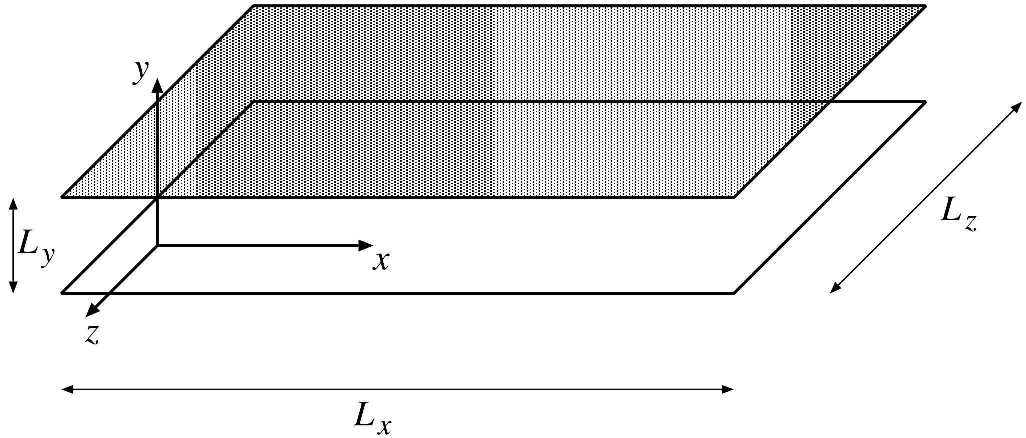
Outline

- Motivation
- Governing equations for LES
- Boundary conditions
- Subfilter-scale modelling
- **Validation of an LES**
- Applications
- Hybrid RANS/LES methods
- Challenges
- Conclusions

Validation of an LES: Case Study

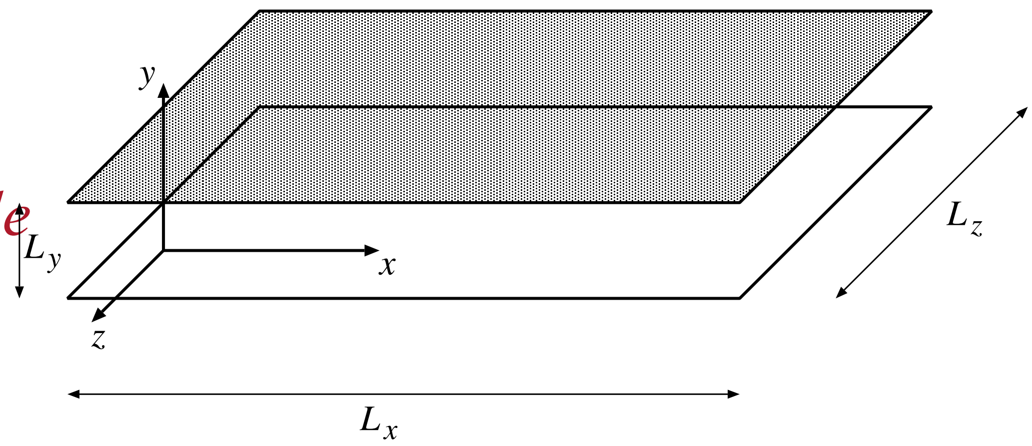
- Plane channel flow:
  - $Re_b = U_b \delta / \nu = 7{,}000$
  - *Periodic bc's in x and z*
  - *Computational domain:* $6\delta \times 2\delta \times 3\delta$



- Numerical method:
  - *2*nd*-order central differences*
  - *Staggered scheme*
  - *2*nd*- order time advancement*
  - *Plane-averaged Dynamic Eddy-Viscosity model*

Validation of an LES: Case Study

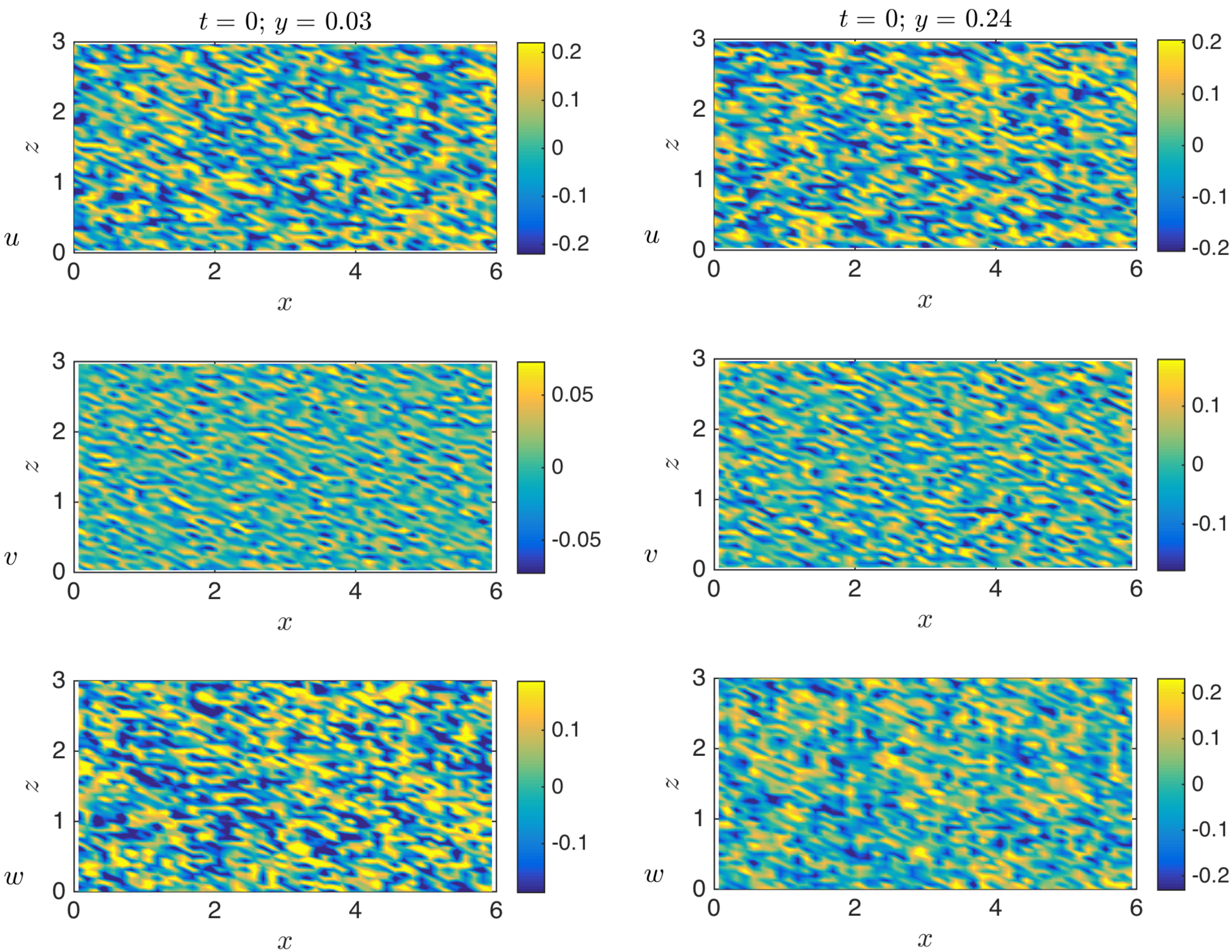
- Process:
  - *Calculation startup*
  - *Verify convergence of the sample*
  - *Verify grid convergence*
  - *Verify domain size*



Validation of an LES: Startup

- Initial field:
  - *Constant velocity* (U=1, V=W=0) + *random fluctuations* (30% *amplitude)*

