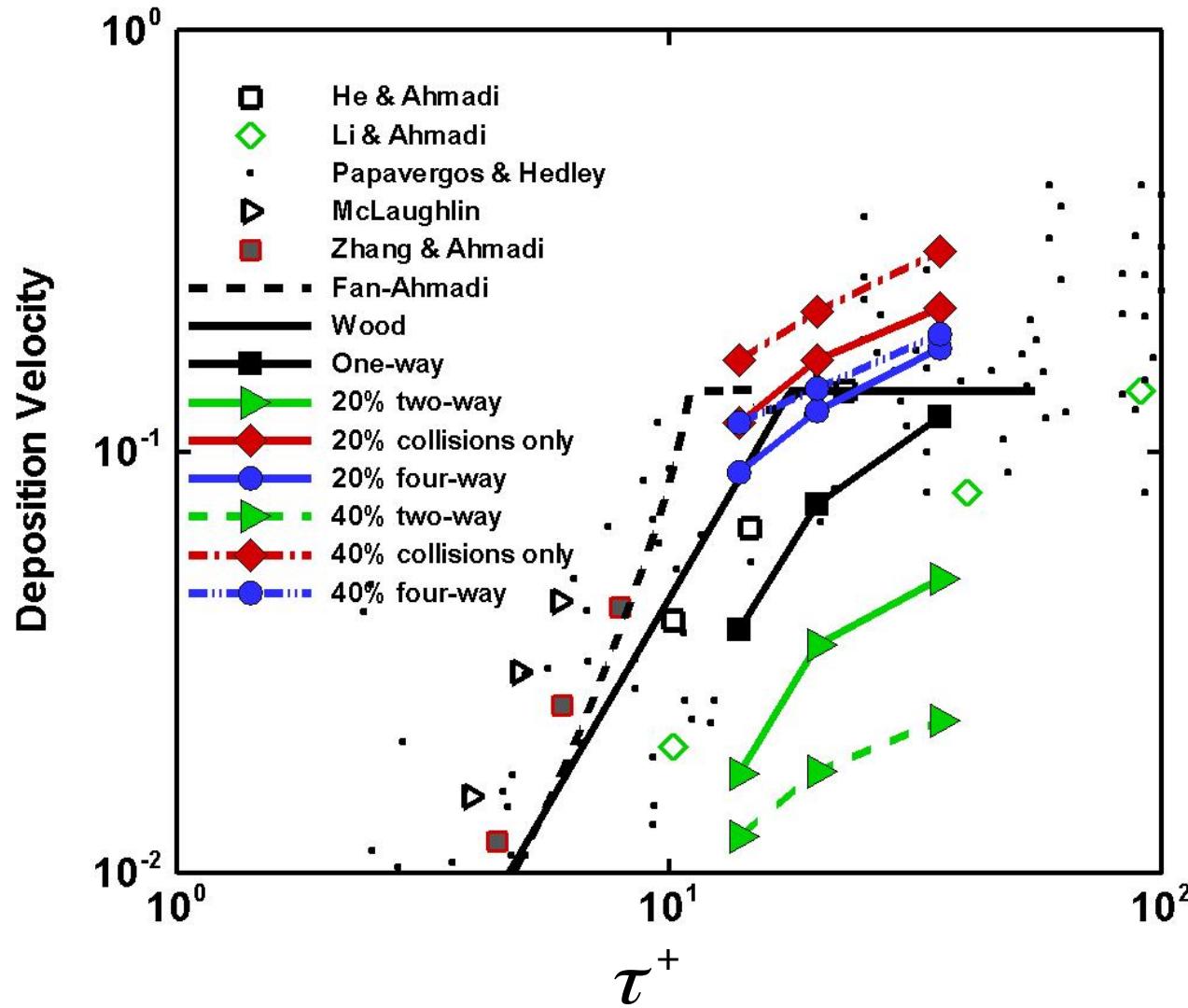
Frame 001 | 08 Dec 2006 |



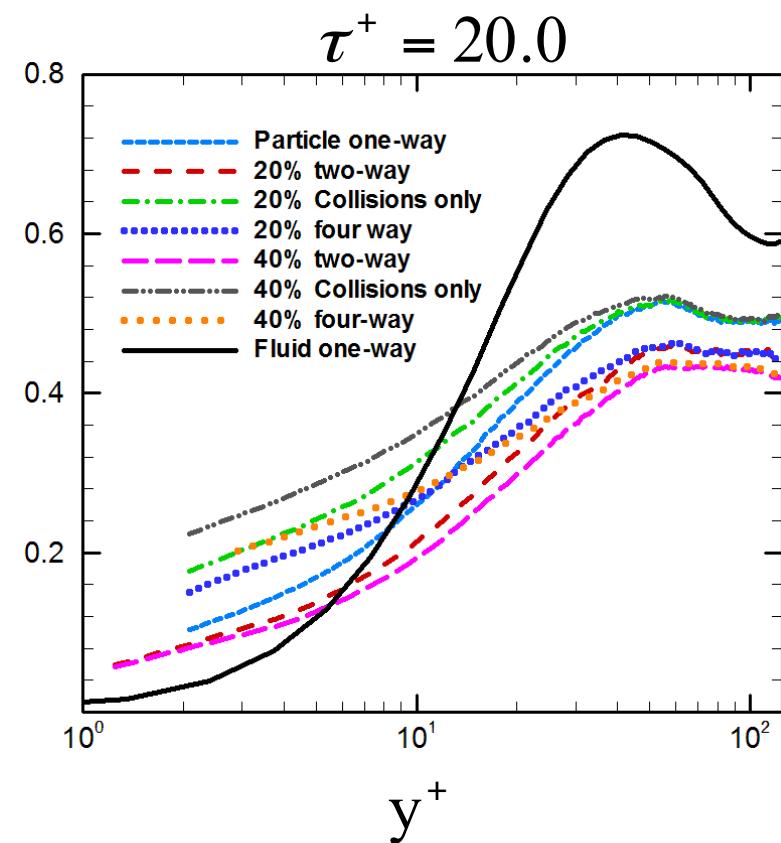
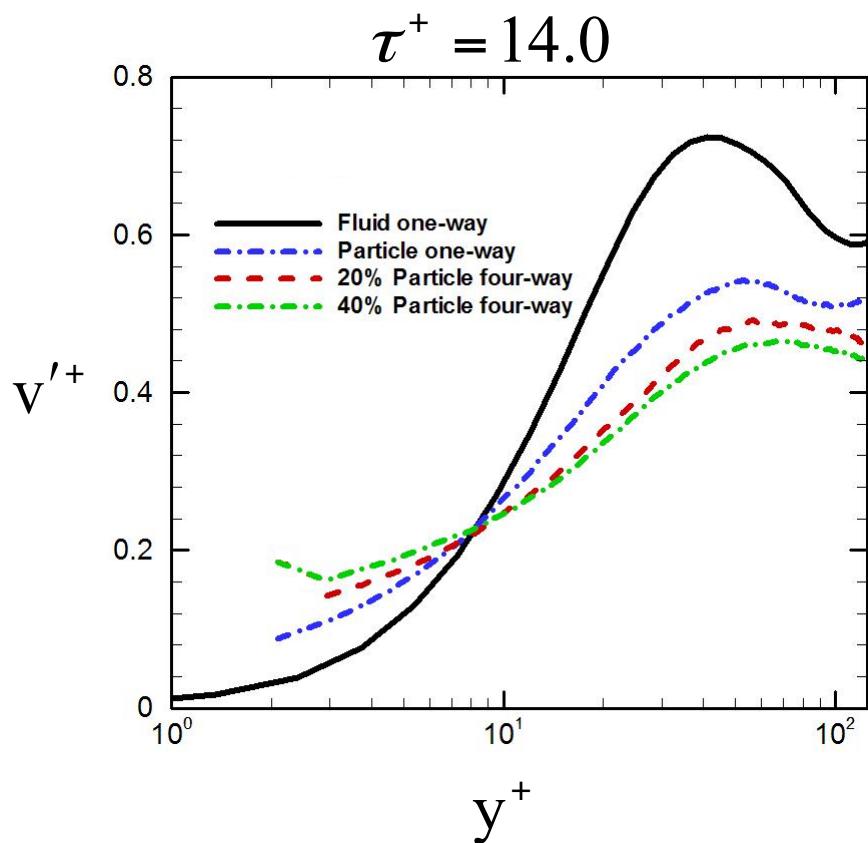
Numerber of Deposited Particles Versus Time



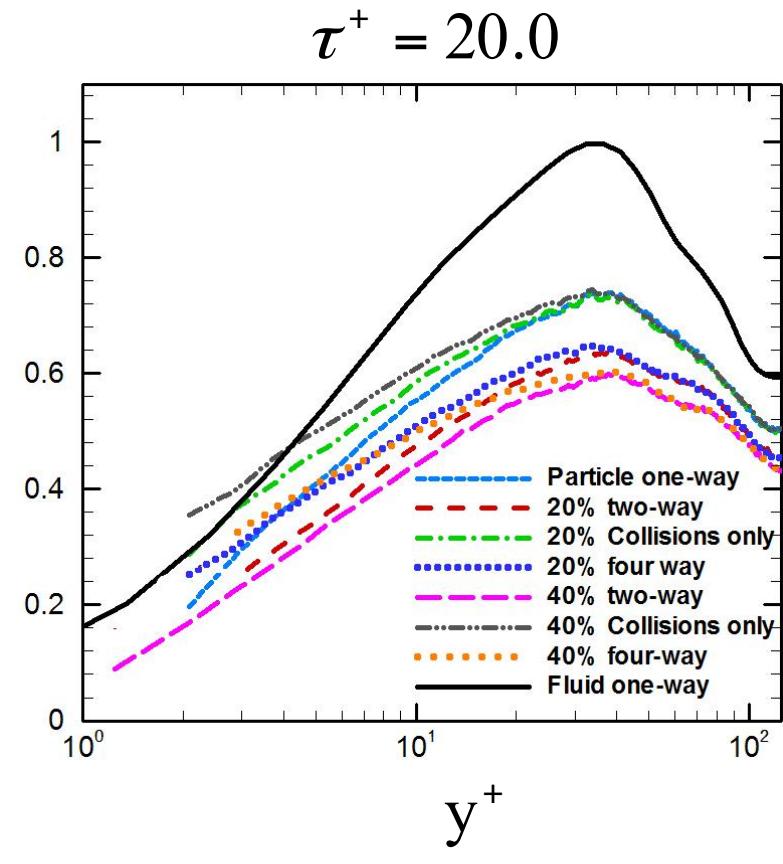
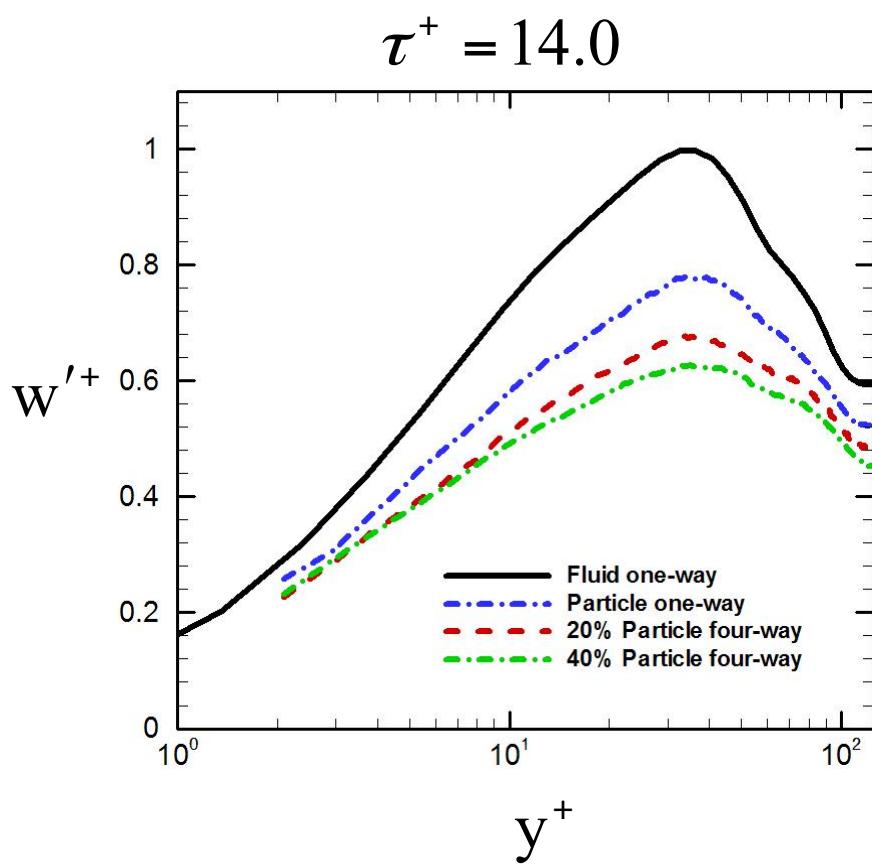
Particle Deposition Velocity