Validation of an LES: Startup

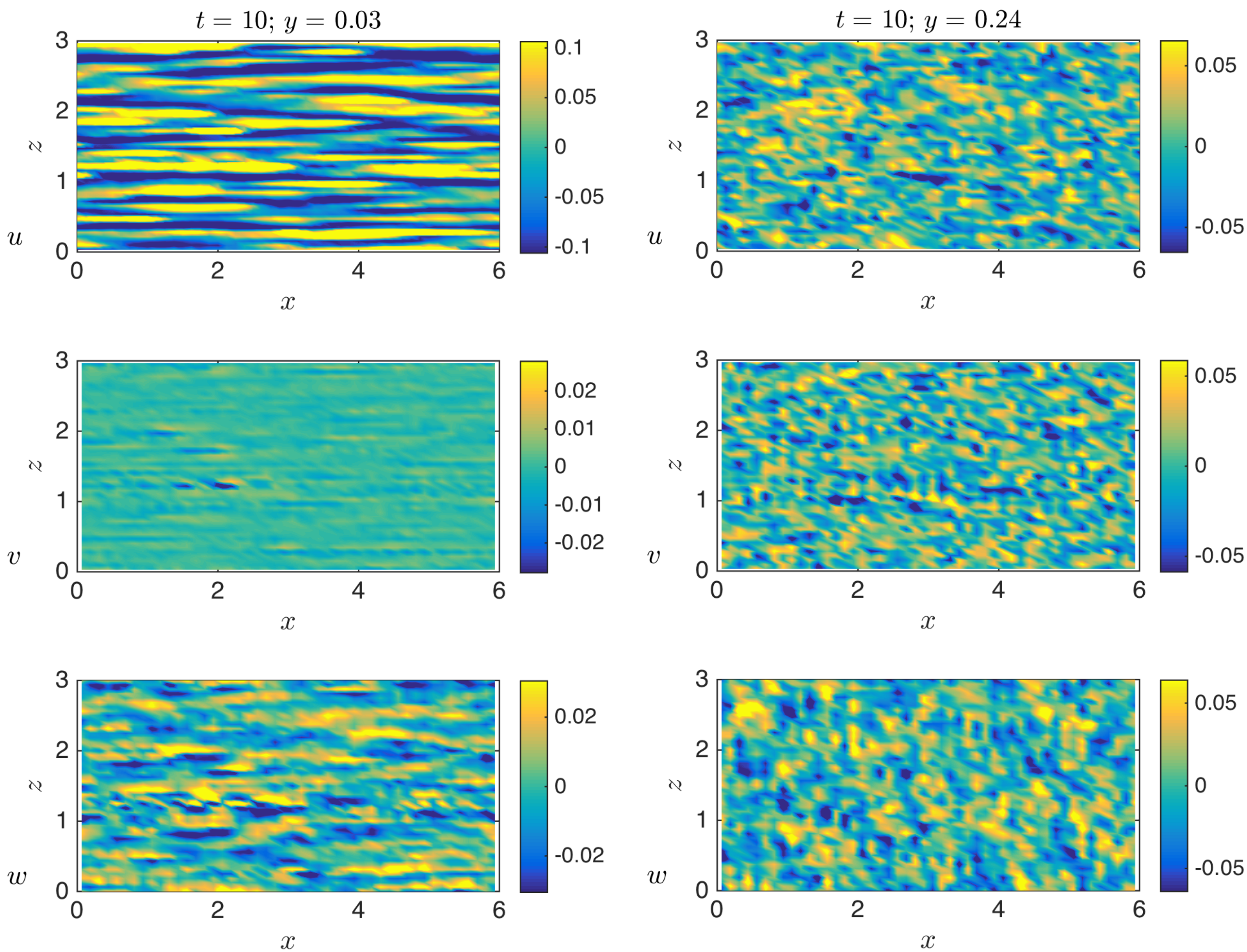


Validation of an LES: Startup

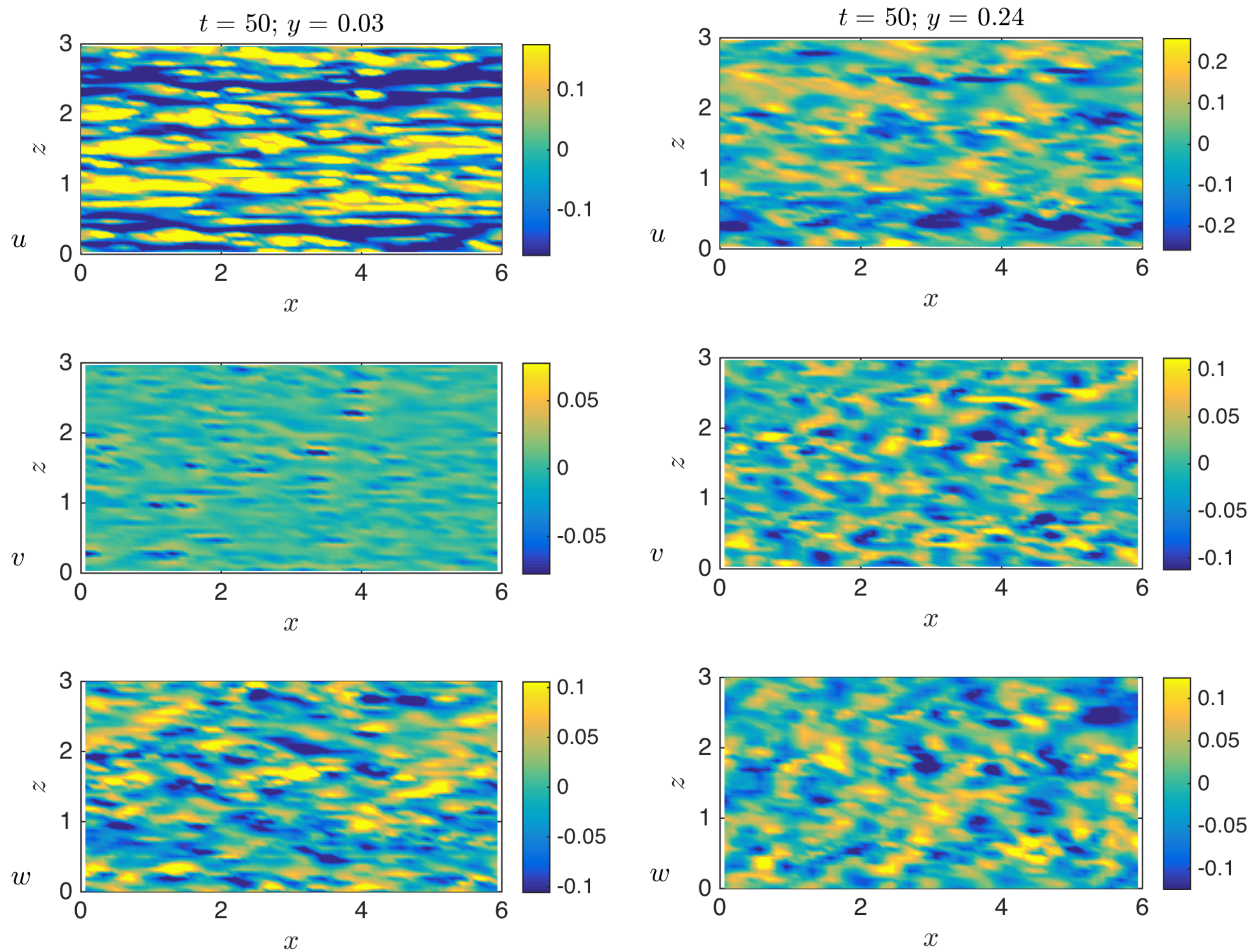
- Initial field:
  - *Constant velocity (U=1, V=W=0) + random fluctuations (30% amplitude)*
  - *Fluctuations are initially dissipated until realistic turbulence is generated by the non-linear interactions.*
  - *Begin the calculation on a coarse grid (48^3) and with no SFS model (Coarse DNS).*
  - *Monitor the average wall stress and the average Turbulent Kinetic Energy:*

$$
\begin{aligned}
\mathrm{TKE}_{avg} &= \frac{1}{L_x L_y L_z} \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} \langle u_i - \langle u_i \rangle_{xz} \rangle \langle u_i - \langle u_i \rangle_{xz} \rangle dx dy dz \\
\langle \tau_w \rangle_{xz} &= \mu \left[\left. \frac{d \langle u \rangle_{xz}}{dy} \right|_{y=0} - \left. \frac{d \langle u \rangle_{xz}}{dy} \right|_{y=2\delta} \right]
\end{aligned}
$$

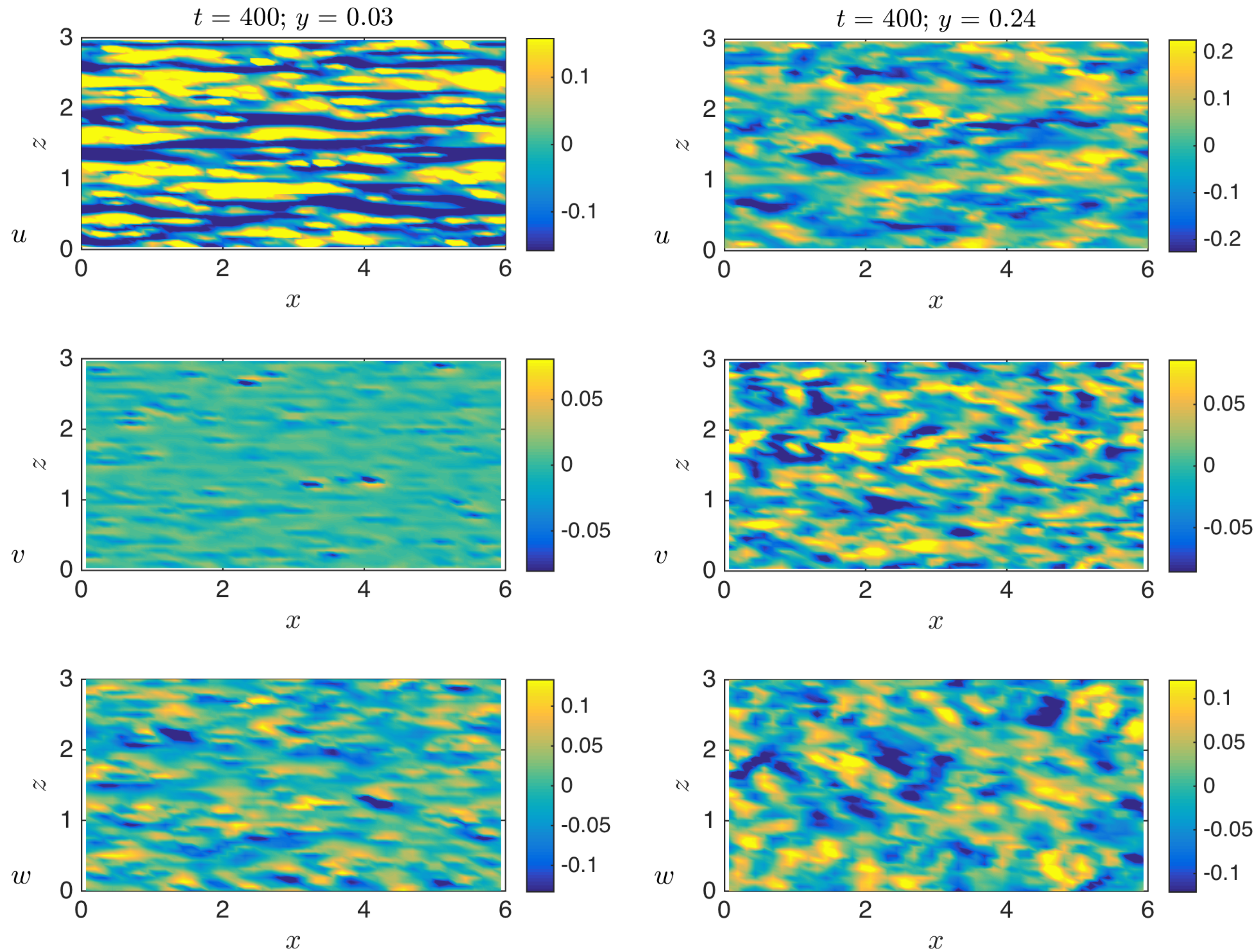
Validation of an LES: Startup



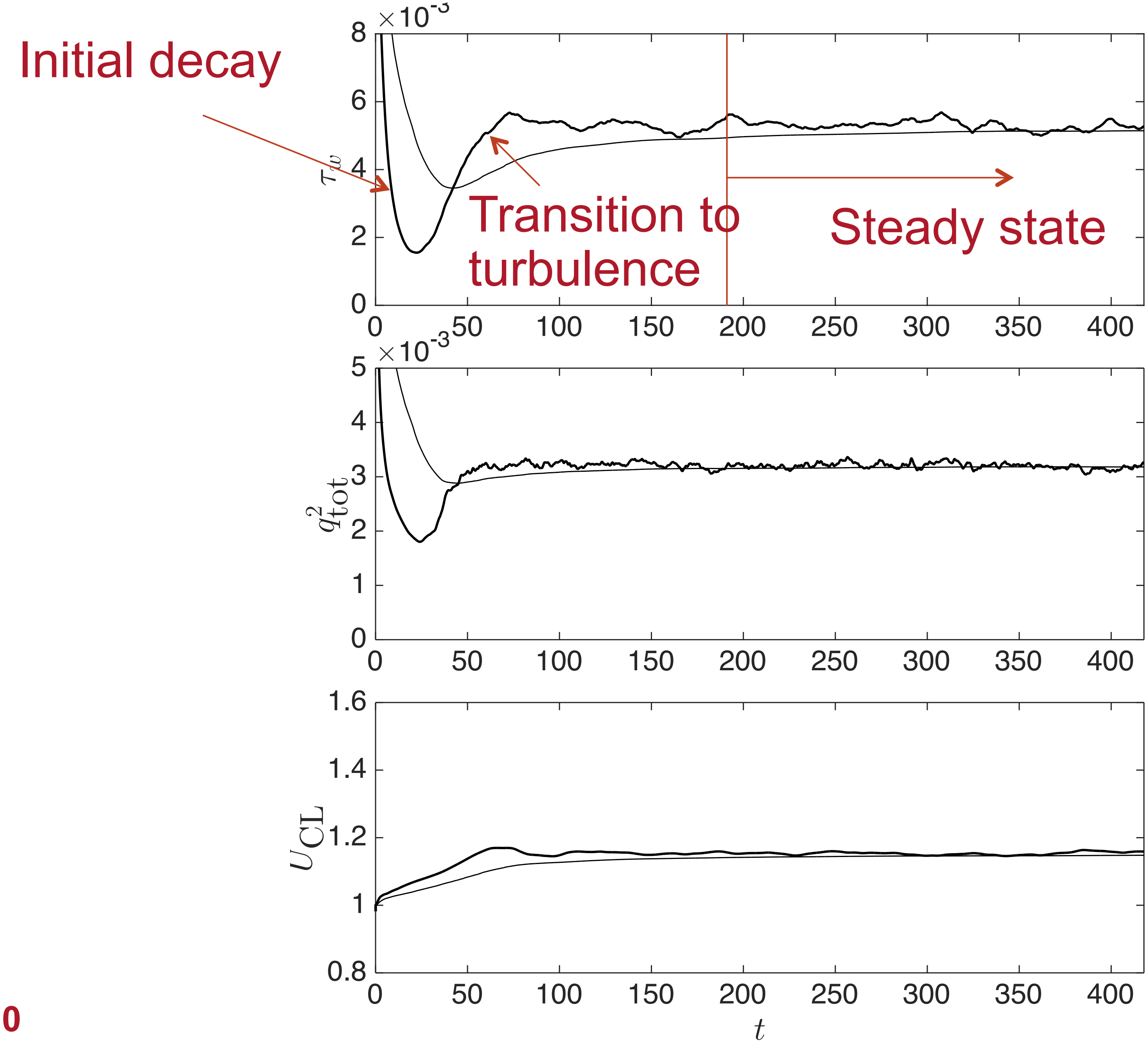
Validation of an LES: Startup



Validation of an LES: Startup

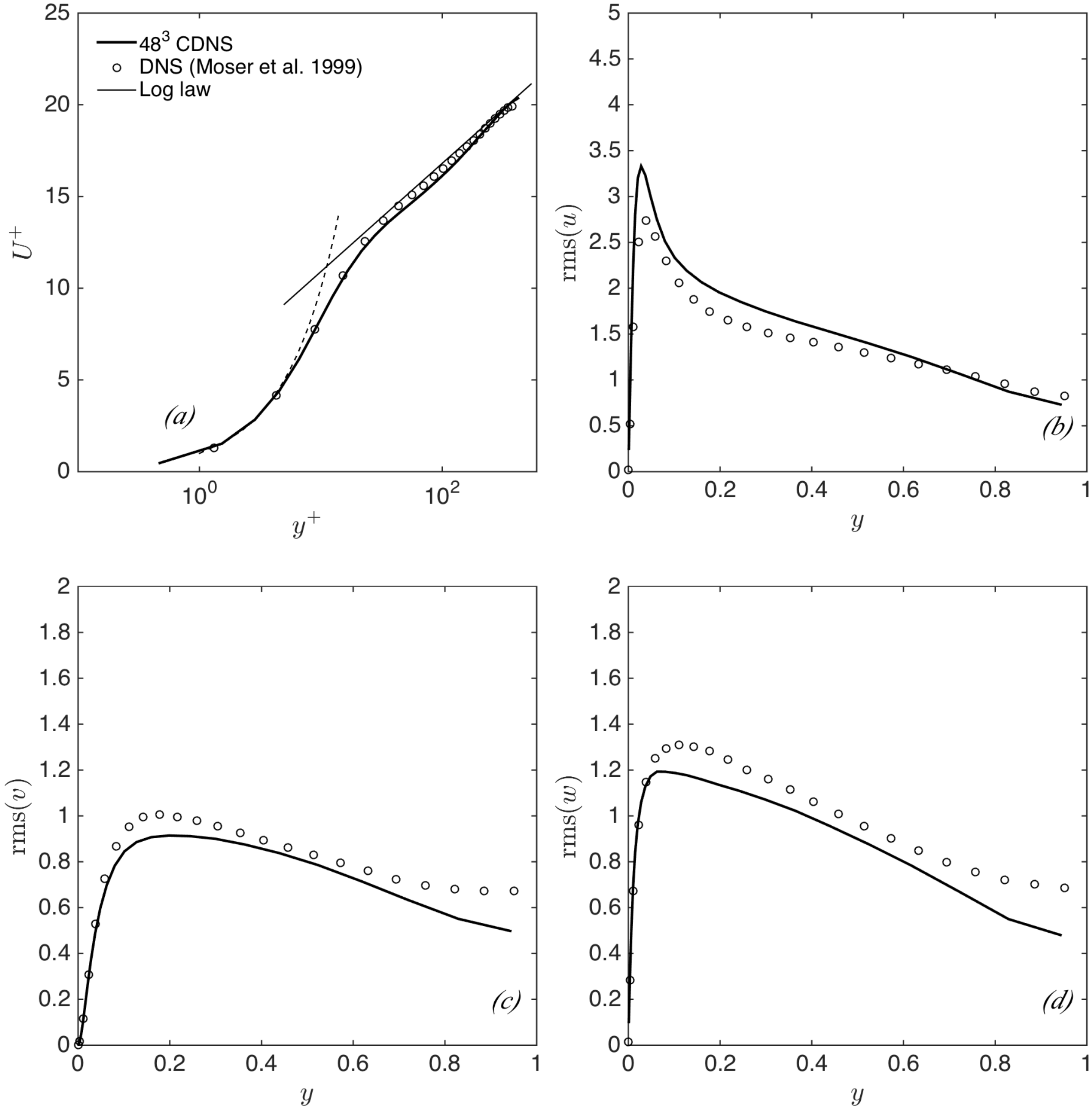


Validation of an LES: Startup



Validation of an LES: Startup

- Statistics:
    - *Not bad, given the coarseness of the grid*

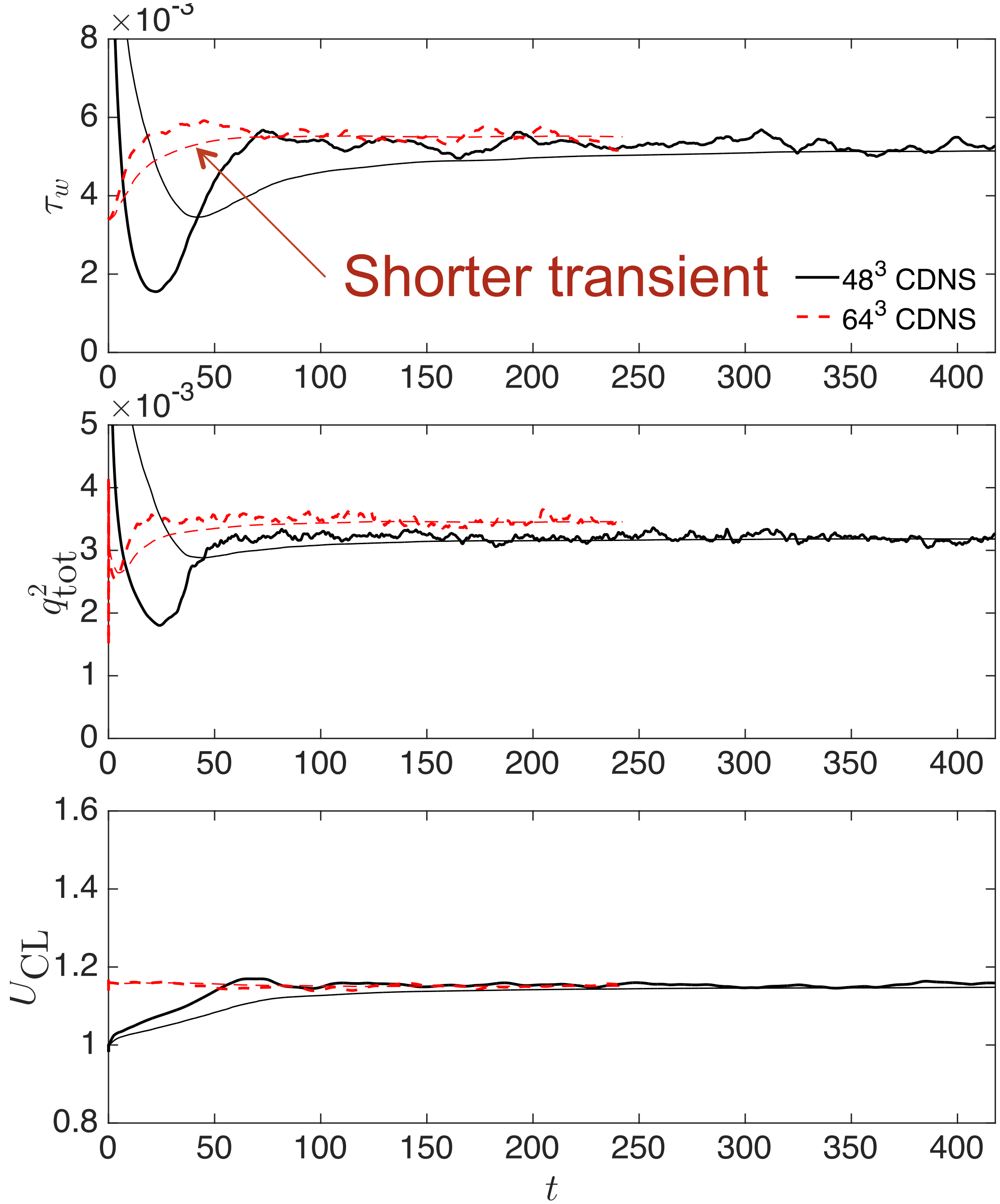


Validation of an LES: Startup

- Now refine the mesh
  - *Interpolate the converged field on the new mesh*
  - *Continue the calculation*