Particle Normal Fluctuating Velocity

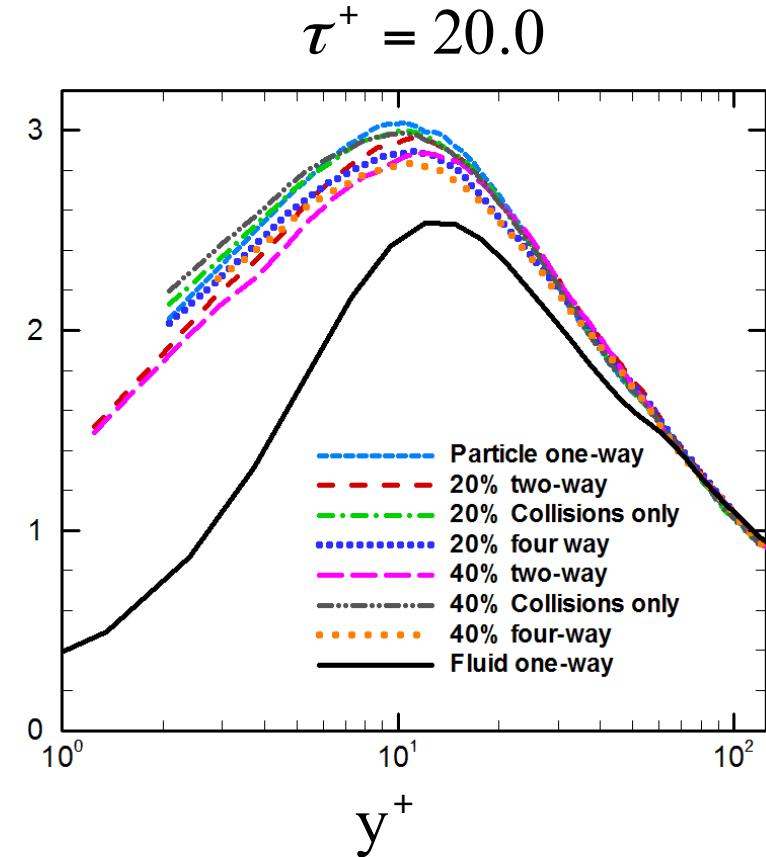
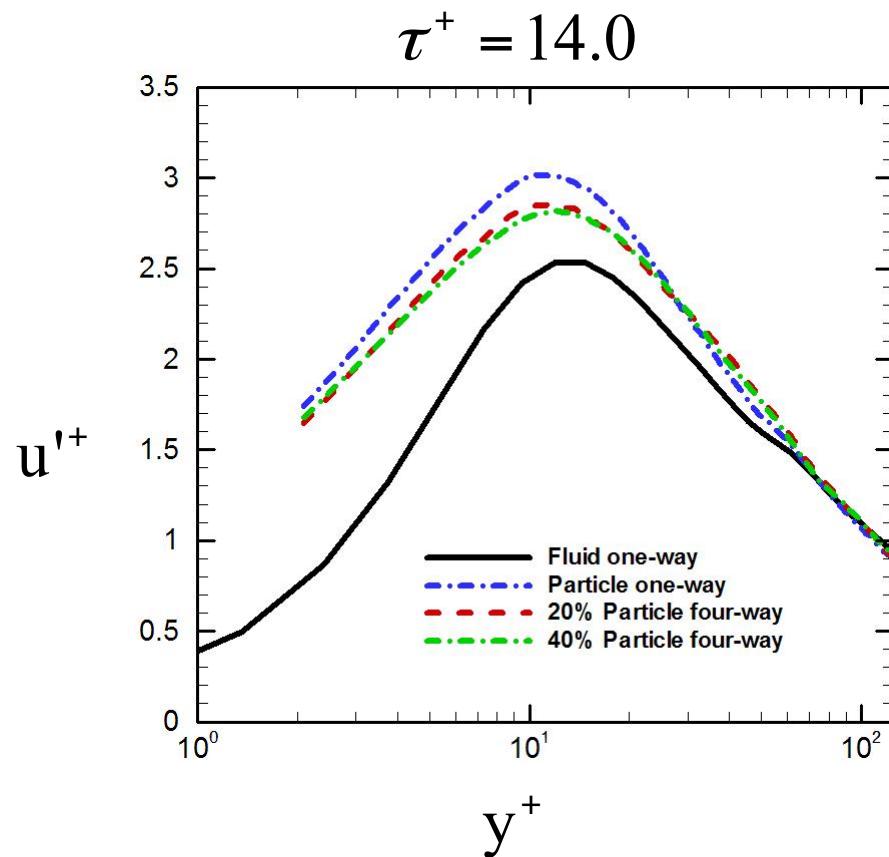


Particle Spanwise Fluctuating Velocity

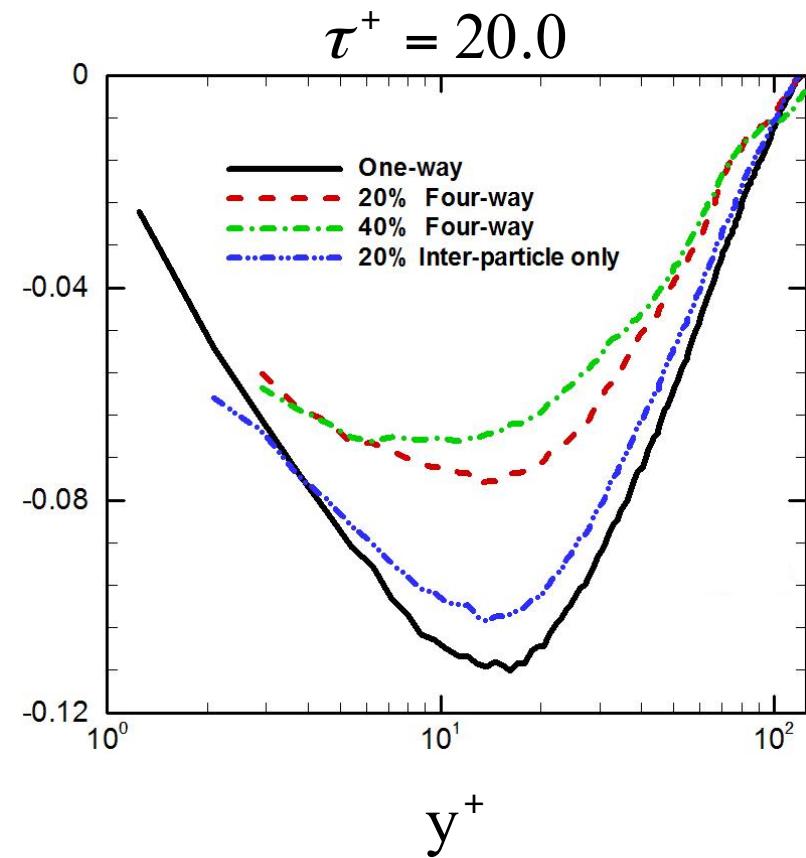
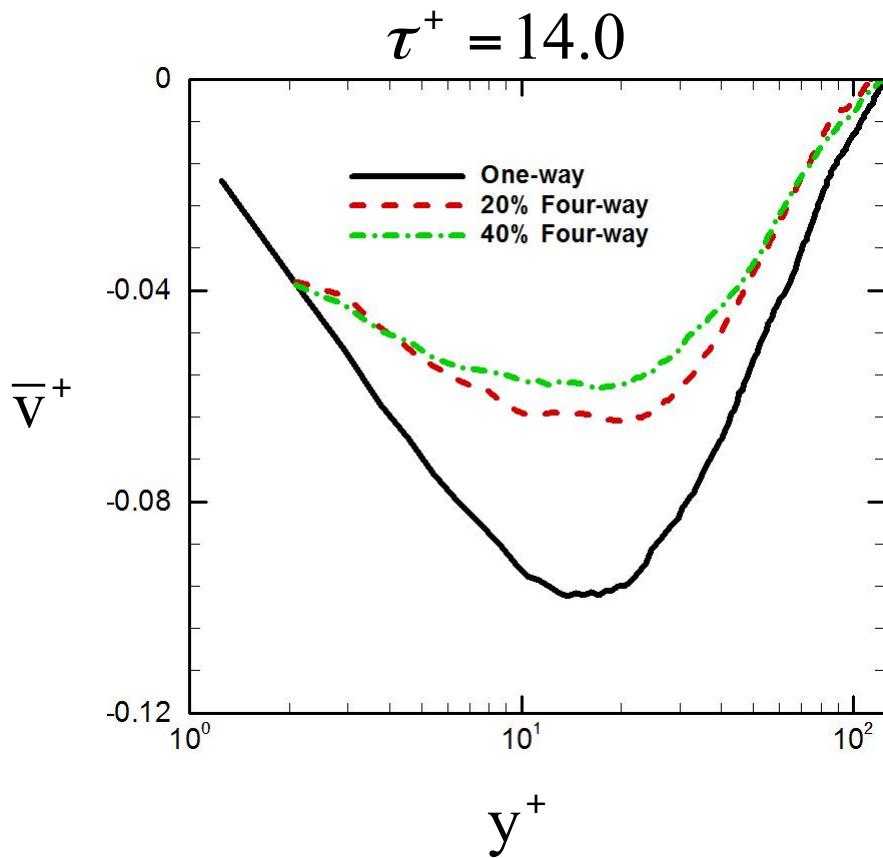


Particle Streamwise Fluctuating Velocity

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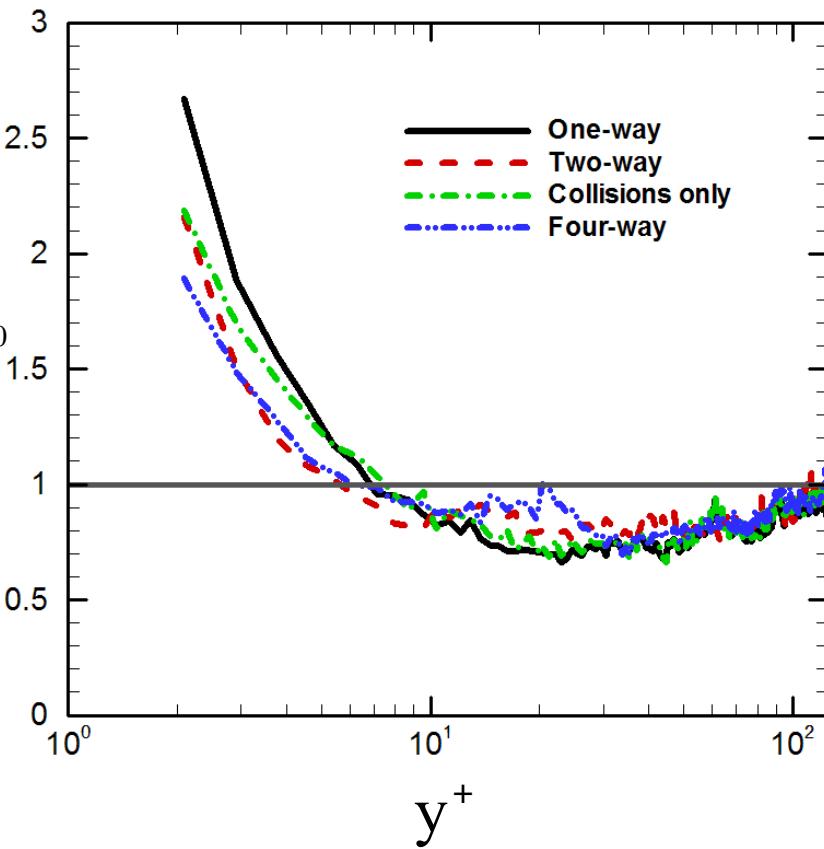
Particle Mean Normal Velocity