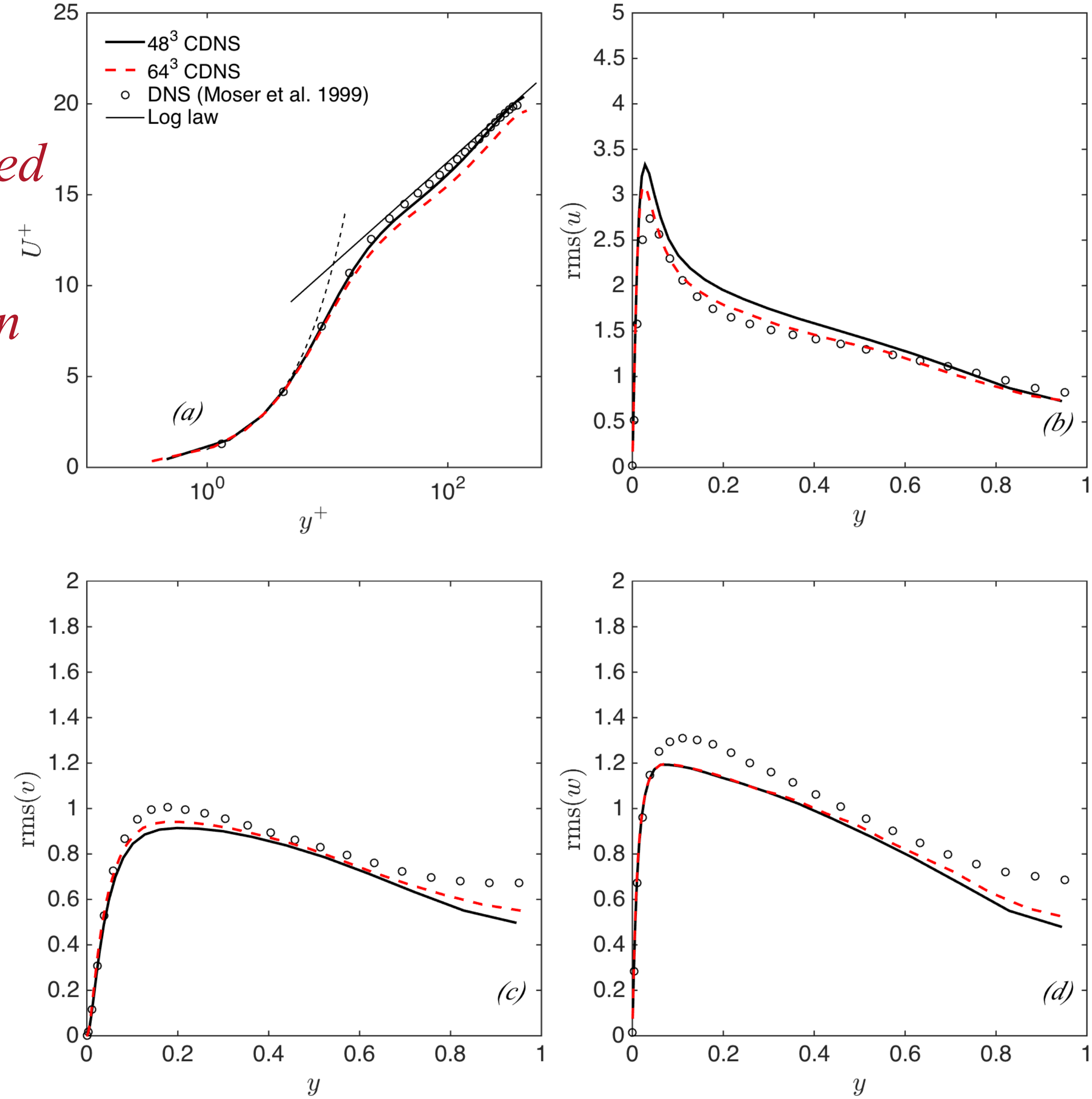
Validation of an LES: Startup

- Now refine the mesh
  - *Interpolate the converged field on the new mesh*
  - *Continue the calculation*
  - *Shorter transient*



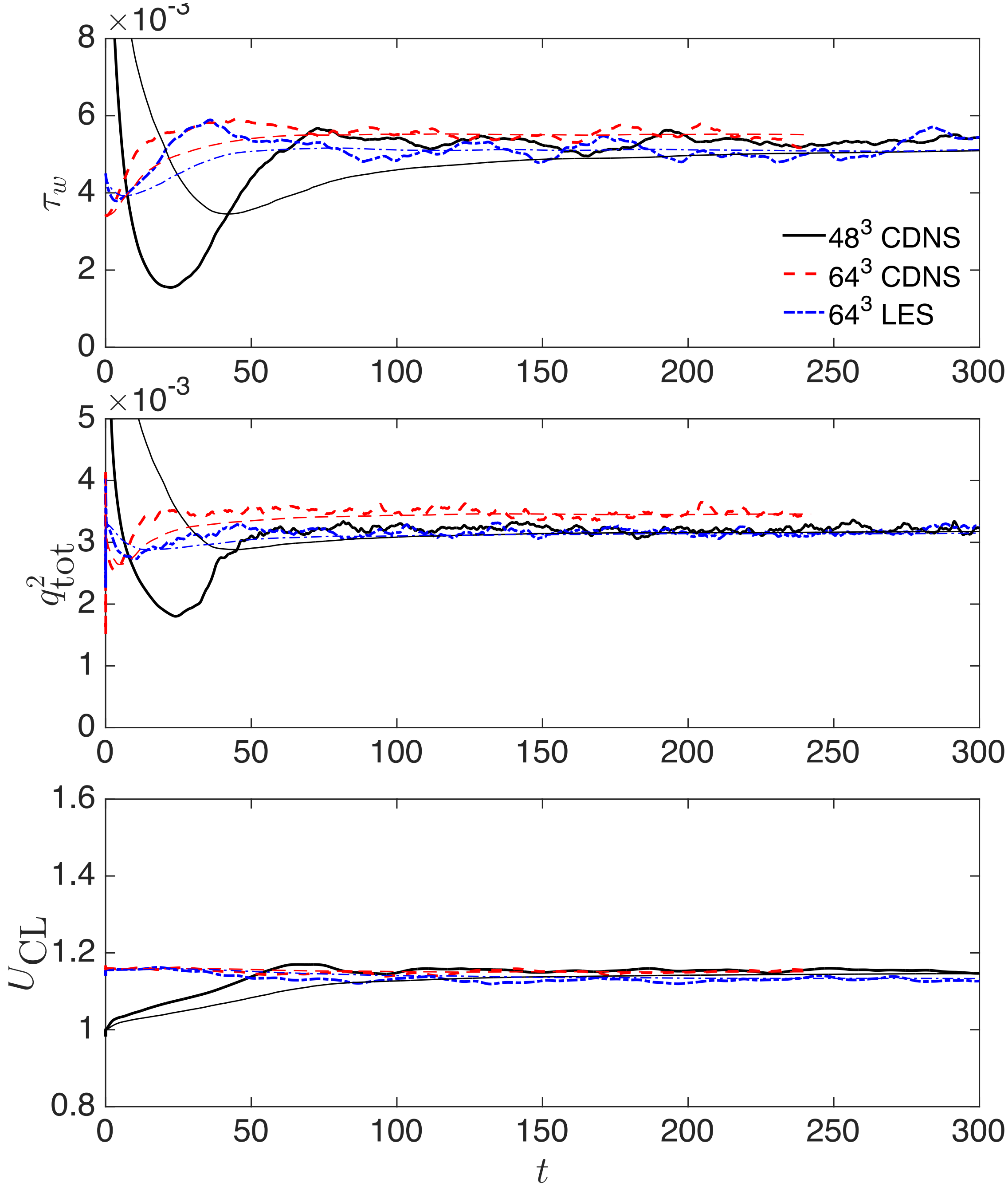
Validation of an LES: Startup

- Now refine the mesh
  - *Interpolate the converged field on the new mesh*
  - *Continue the calculation*
  - *Shorter transient*
  - *Worse agreement with data*