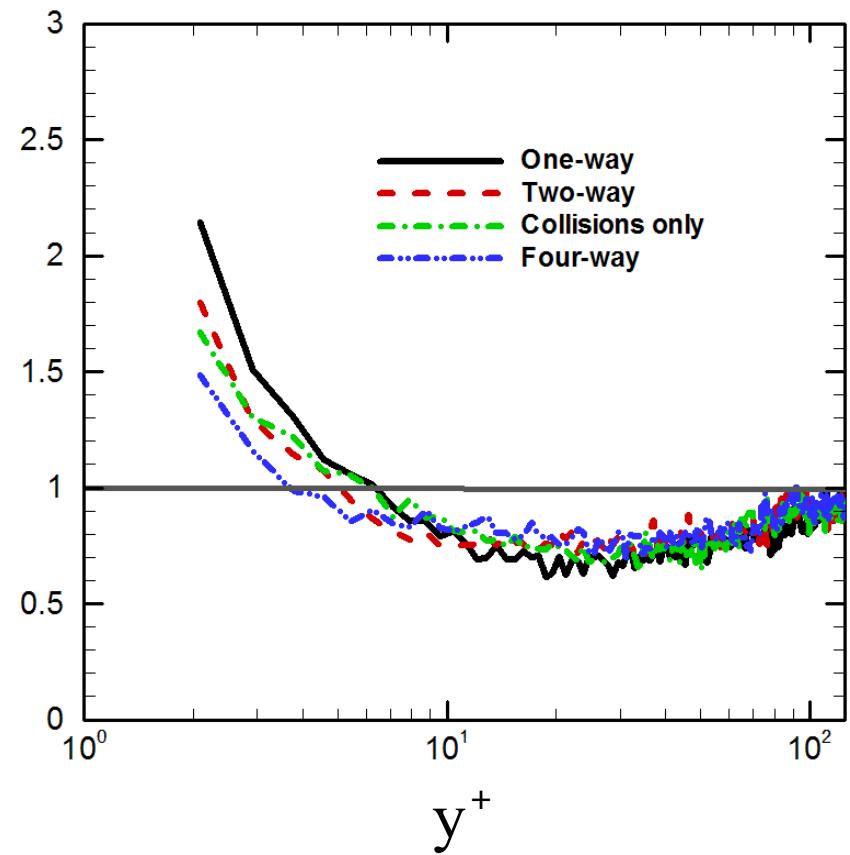
Particle Concentration Profiles

Mass Loading = 20%

$\tau^+ = 14.0$



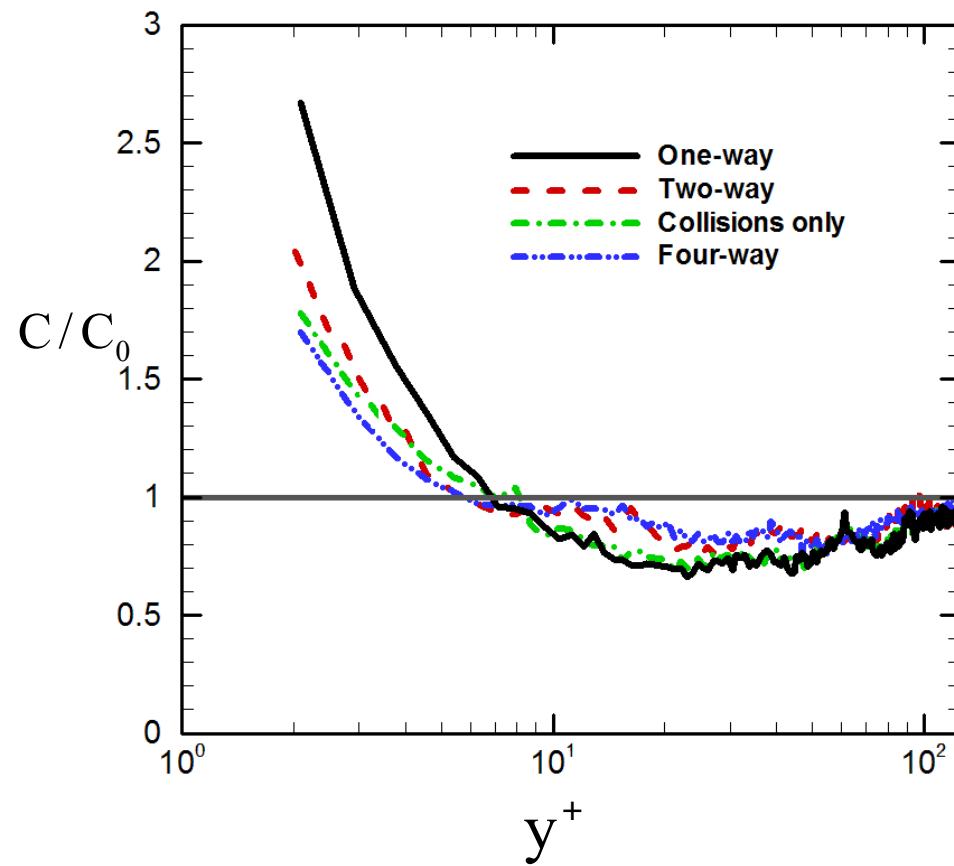
$\tau^+ = 20.0$



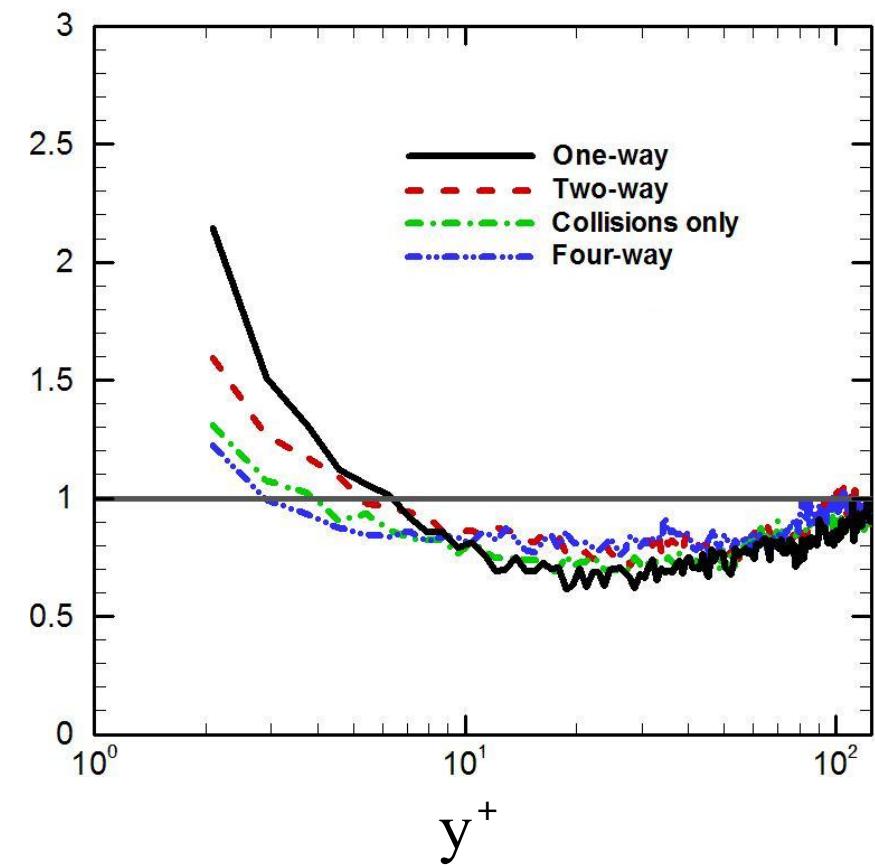
Particle Concentration Profiles

Mass Loading = 40%

$$\tau^+ = 14.0$$



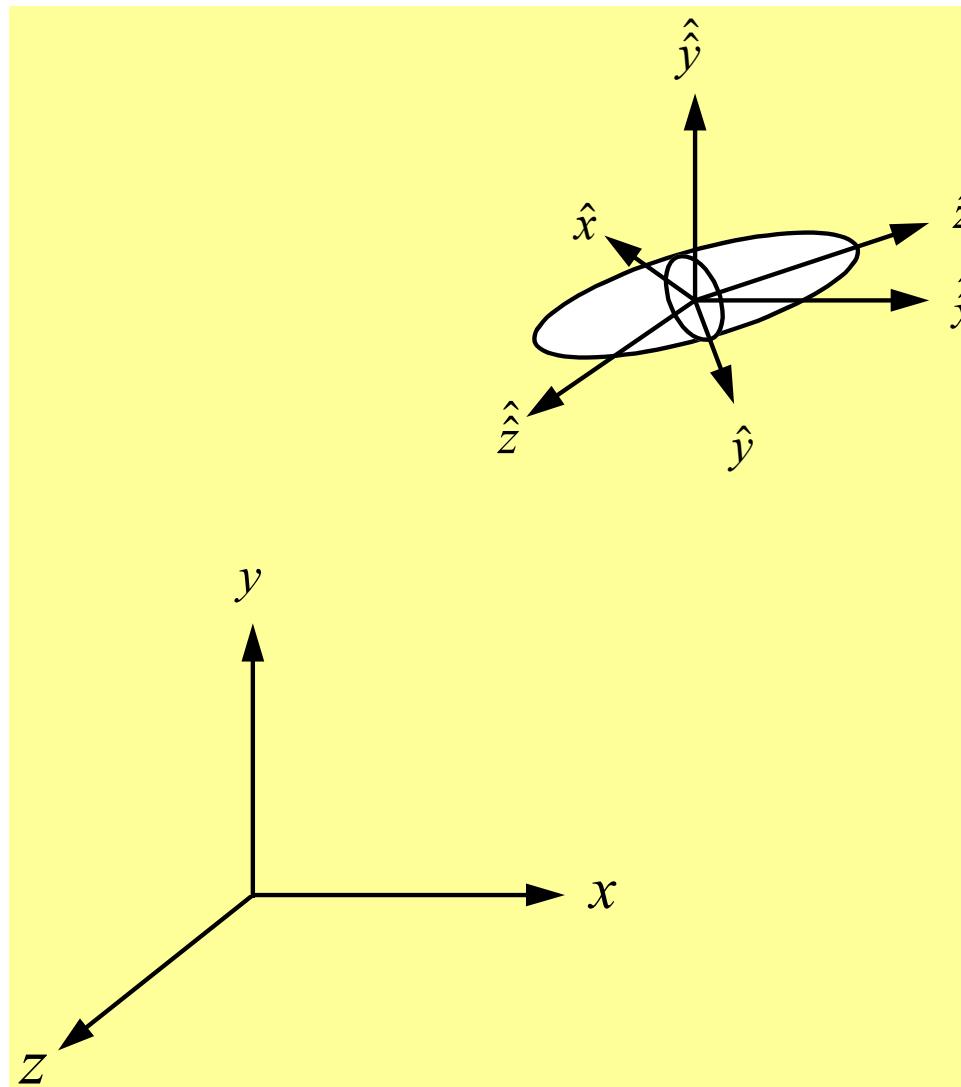
$$\tau^+ = 20.0$$



Fiber Transport and Deposition

Schematics of Ellipsoidal Fiber

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Fiber Equation of Motion

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$$m^p \frac{d\mathbf{v}}{dt} = (m^p - m^f)g + \mathbf{f}^h + \mathbf{f}^L$$

$$I_{\hat{x}} \frac{d\omega_{\hat{x}}}{dt} - \omega_{\hat{y}}\omega_{\hat{z}}(I_{\hat{y}} - I_{\hat{z}}) = T_{\hat{x}}^h$$

$$I_{\hat{y}} \frac{d\omega_{\hat{y}}}{dt} - \omega_{\hat{z}}\omega_{\hat{x}}(I_{\hat{z}} - I_{\hat{x}}) = T_{\hat{y}}^h$$

$$I_{\hat{z}} \frac{d\omega_{\hat{z}}}{dt} - \omega_{\hat{x}}\omega_{\hat{y}}(I_{\hat{x}} - I_{\hat{y}}) = T_{\hat{z}}^h$$

Fiber Equation of Motion

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Drag

$$\mathbf{f}^h = \mu \pi a \hat{\mathbf{K}} \cdot (\mathbf{u} - \mathbf{v})$$

Shear Lift

$$\mathbf{f}^L = \frac{\pi^2 \mu a^2}{\mathbf{v}^{1/2}} \frac{\partial \mathbf{u}_x / \partial y}{|\partial \mathbf{u}_x / \partial y|^{1/2}} \left(\hat{\mathbf{K}} \cdot \mathbf{L} \cdot \hat{\mathbf{K}} \right) \cdot (\mathbf{u} - \mathbf{v})$$

$$\hat{\hat{\mathbf{K}}} = \mathbf{A}^{-1} \hat{\mathbf{K}} \mathbf{A}$$

$$k_{\hat{x}\hat{x}} = k_{\hat{y}\hat{y}} = \left[\frac{16(\beta^2 - 1)}{(2\beta^2 - 3) \ln(\beta + \sqrt{\beta^2 - 1}) / \sqrt{\beta^2 - 1}} \right] + \beta$$

$$k_{\hat{z}\hat{z}} = \left[\frac{8(\beta^2 - 1)}{(2\beta^2 - 1) \ln(\beta + \sqrt{\beta^2 - 1}) / \sqrt{\beta^2 - 1}} \right] - \beta$$

Equivalent Relaxation Time Shapiro-Goldenberg

$$\tau_{eq}^+ = \frac{4\beta Sa^{+2}}{9} \left(\frac{1}{k_{\hat{x}\hat{x}}} + \frac{1}{k_{\hat{y}\hat{y}}} + \frac{1}{k_{\hat{z}\hat{z}}} \right) = \frac{2\beta Sa^{+2}}{9} \frac{\ln(\beta + \sqrt{\beta^2 - 1})}{\sqrt{\beta^2 - 1}}$$

Fiber Equation of Motion

Equivalent Relaxation Time (Fan-Ahmadi)

$$\tau_{eq}^+ = \frac{4\beta Sa^{+2}}{k_{\hat{x}\hat{x}} + k_{\hat{y}\hat{y}} + k_{\hat{z}\hat{z}}}$$

Hydrodynamic Torque

$$T_x^h = \frac{16\pi\mu a^3 \beta}{3(\beta_0 + \beta^2 \gamma_0)} [(1 - \beta^2) d_{\hat{z}\hat{y}} + (1 + \beta^2) (w_{\hat{z}\hat{y}} - \omega_{\hat{x}})]$$

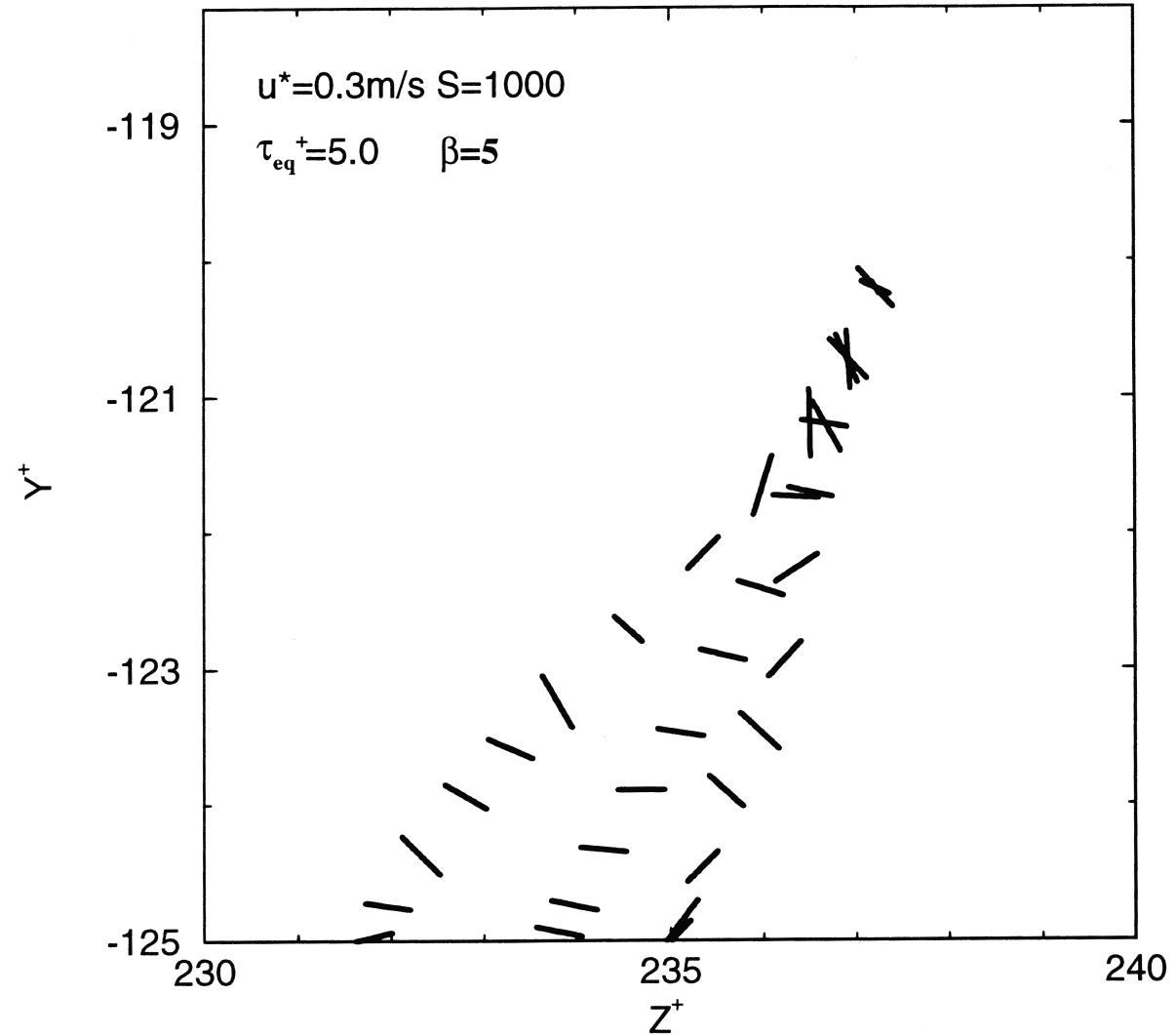
$$T_y^h = \frac{16\pi\mu a^3 \beta}{3(\alpha_0 + \beta^2 \gamma_0)} [(\beta^2 - 1) d_{\hat{x}\hat{z}} + (1 + \beta^2) (w_{\hat{x}\hat{z}} - \omega_{\hat{y}})]$$

$$T_z^h = \frac{32\pi\mu a^3 \beta}{3(\alpha_0 + \beta_0)} (w_{\hat{y}\hat{z}} - \omega_{\hat{z}})$$

Sample Fiber Trajectories

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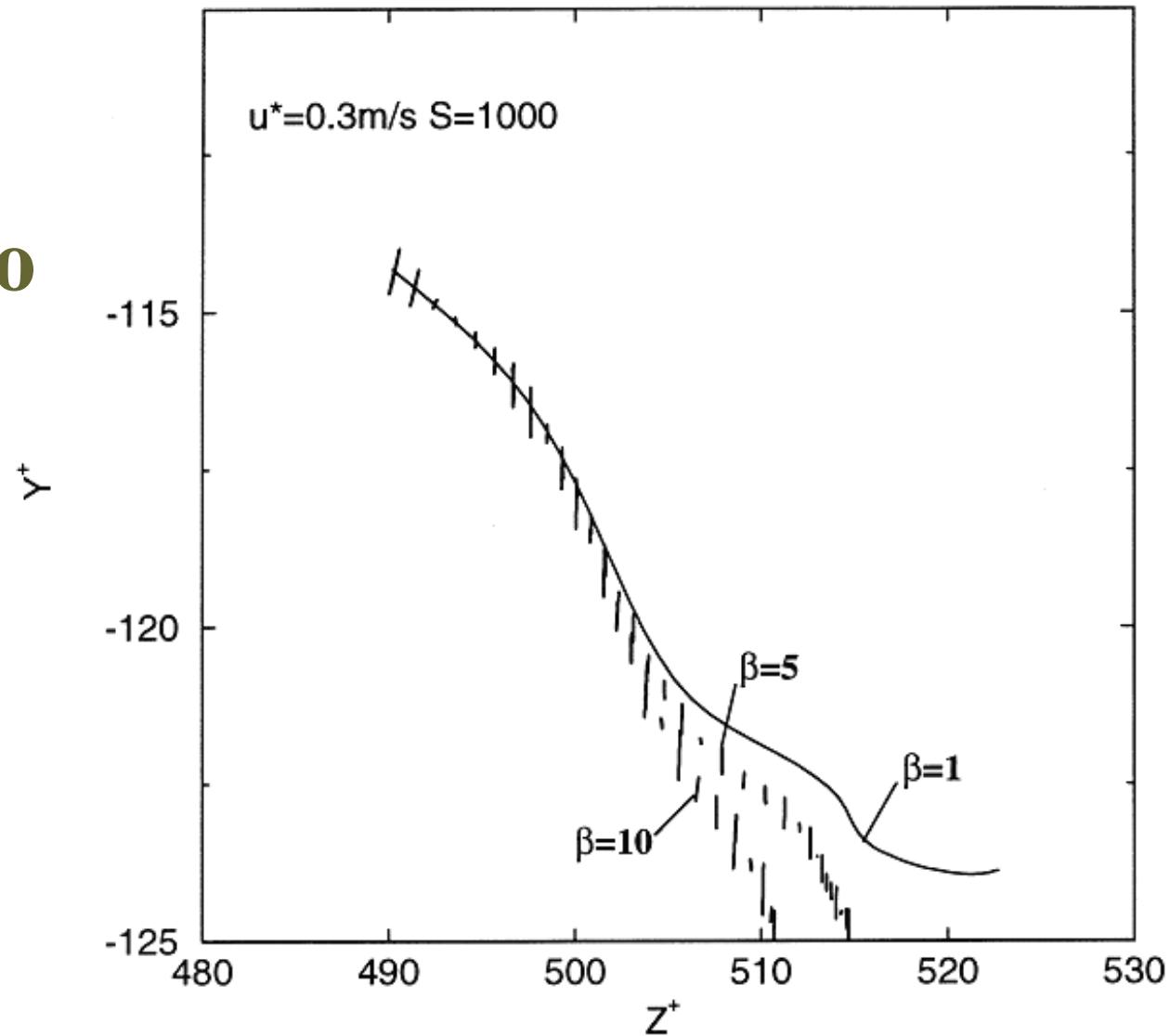
Effects of Initial Orientation



Sample Fiber Trajectories

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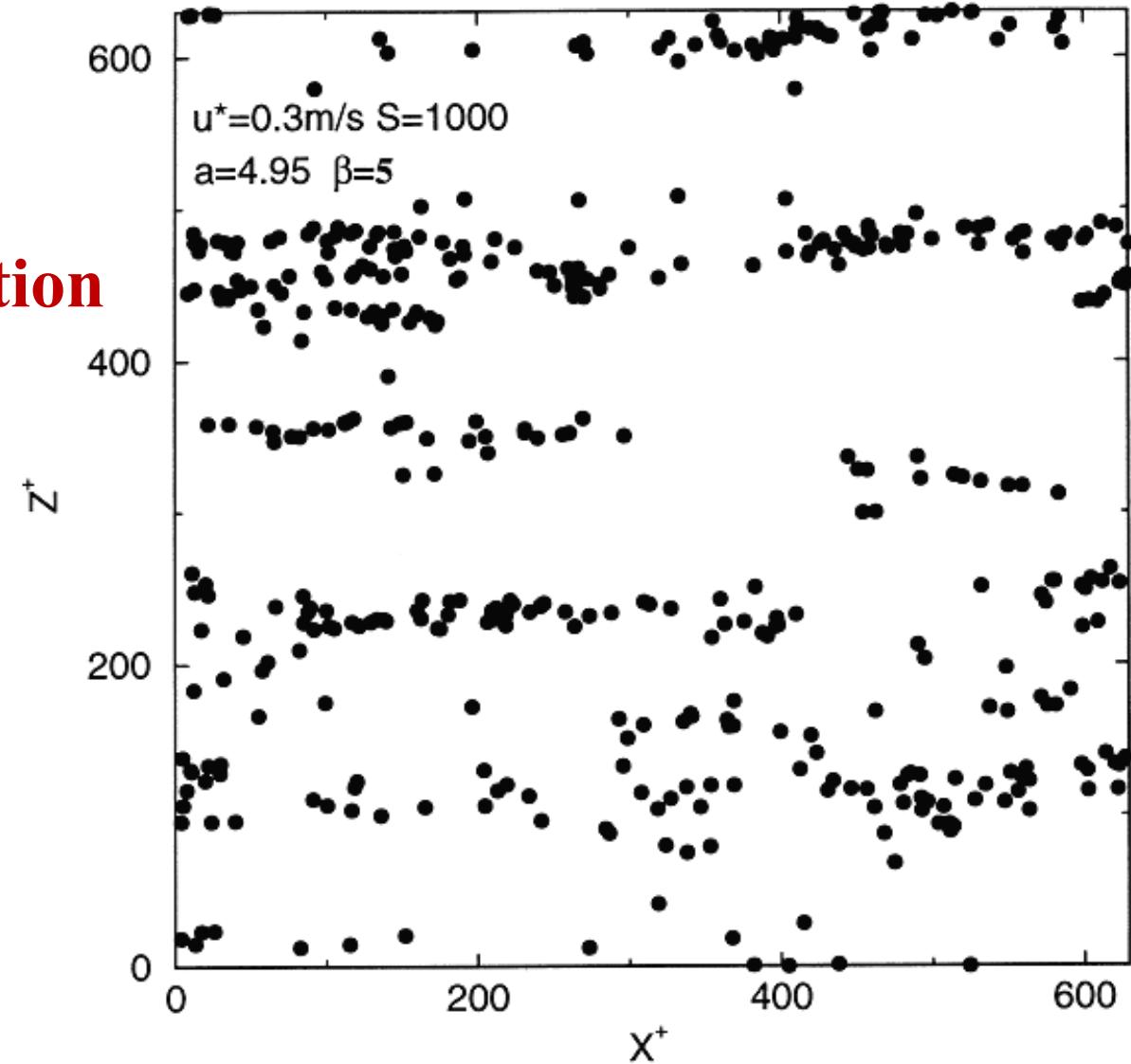
Effects of Aspect Ratio