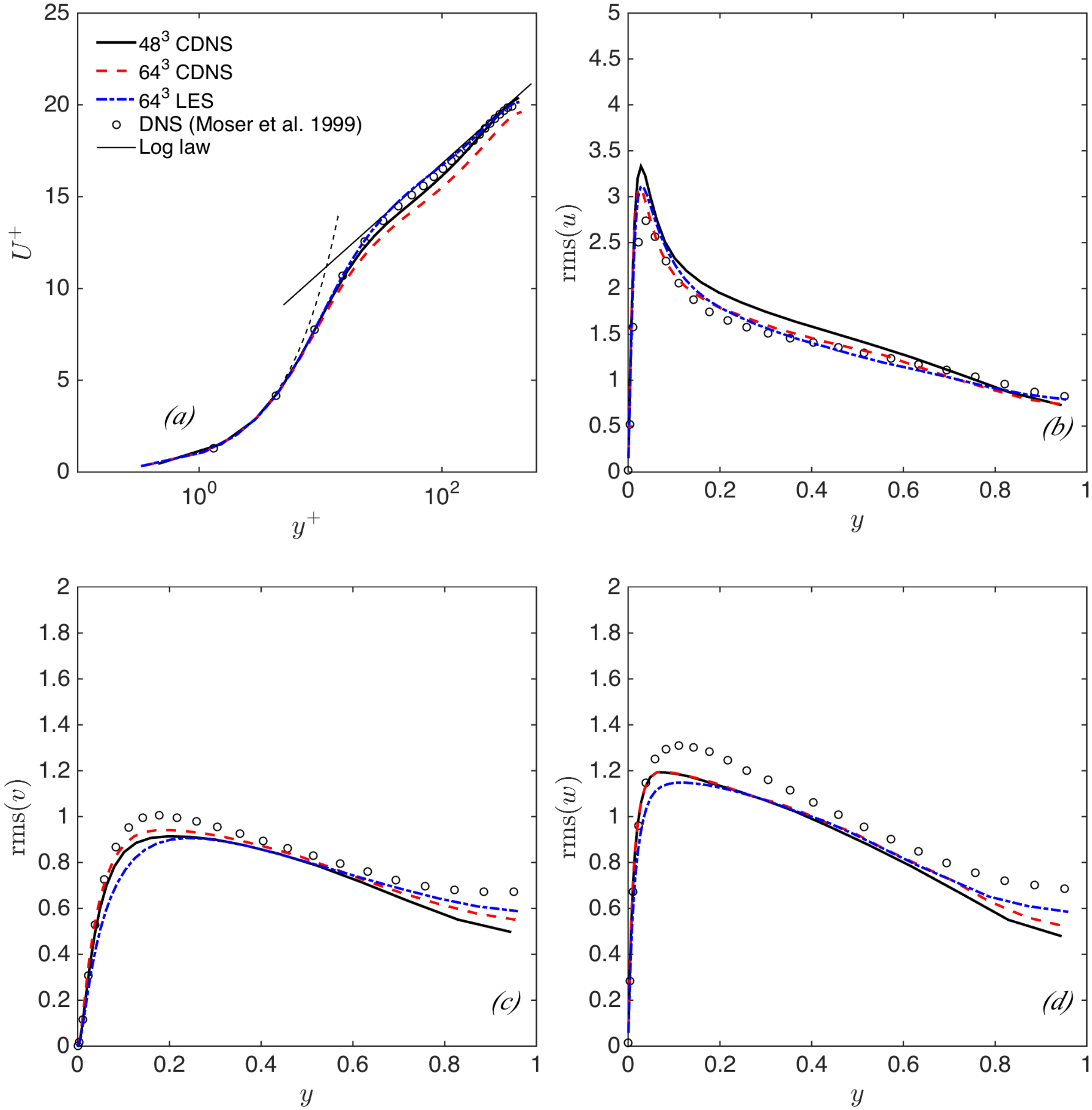
Validation of an LES: Startup

- Now add the SFS model



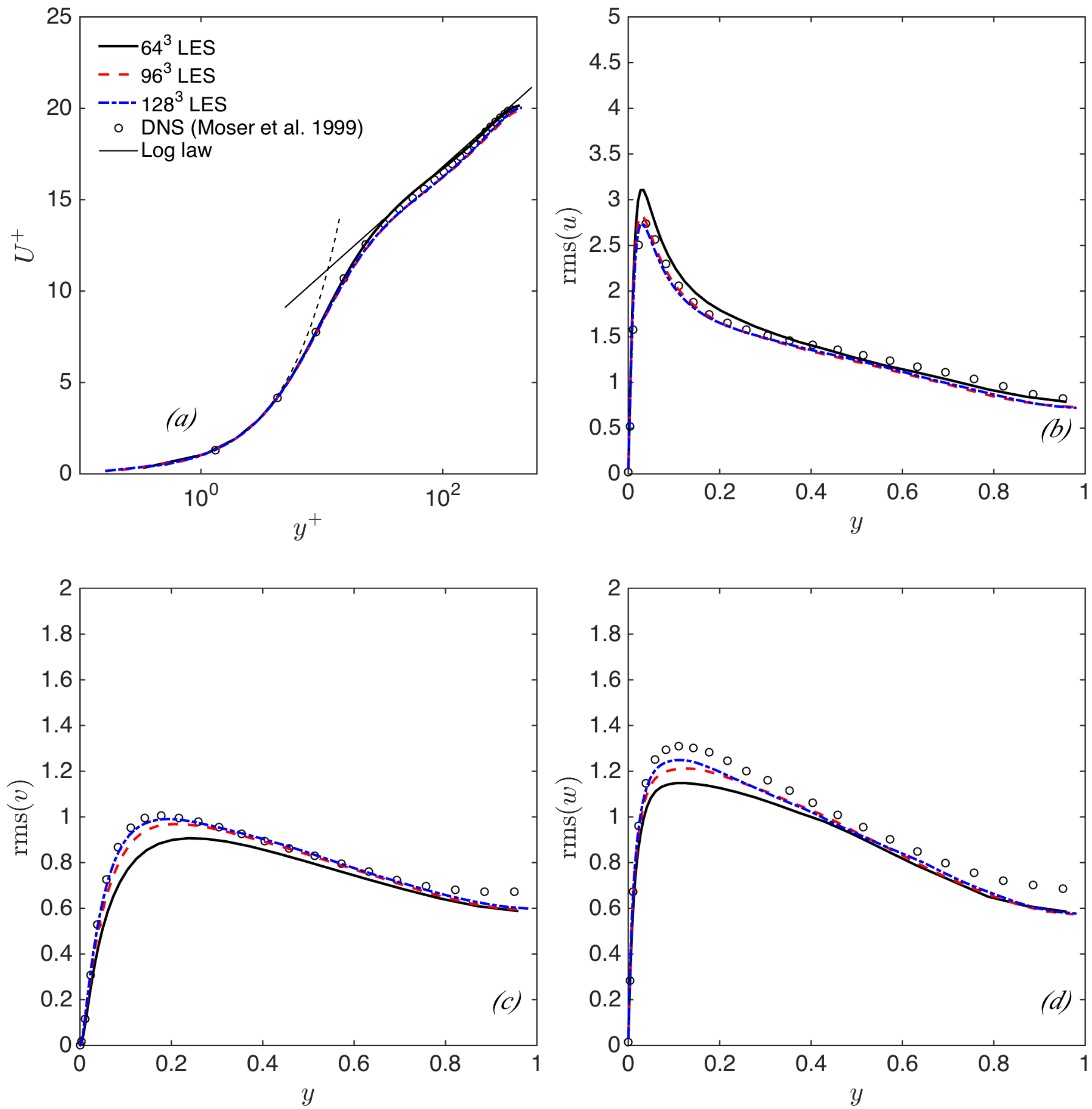
Validation of an LES: Startup

- Now add the SFS model
  - *Improved agreement with data*
  - *Is it good enough?*



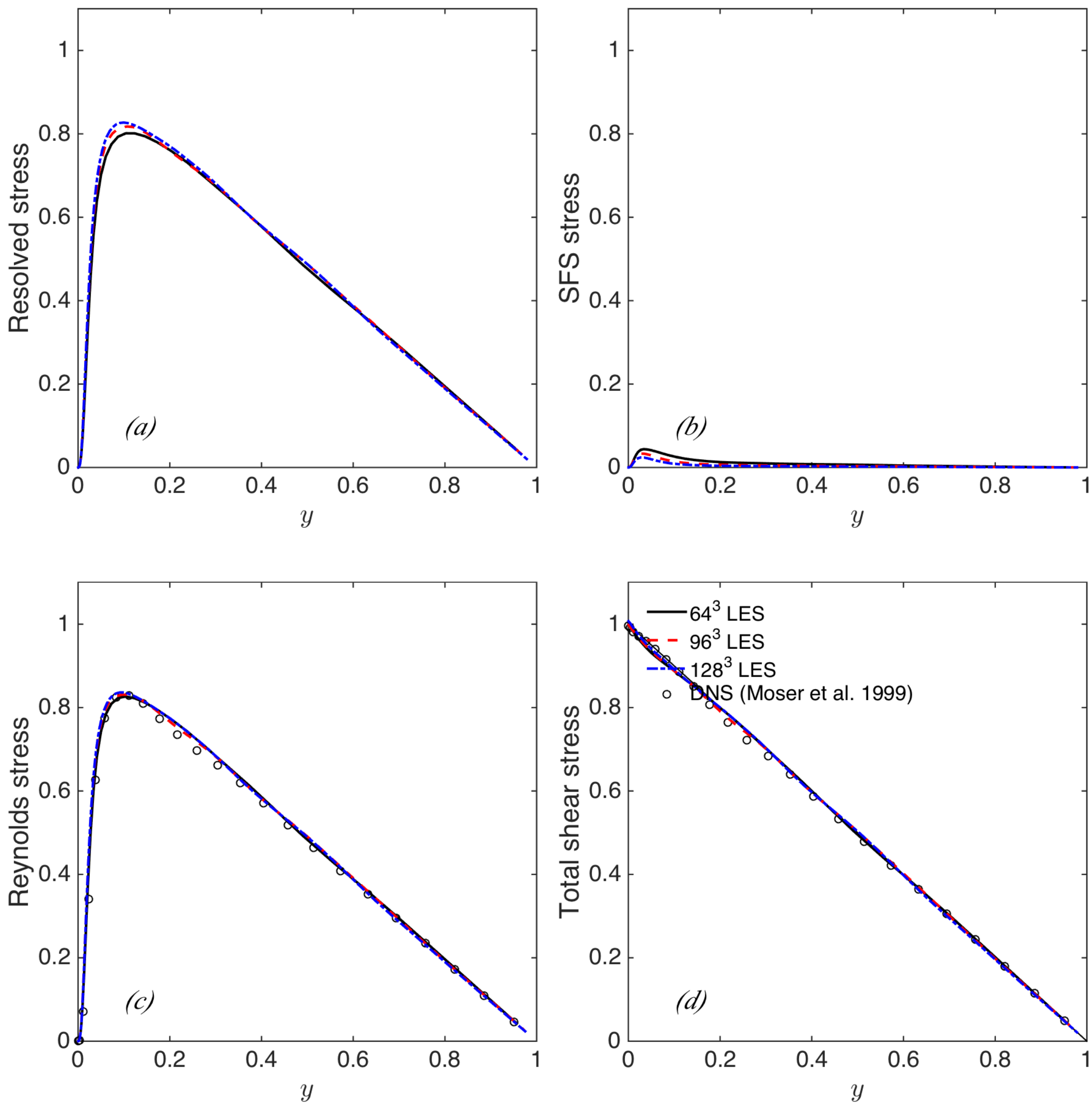
Validation of an LES: Startup

- Now add the SFS model
  - *Improved agreement with data*
  - *Is it good enough?*
- Perform calculations on finer grids
  - *Grid convergence with 96^3 points*



Validation of an LES: Startup

- Now add the SFS model
  - *Improved agreement with data*
  - *Is it good enough?*
- Perform calculations on finer grids
  - *Grid convergence with 96^3 points*
  - *SFS contribution is small*