Fiber Deposition Pattern

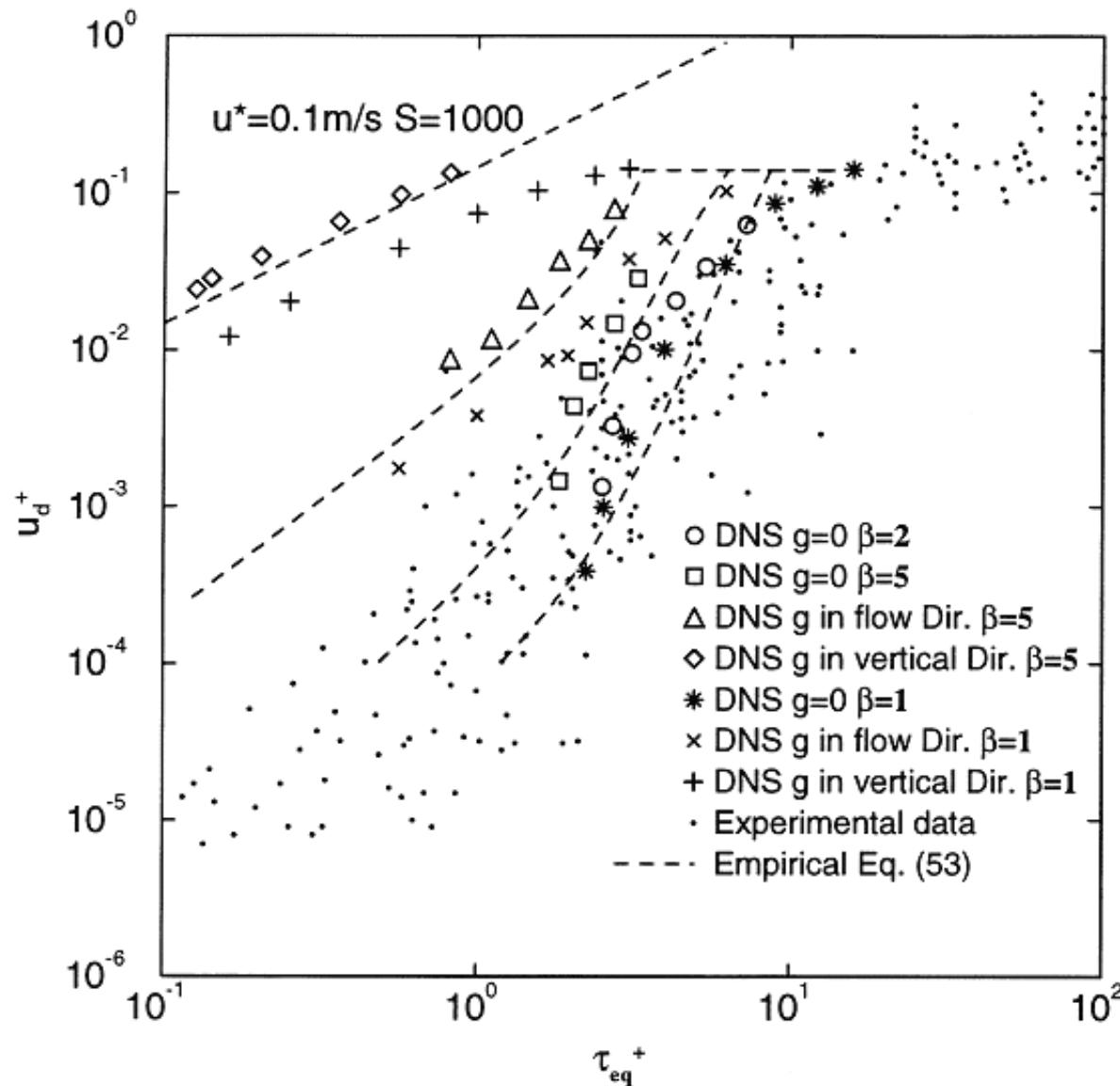
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Mean Flow Direction



Fiber Deposition Velocity

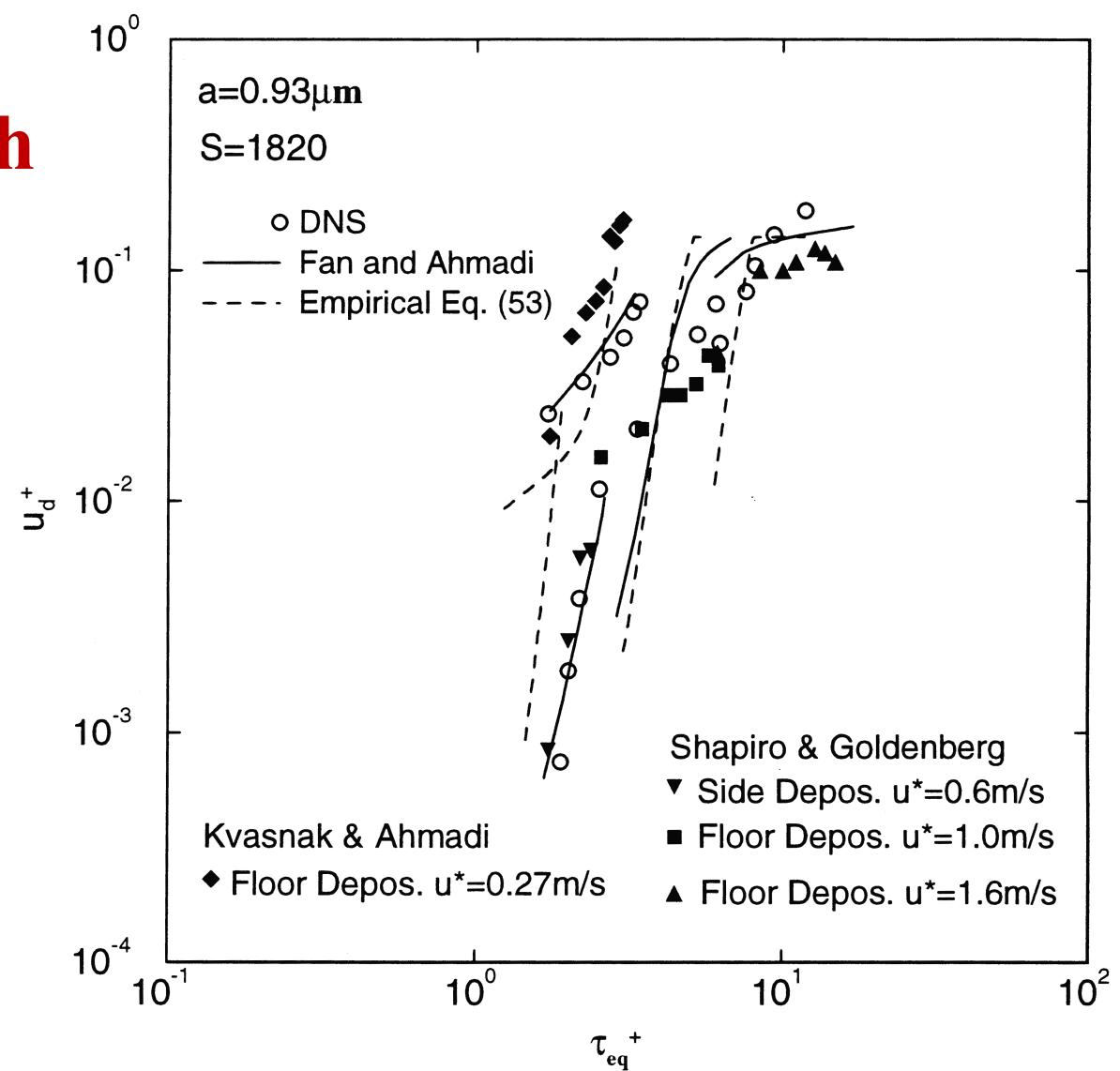
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Fiber Deposition Velocity

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Comparison with Experimental Data



Empirical Equation for Fiber Deposition Velocity

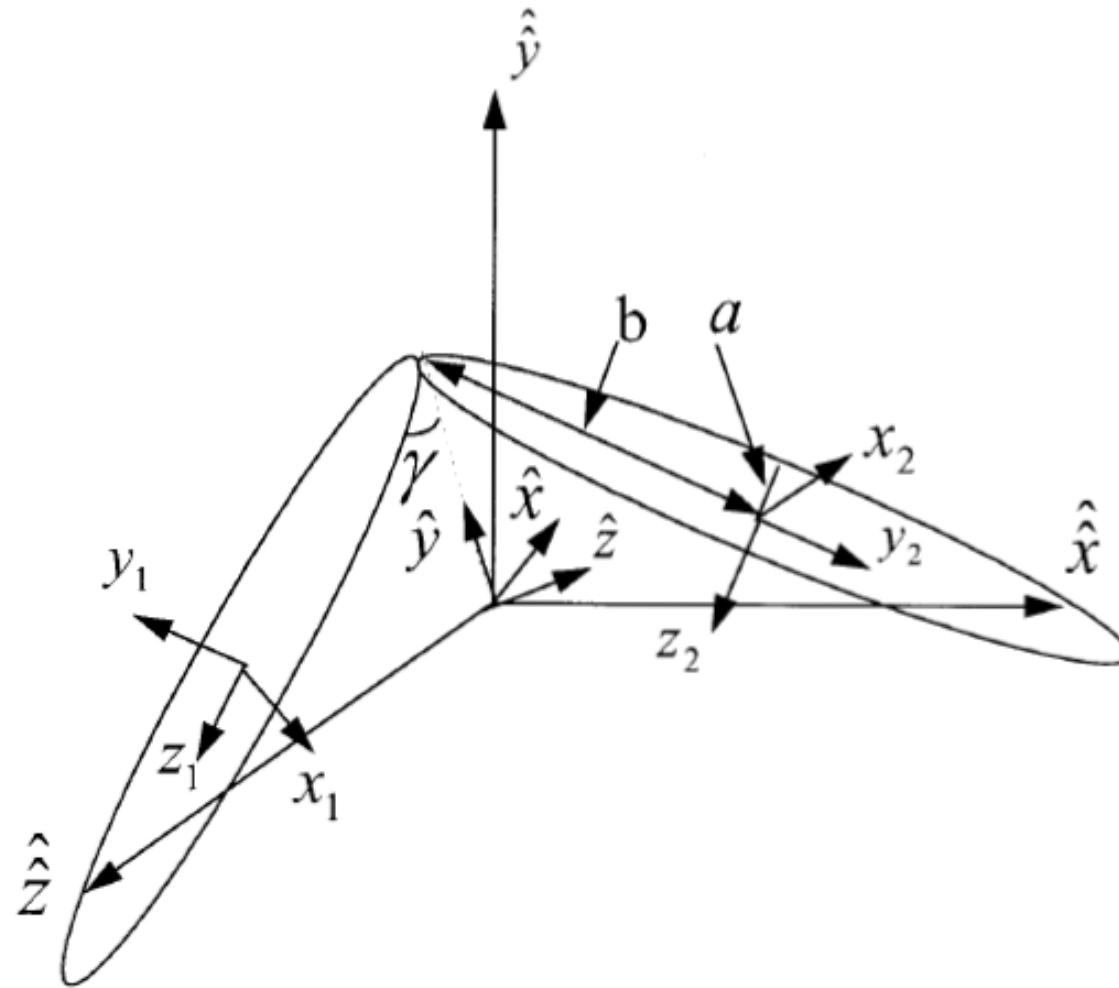
$$u_d^+ = \begin{cases} 0.0185 \times \left[\frac{\frac{\beta L^{+2}}{\beta + 3} + \frac{4\beta \tau_{eq}^{+2} g^+ L_1^+}{0.01085(\beta + 3)(1 + \tau_{eq}^{+2} L_1^+)} }{3.42 + \frac{\tau_{eq}^{+2} g^+ L_1^+}{0.01085(1 + \tau_p^{+2} L_1^+)} } \right] & \text{if } u_d^+ < 0.14 \\ \times \left[1 + 8e^{-(\tau_{eq}^+ - 10)^2 / 32} \right] \frac{1}{1 - \tau_{eq}^{+2} L_1^+ \left(1 + \frac{g^+}{0.037} \right)} & \\ 0.14 & \text{otherwise} \end{cases}$$

$$L_1^+ = \frac{3.08}{S d_{eq}^+} = \frac{0.725}{\sqrt{S \tau_{eq}^+}}$$

$$d_{eq}^+ = \sqrt{\frac{18 \tau_{eq}^+}{S}}$$

Angular Fiber

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Angular Fiber Parameters

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$$\tau_{\text{eq}}^+ = \frac{8\beta S(a^+)^2}{R_{xx} + R_{yy} + R_{zz}}$$

$$\mathbf{R} = \mathbf{A}_1^{-1} \mathbf{K}_1 \mathbf{A}_1 + \mathbf{A}_2^{-1} \mathbf{K}_2 \mathbf{A}_2$$

$$\mathbf{x}_1 = \begin{pmatrix} 0 \\ 0 \\ -b \sin(\gamma) \end{pmatrix} + \mathbf{A}_1 \hat{\mathbf{x}}$$

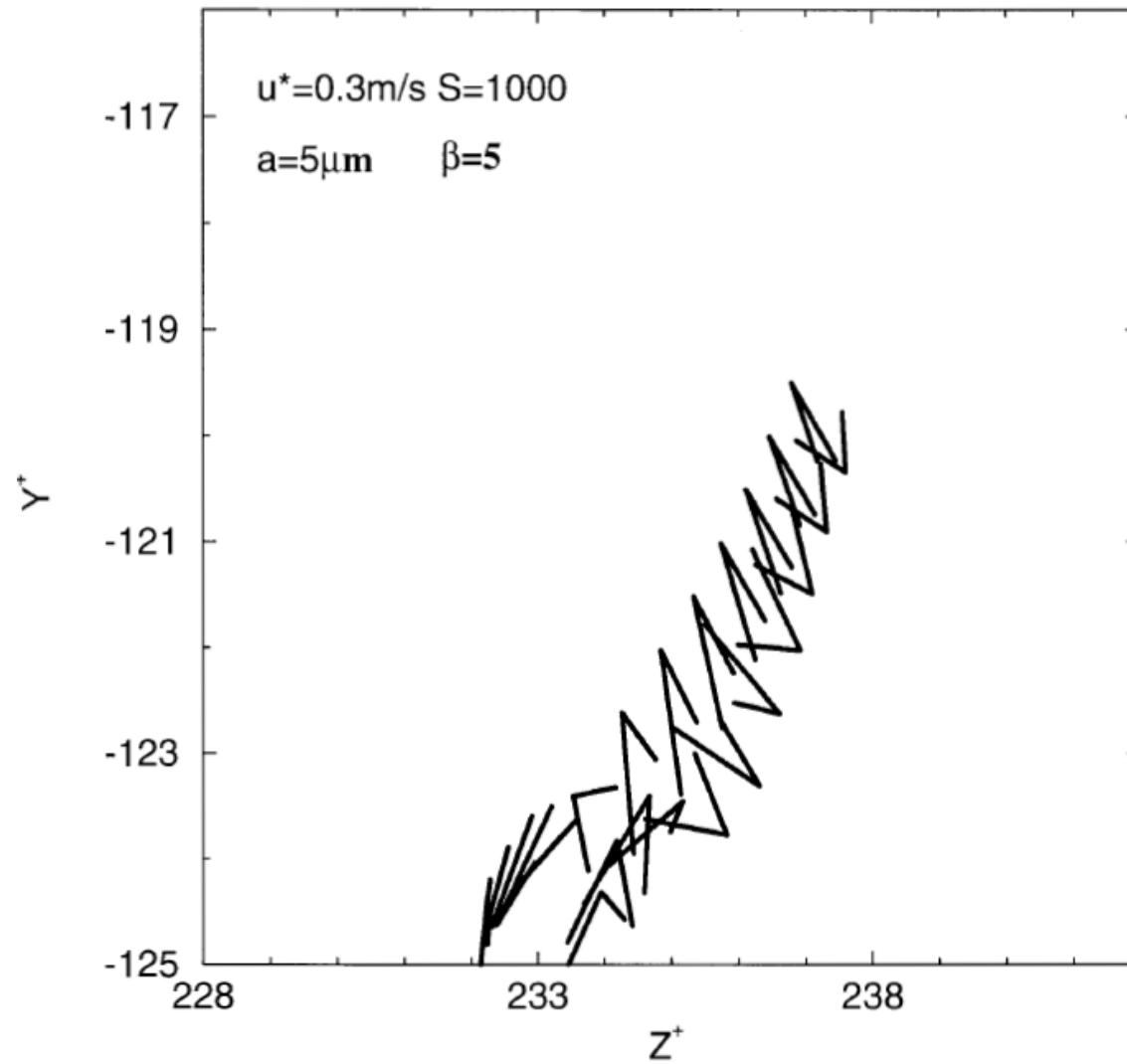
$$\mathbf{A}_1 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & \sin \gamma & -\cos \gamma \\ 0 & -\cos \gamma & -\sin \gamma \end{bmatrix}$$

$$\mathbf{x}_2 = \begin{pmatrix} 0 \\ 0 \\ b \sin(\gamma) \end{pmatrix} + \mathbf{A}_2 \hat{\mathbf{x}}$$

$$\mathbf{A}_2 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\sin \gamma & -\cos \gamma \\ 0 & -\cos \gamma & \sin \gamma \end{bmatrix}$$

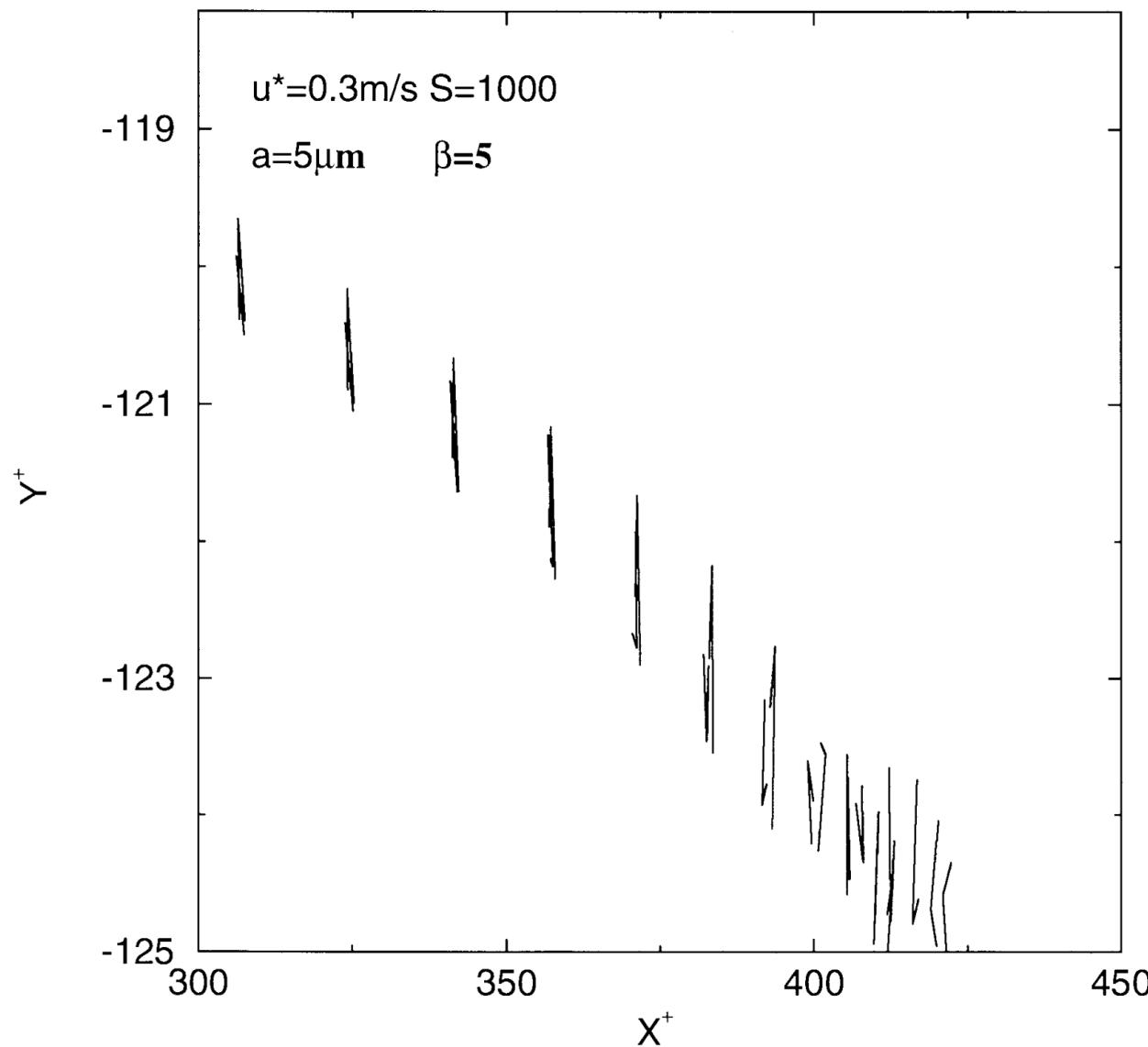
Angular Fiber Sample Trajectories

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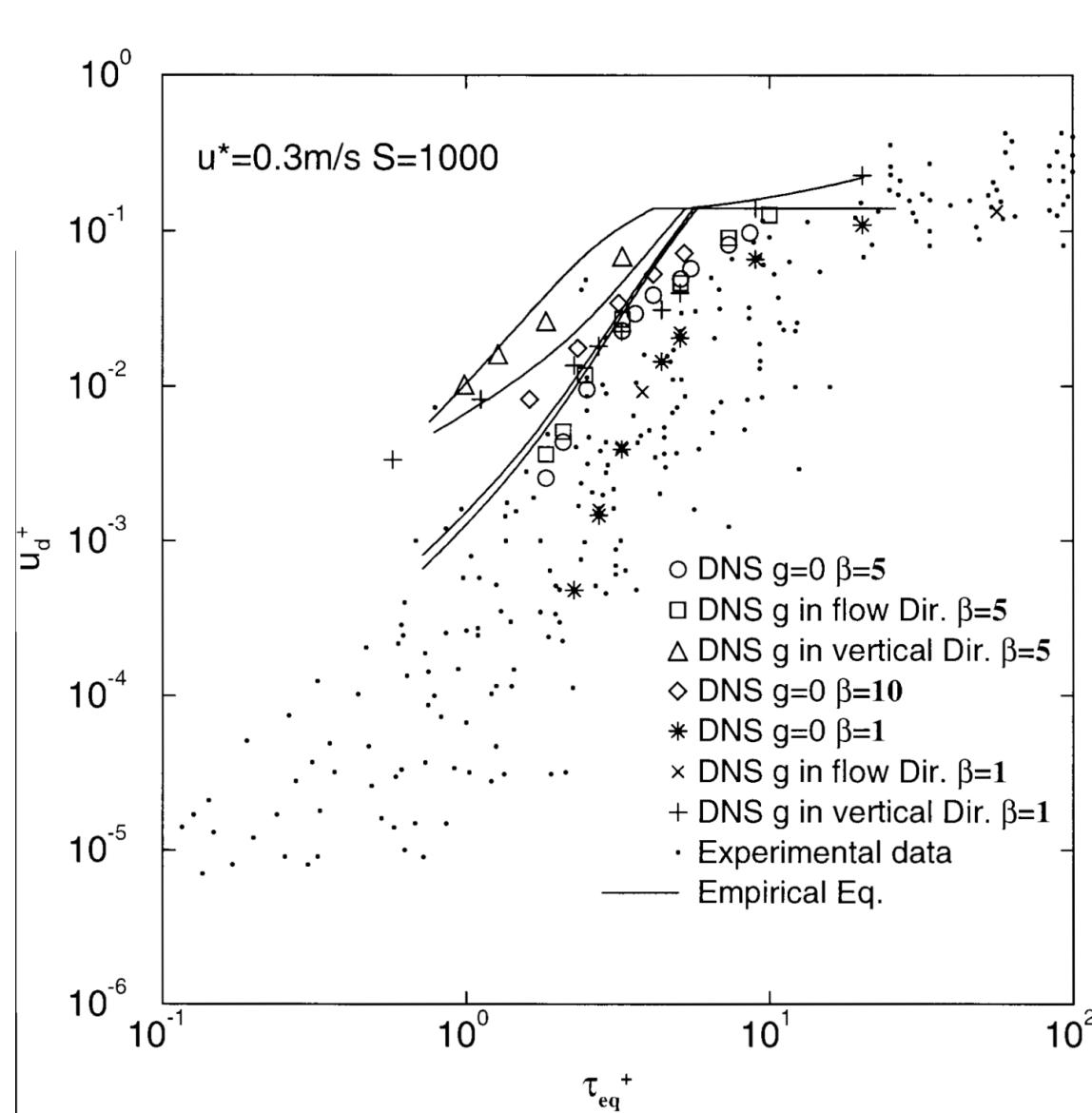
Angular Fiber Sample Trajectories

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Angular Fiber Deposition Velocity