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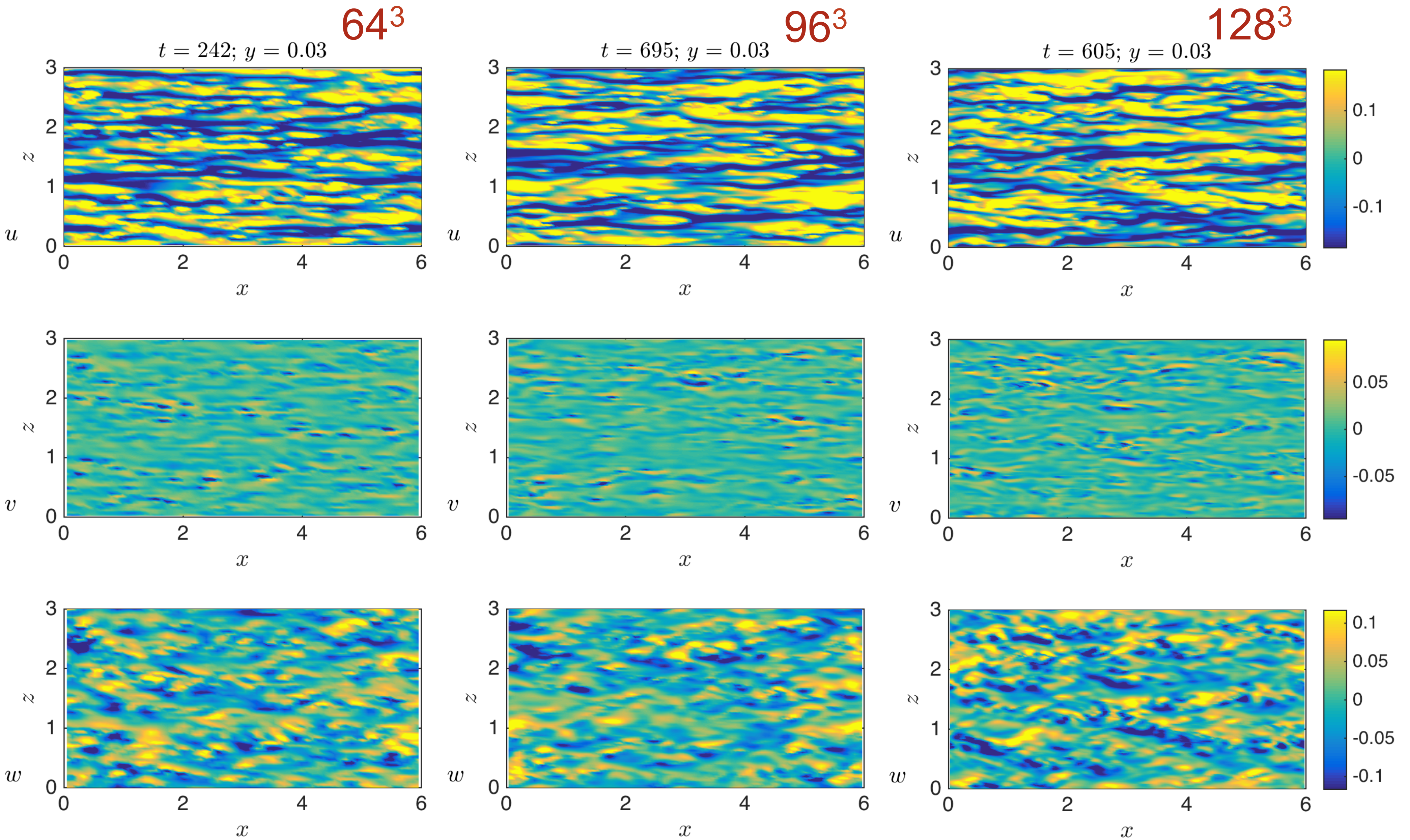

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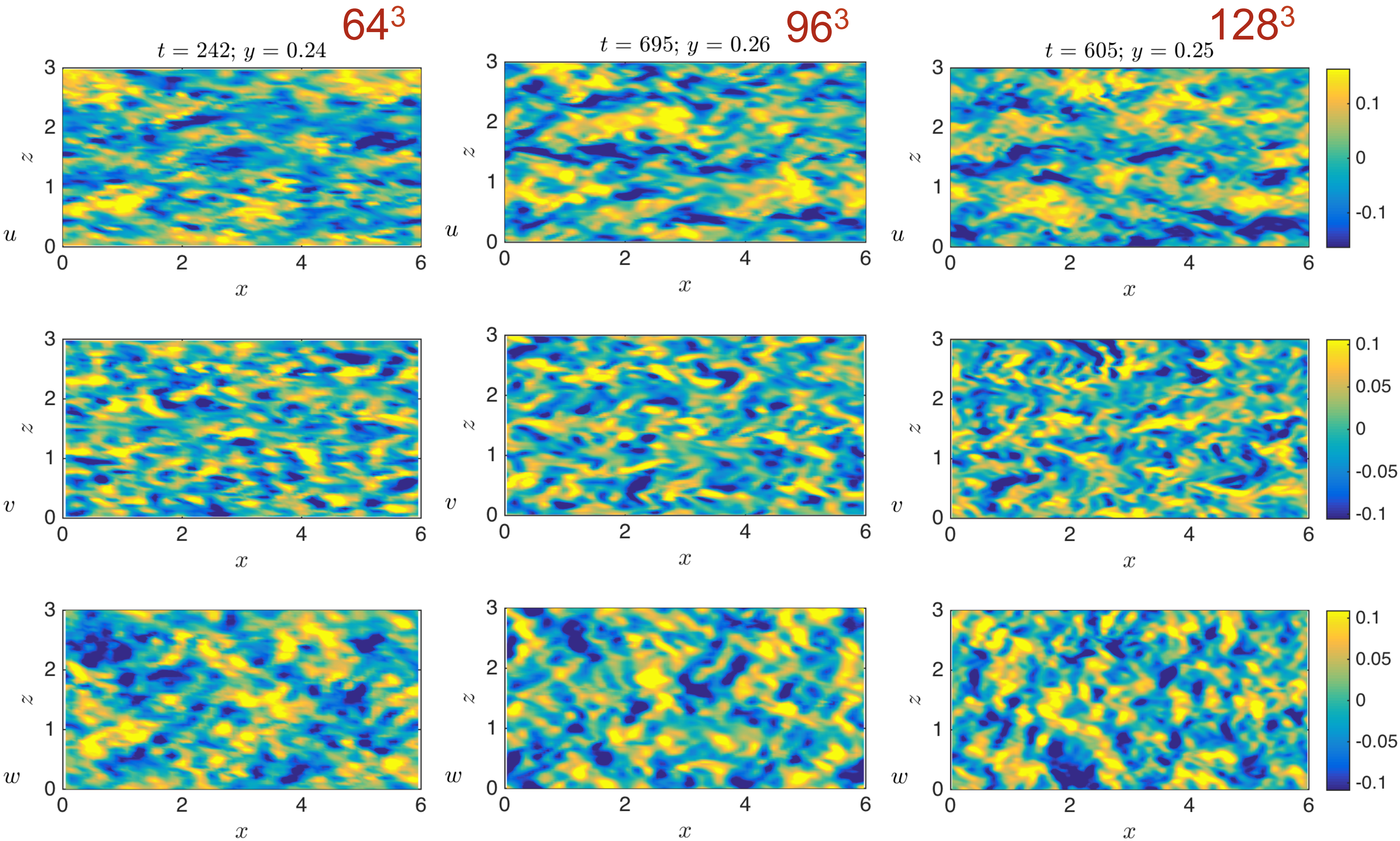
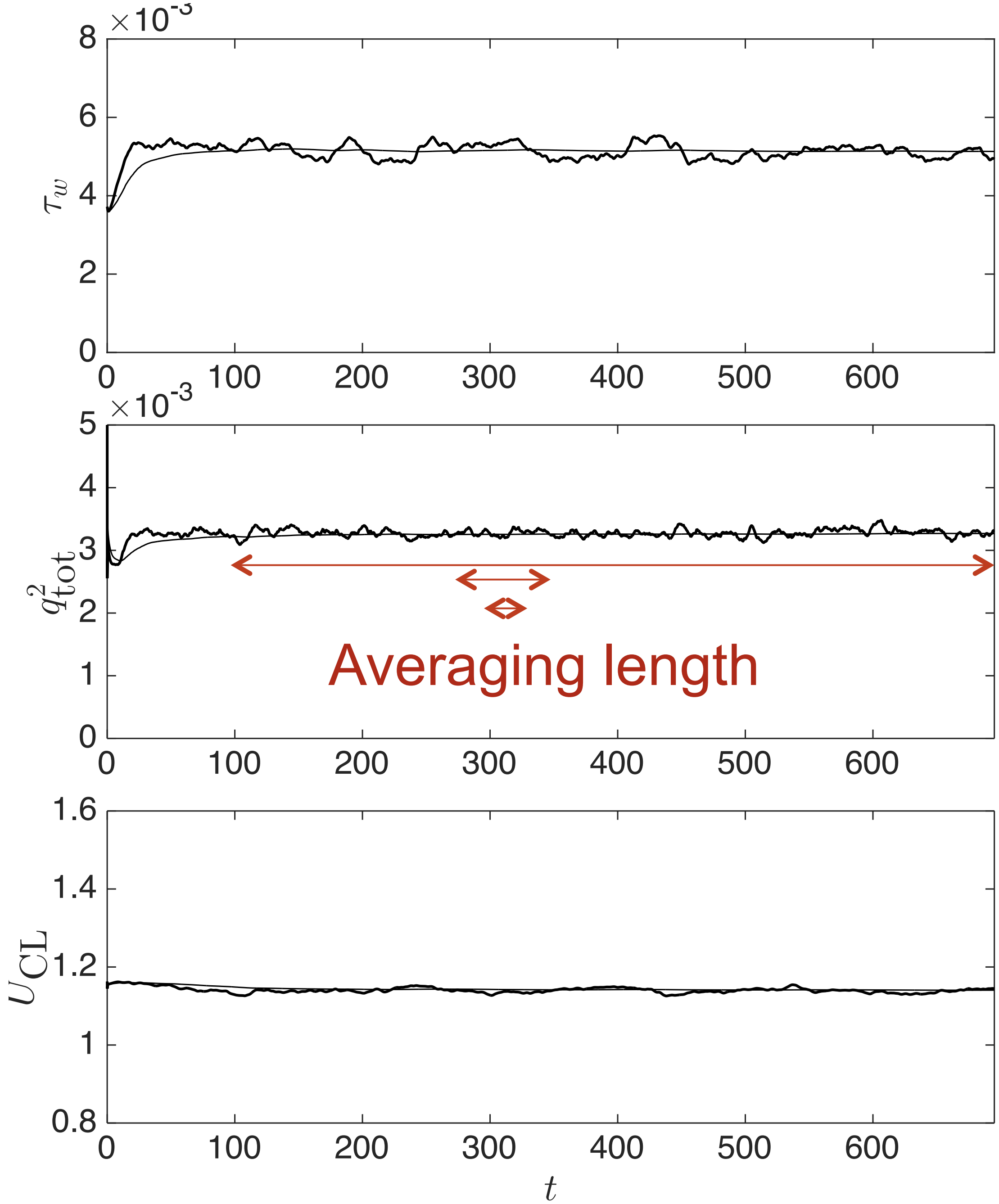
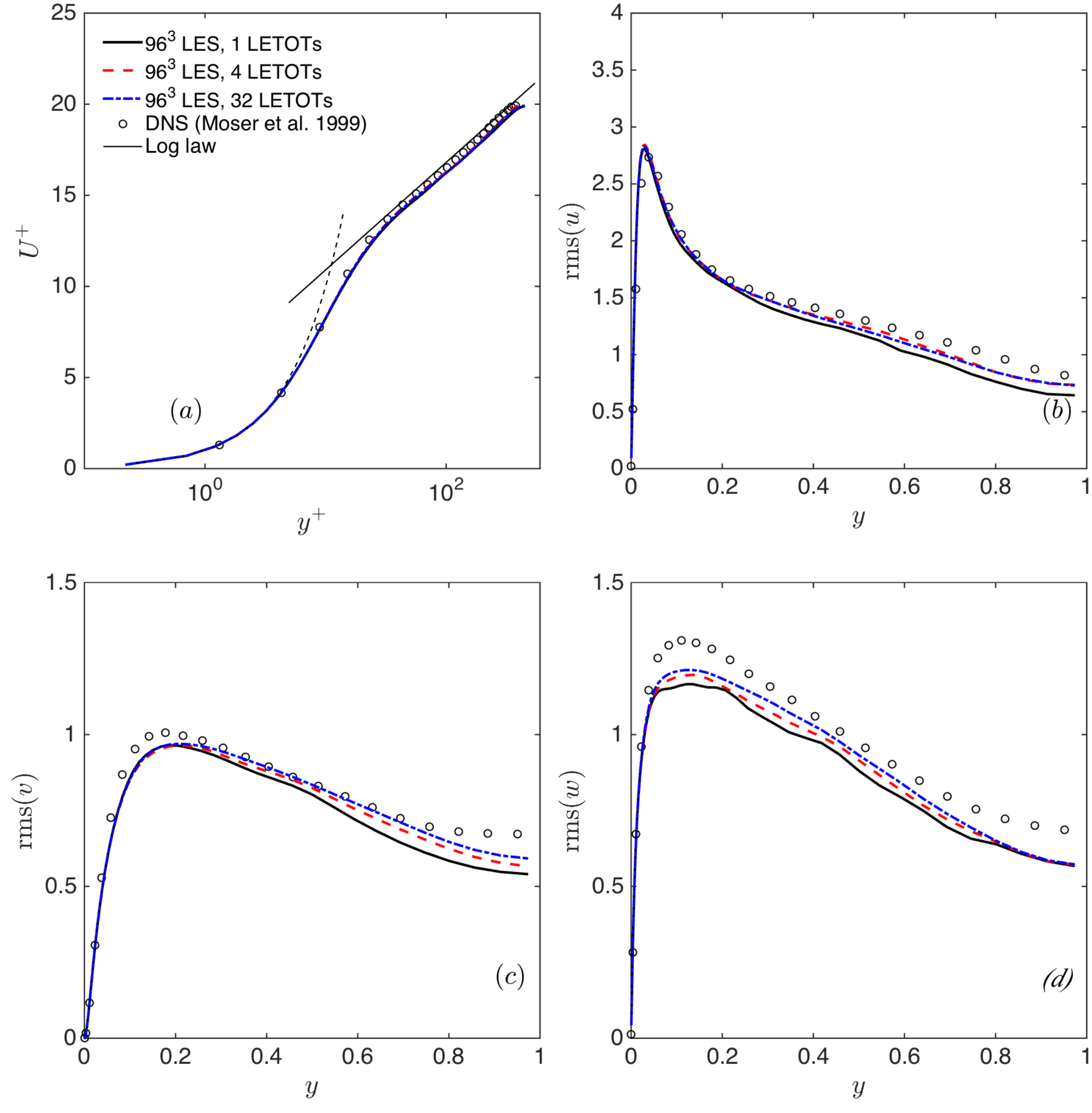