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Empirical Equation for Angular Fiber Deposition Velocity

$$u_d^+ = \begin{cases} 0.0185 \times \left[\frac{\frac{\beta L_1^{+2}}{\beta + 3} + \frac{4\beta \tau_{eq}^{+2} g^+ L_1^+}{0.01085(\beta + 3)(1 + \tau_{eq}^{+2} L_1^+)} }{3.42 + \frac{\tau_{eq}^{+2} g^+ L_1^+}{0.01085(1 + \tau_p^{+2} L_1^+)}} \right] & 1 / \left(1 + \tau_{eq}^{+4\beta/(1+\beta)} L_1^+ \right) \\ \times \left[1 + 8e^{-(\tau_{eq}^+ - 10)^2 / 32} \right] \frac{1}{1 - \tau_{eq}^{+2} L_1^+ \left(1 + \frac{g^+}{0.037} \right)} & \text{if } u_d^+ < 0.14 \\ 0.14 & \text{otherwise} \end{cases}$$

$$L_1^+ = \frac{3.08}{Sd_{eq}^+} = \frac{0.725}{\sqrt{S\tau_{eq}^+}}$$

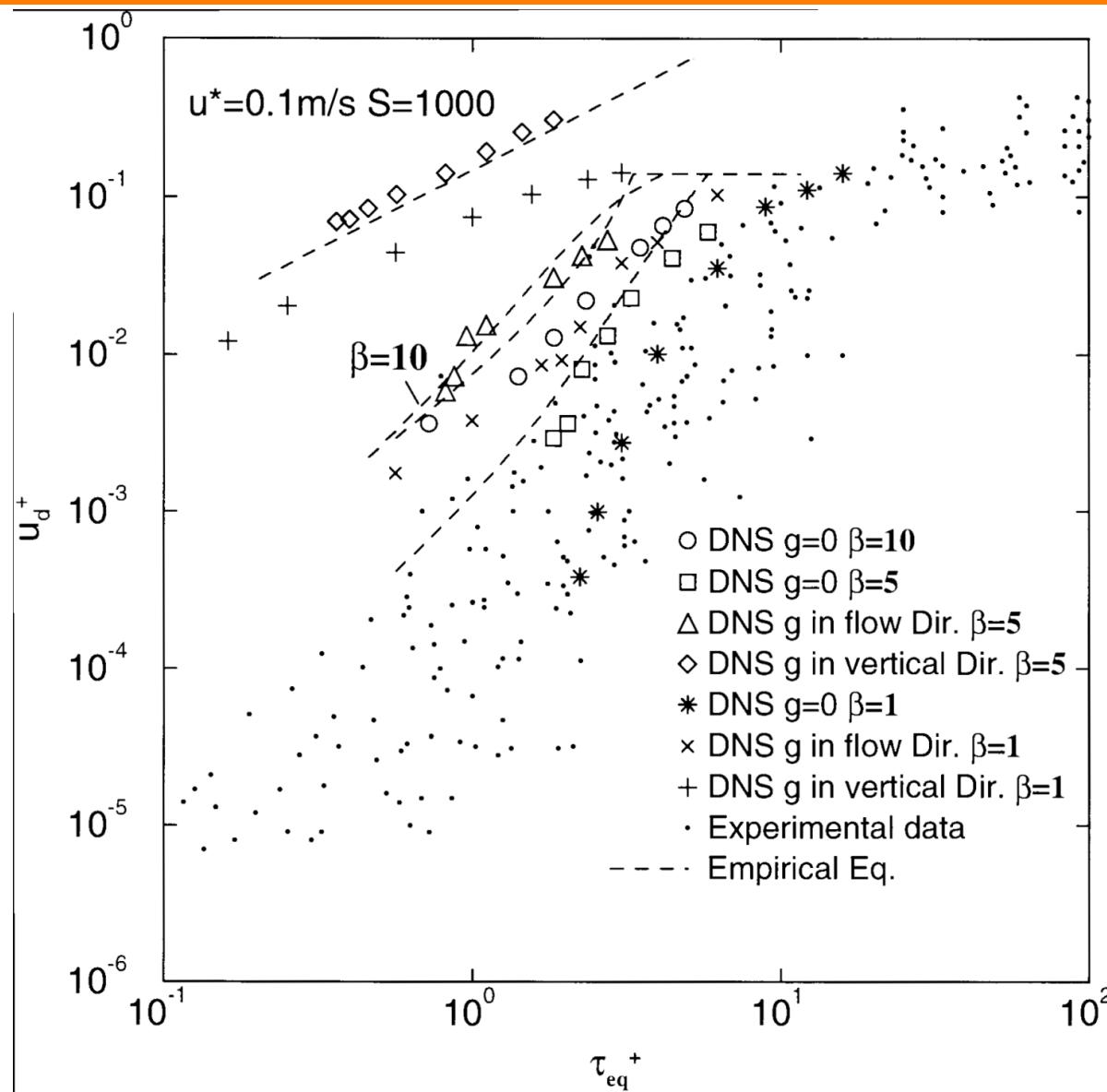
$$d_{eq}^+ = \sqrt{\frac{18\tau_{eq}^+}{S}}$$

$$\mathbf{g}^+ = \frac{\mathbf{v}}{u^{*3}} \mathbf{g}$$

$$L^+ = \begin{cases} 2a^+\beta & \gamma < \frac{\pi}{6} \\ 4a^+\beta \sin\gamma & \gamma \geq \frac{\pi}{6} \end{cases}$$

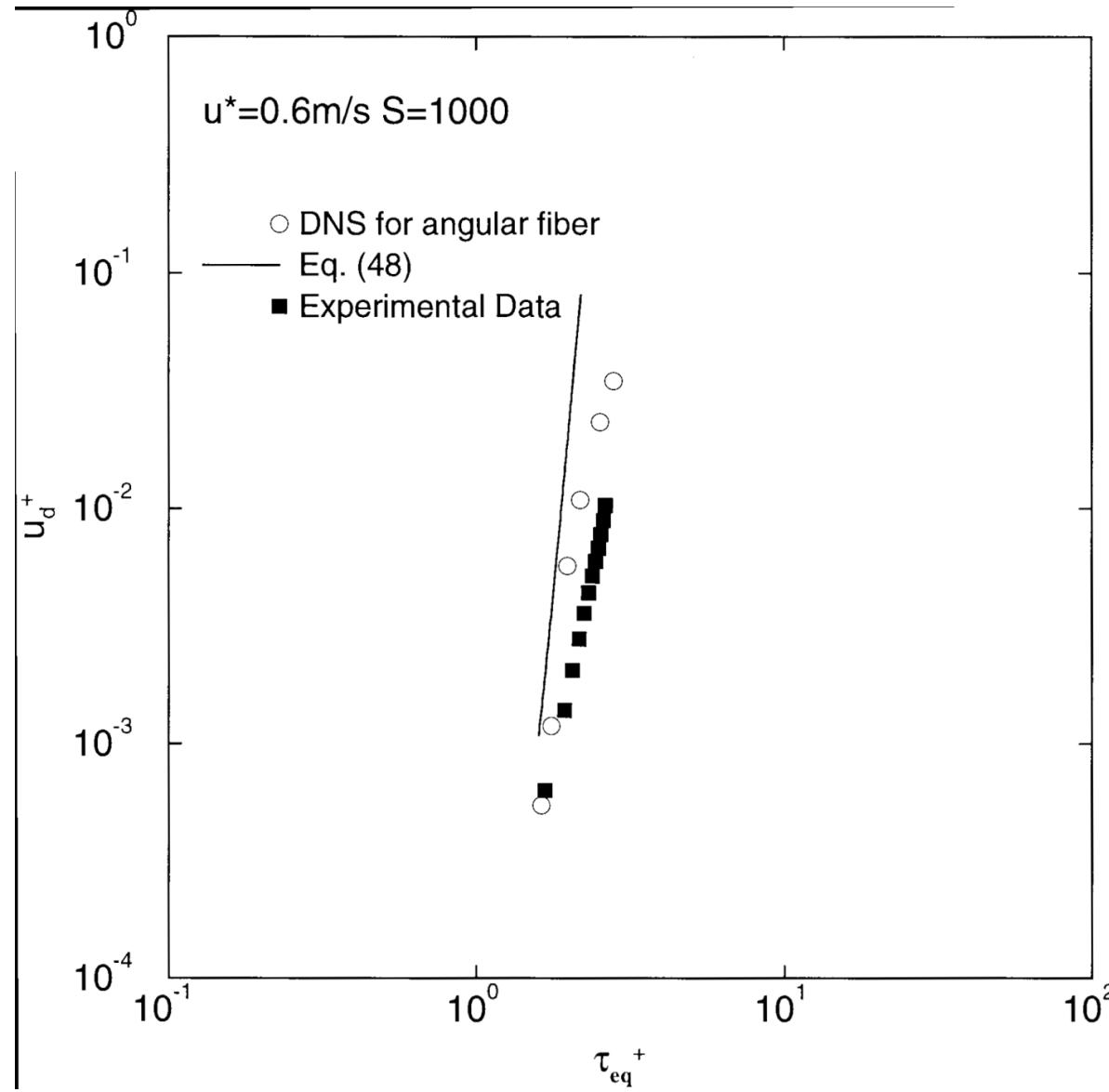
Angular Fiber Deposition Velocity

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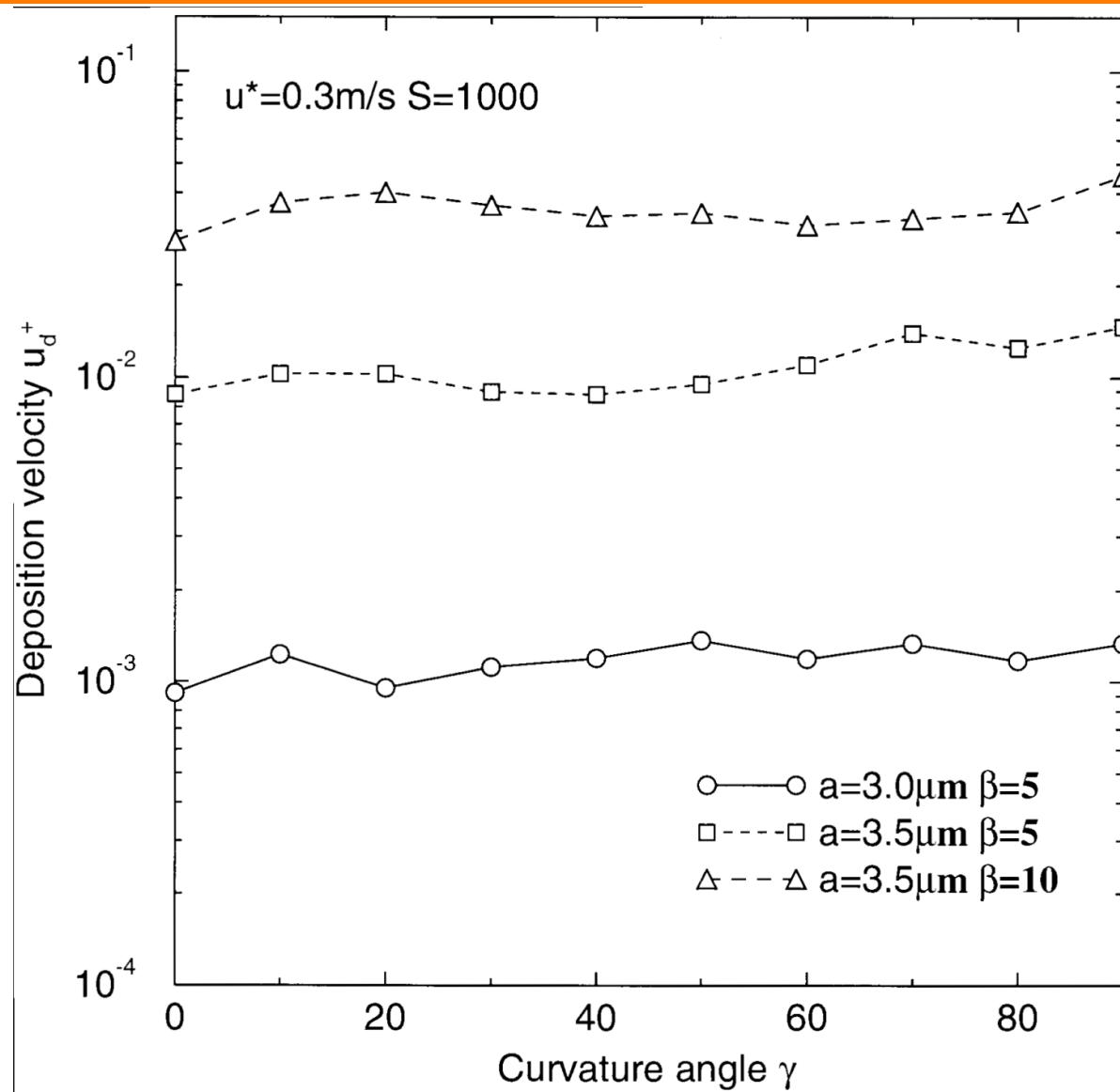
Angular Fiber Deposition Velocity

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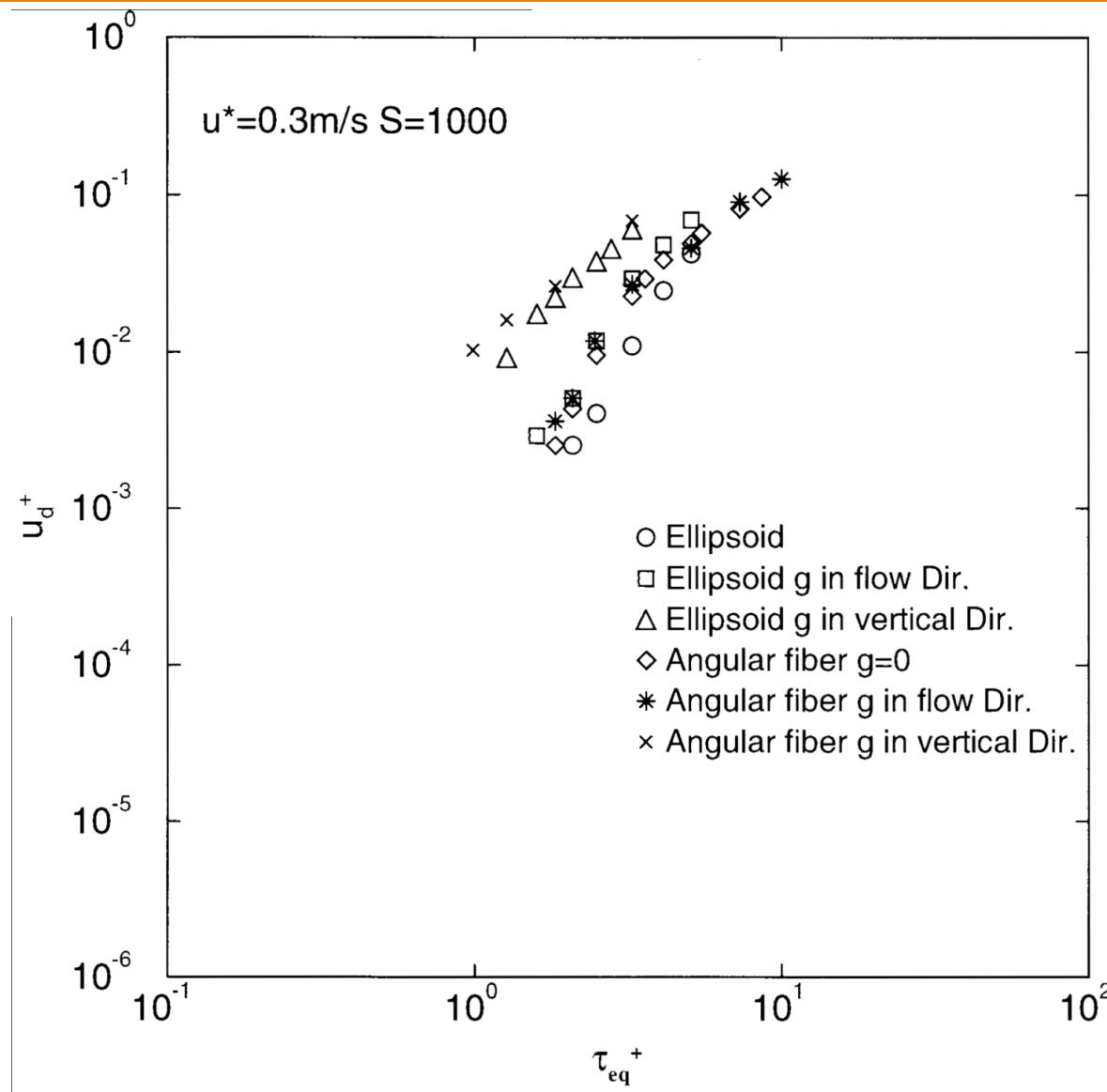


Angular Fiber Deposition Velocity

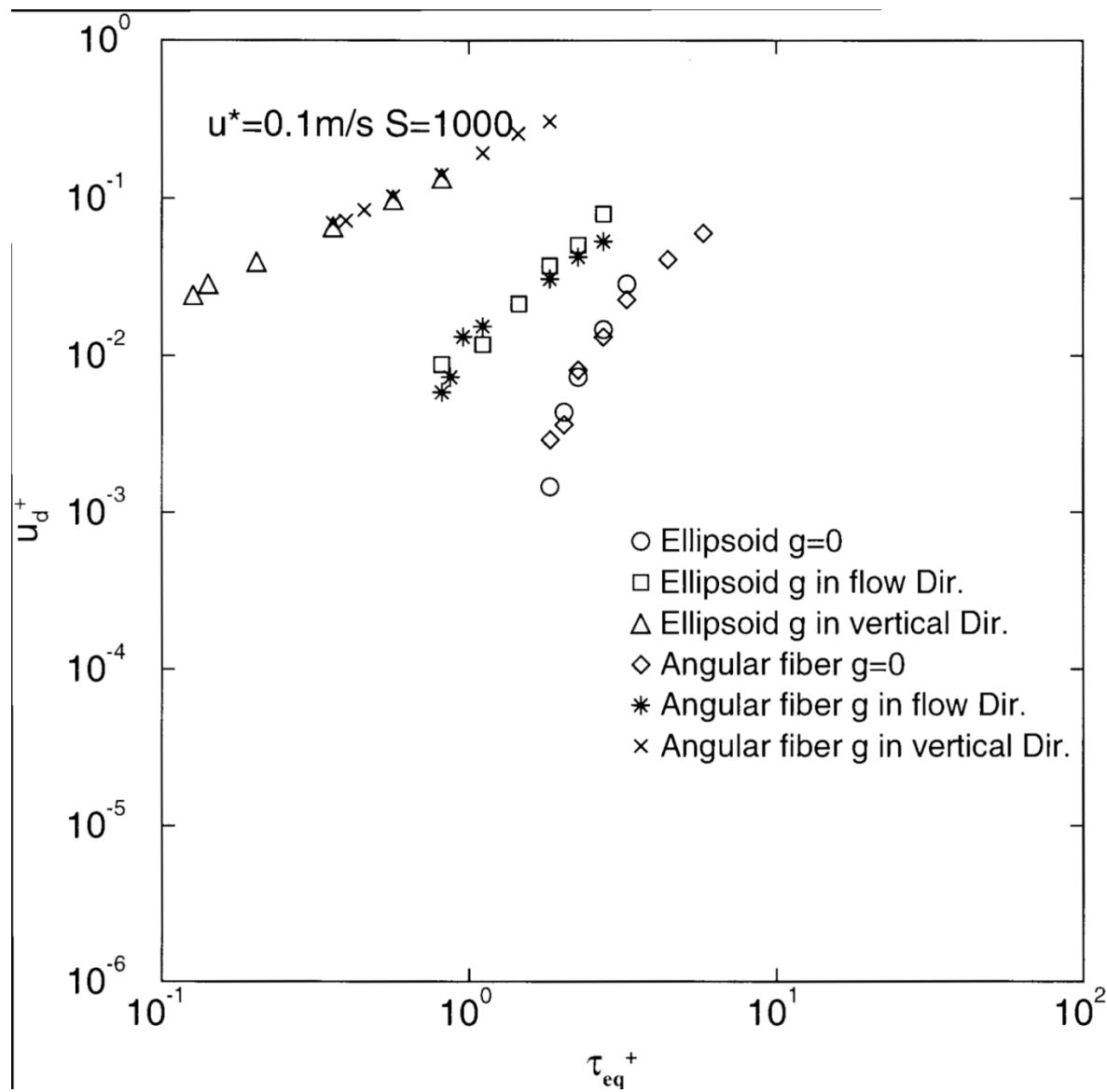
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Comparison of Deposition Velocities

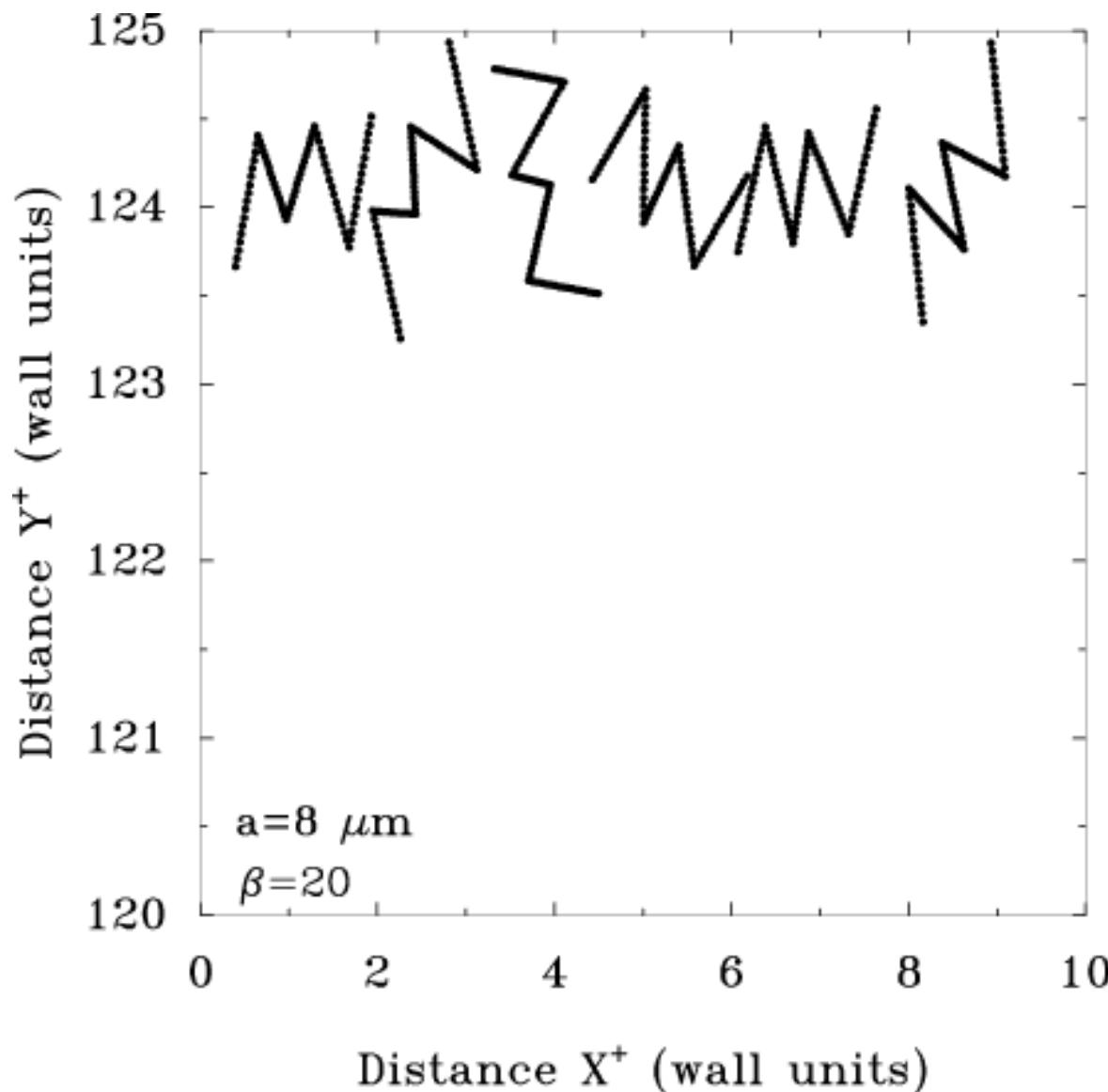


Comparison of Deposition Velocities



Curly Fiber Motions

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Concluding Remarks

- **Turbulence coherent vortical near-wall structure plays an important role on the particle deposition concentration profiles.**
- **Presence of particles attenuates the intensity of the fluid fluctuations, and as particle mass loading increases, the level of attenuation increases.**
- **Inter-particle collisions increases the particle deposition velocity while two-way coupling decreases it.**
- **Inter-particle collisions and two-way coupling reduce the particle accumulation near the wall.**

Conclusions

- **Aspect ratio plays an important role on ellipsoidal particle deposition rate.**
- **The simulation results for deposition velocity are in good agreement with the experimental data.**
- **Deposition velocity increases with fiber aspect ratio.**
- **Effect of gravity on particle deposition velocity depends on the magnitude of shear velocity.**
- **The gravitational sedimentation enhances the deposition rate on the lower wall in horizontal duct flows.**

Thank You!

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Questions?