Validation of an LES: Statistical Convergence

- Now we must verify that the statistics are accumulated over a long enough sample.
- Important time scale: Large-eddy Turnover Time (LETOT) tu_τ/δ
- Compare statistics obtained over 1, 4 and 32 LETOTs



Validation of an LES: Statistical Convergence



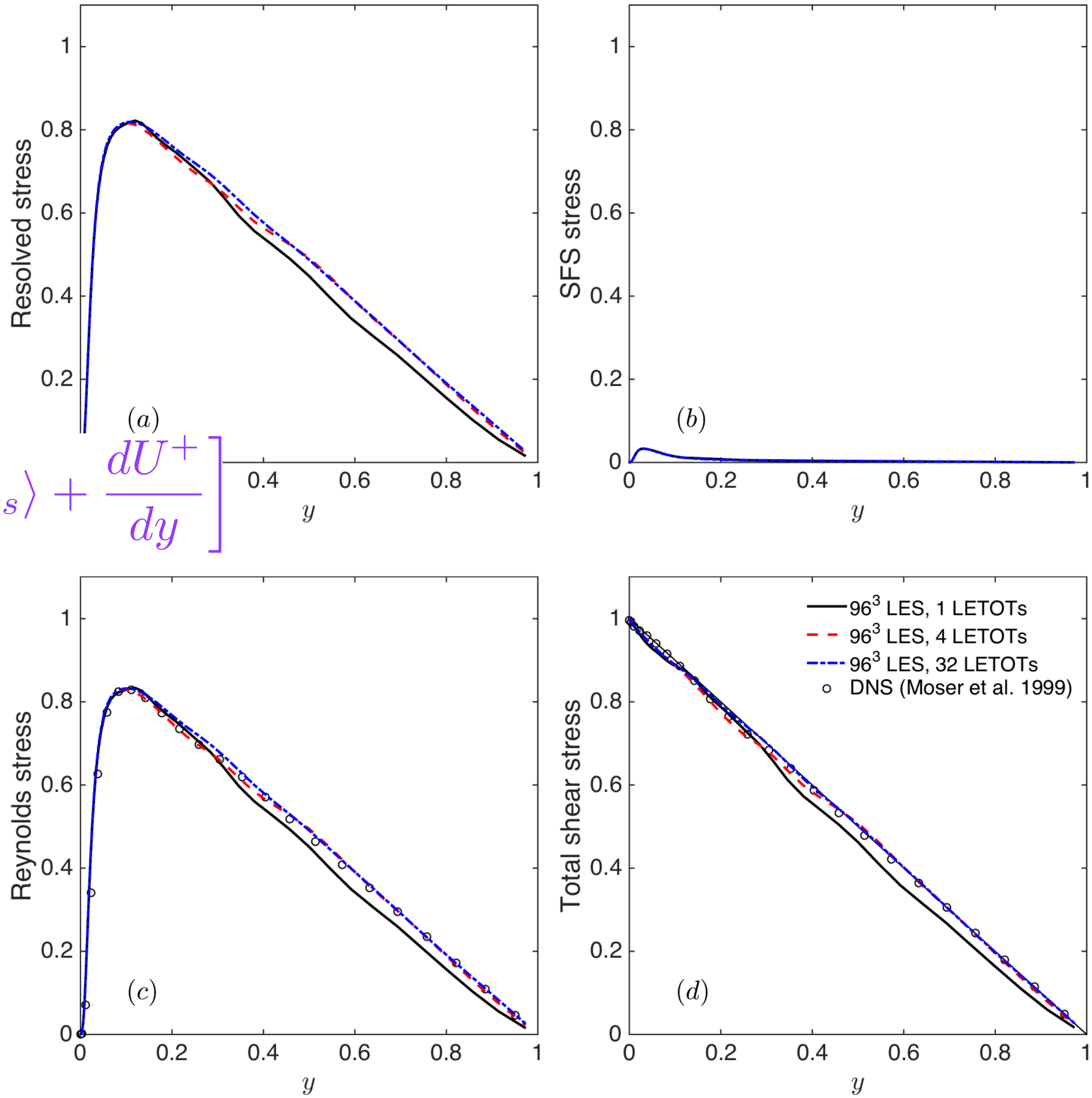
Validation of an LES: Statistical Convergence

Mean momentum equation:

$$0 = -1 + \frac{d}{dy}\boxed{\left[-\langle u'v'\rangle + \langle \tau_{12,sgs}\rangle + \frac{dU^+}{dy}\right]}$$

$\Rightarrow$ $\boxed{\text{total stress}}$ is linear in y

Validation of an LES: Statistical Convergence



$$0 = -1 + \frac{d}{dy}\left[-\langle u'v'\rangle + \langle \tau_{12,sgs}\rangle + \frac{dU^+}{dy}\right]$$

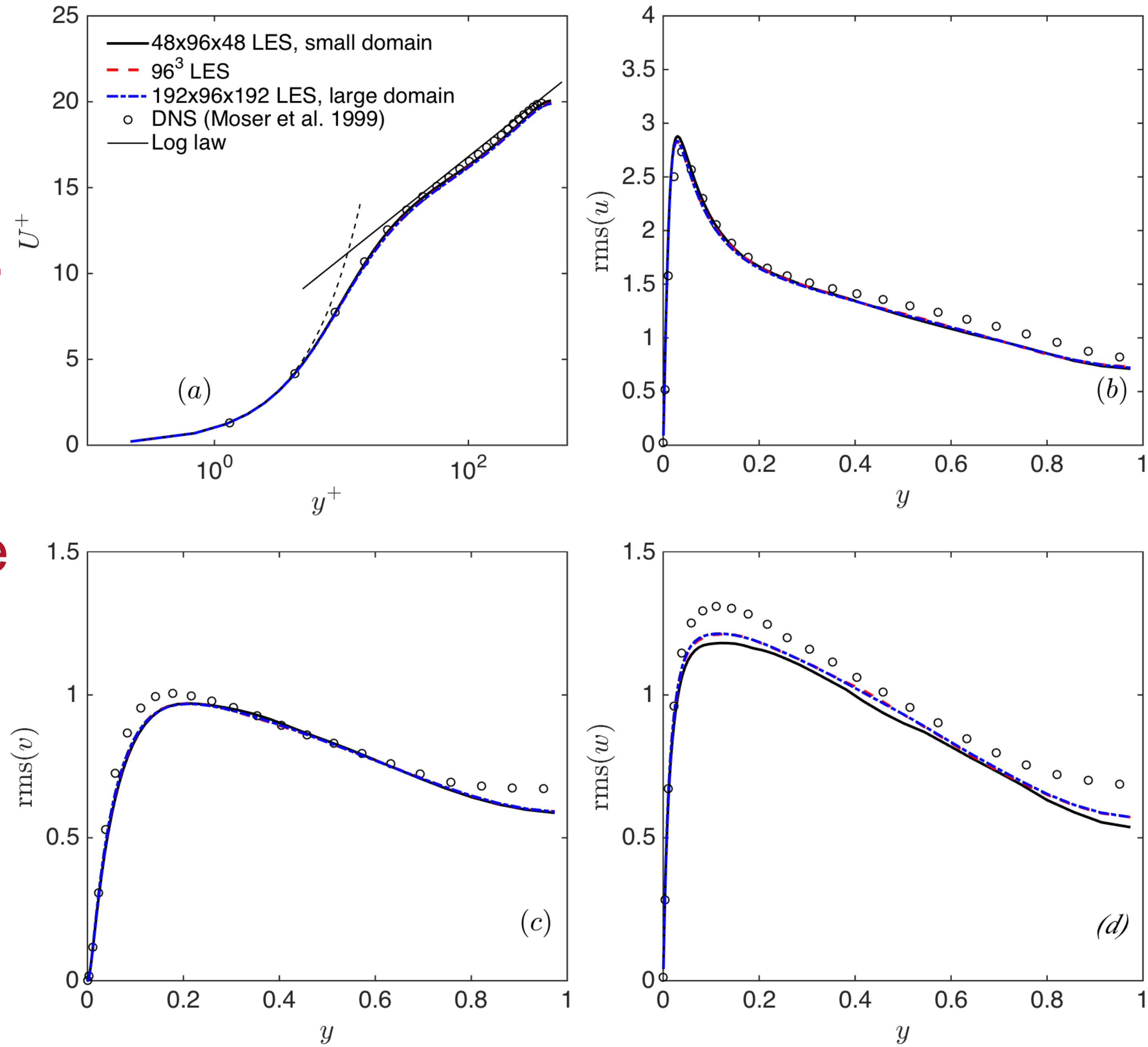
⇒ A linear total stress is achieved after sufficient averaging time.

Validation of an LES: Domain size verification

- When periodic bc's are used, it must be verified that the domain is twice as large as the largest structure present in the flow.
- Perform simulations with three domain sizes:
  - $3\times2\times1.5$, $48\times96\times48$ *points*
  - $6\times2\times3$, $96\times96\times96$ *points*
  - $12\times2\times6$, $192\times96\times192$ *points*

Validation of an LES: Domain size verification

- The low-order statistics are insensitive to the domain size.
- The turbulence structure, however, can be unrealistic if the domain is too small.



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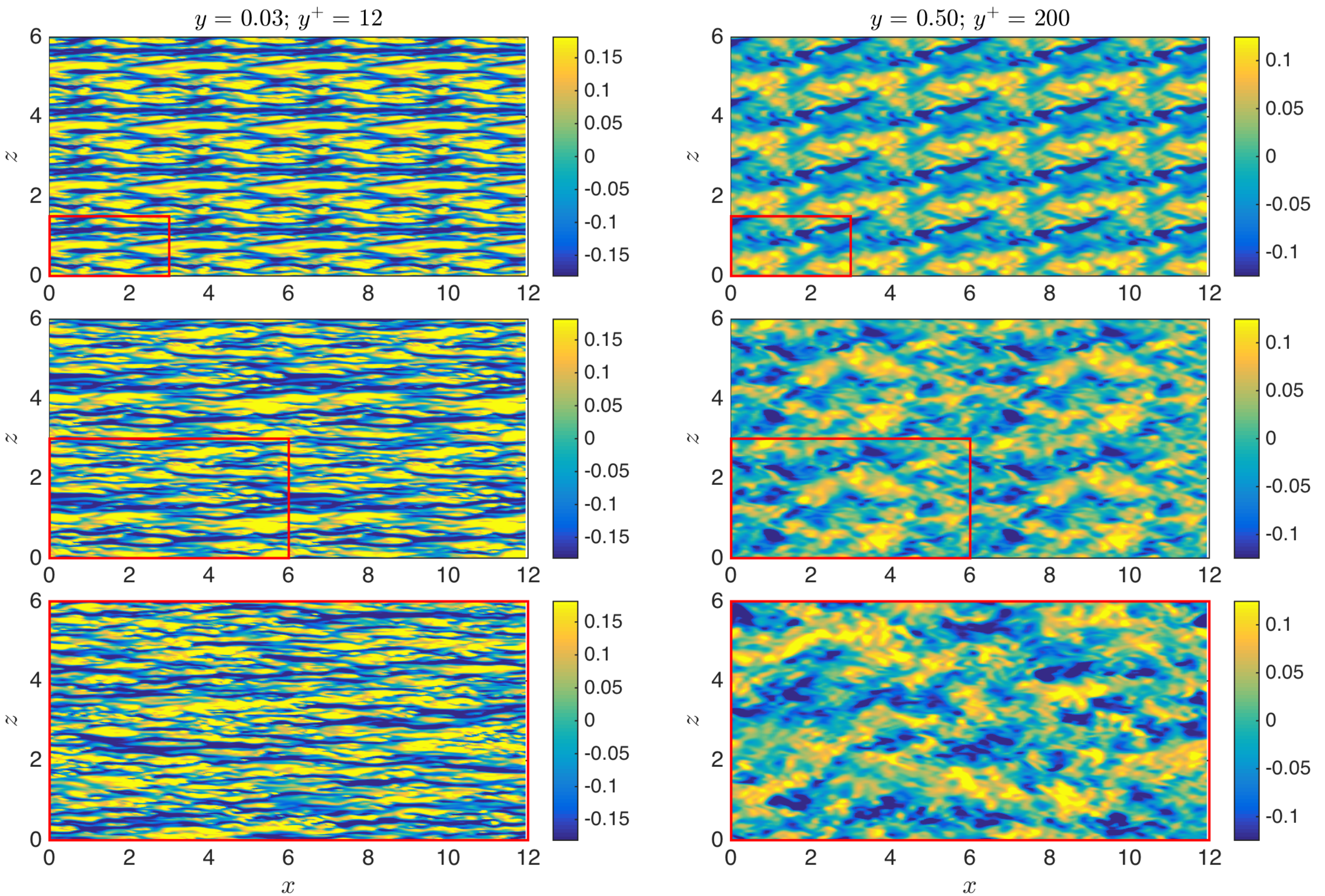
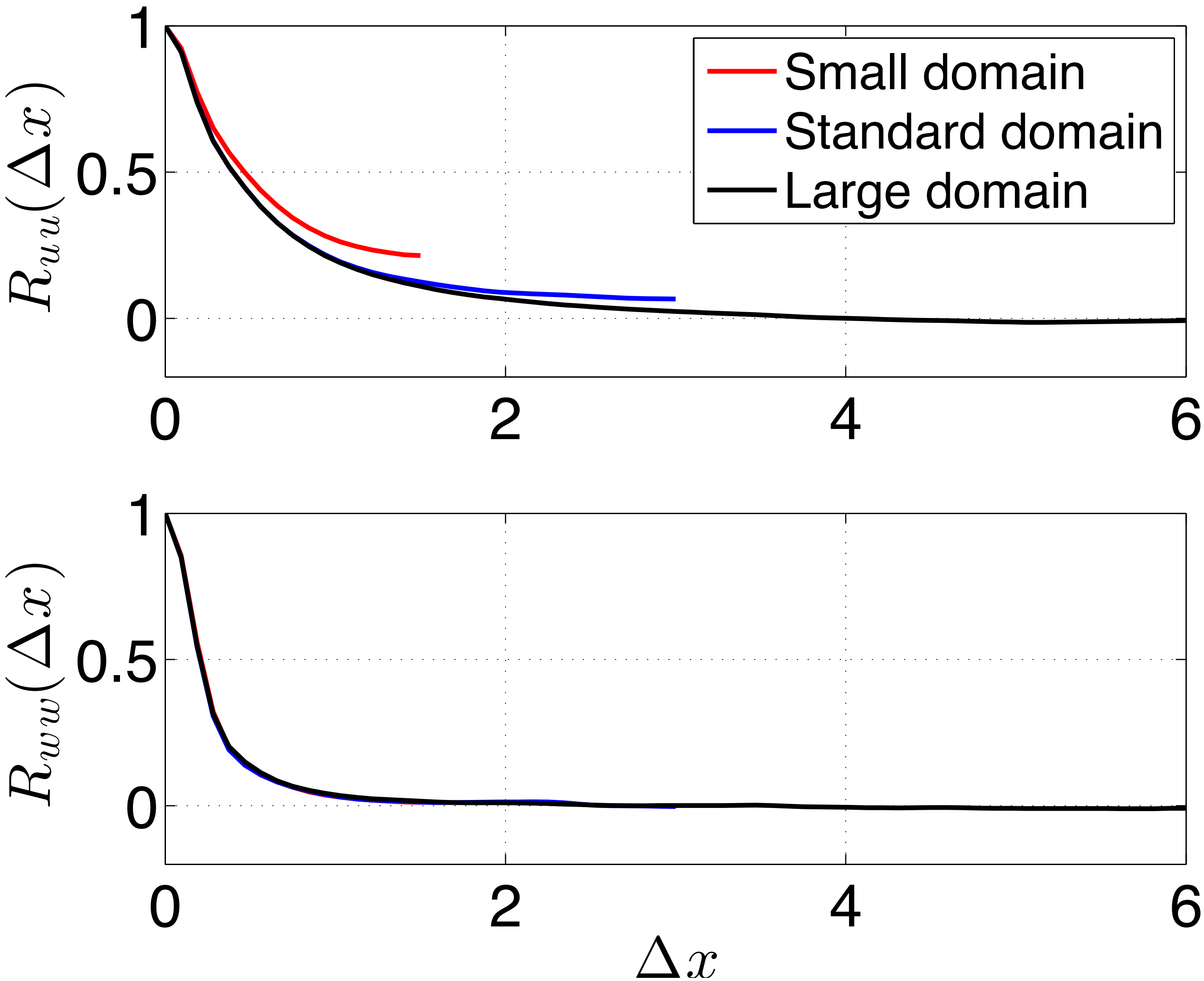


Validation of an LES: Domain size verification

- When periodic bc's are used, it must be verified that the domain is twice as large as the largest structure present in the flow.
- Low order statistics are not very sensitive to finite-size effects.
- Local transport (i.e., maxima and minima) is more strongly affected by an accurate prediction of the eddy scale and shape.
- Perform simulations on three computational domains:
  - *Small:* $3\delta \times 2\delta \times 1.5\delta$
  - *Standard:* $6\delta \times 2\delta \times 3\delta$
  - *Large:* $12\delta \times 2\delta \times 6\delta$
- Calculate the two-point autocorrelation:

$$R_{ff}(\Delta x) = \frac{\langle f'(x,y,z) f'(x+\Delta x, y, z)\rangle}{\langle f' f' \rangle}$$

Validation of an LES: Domain size verification

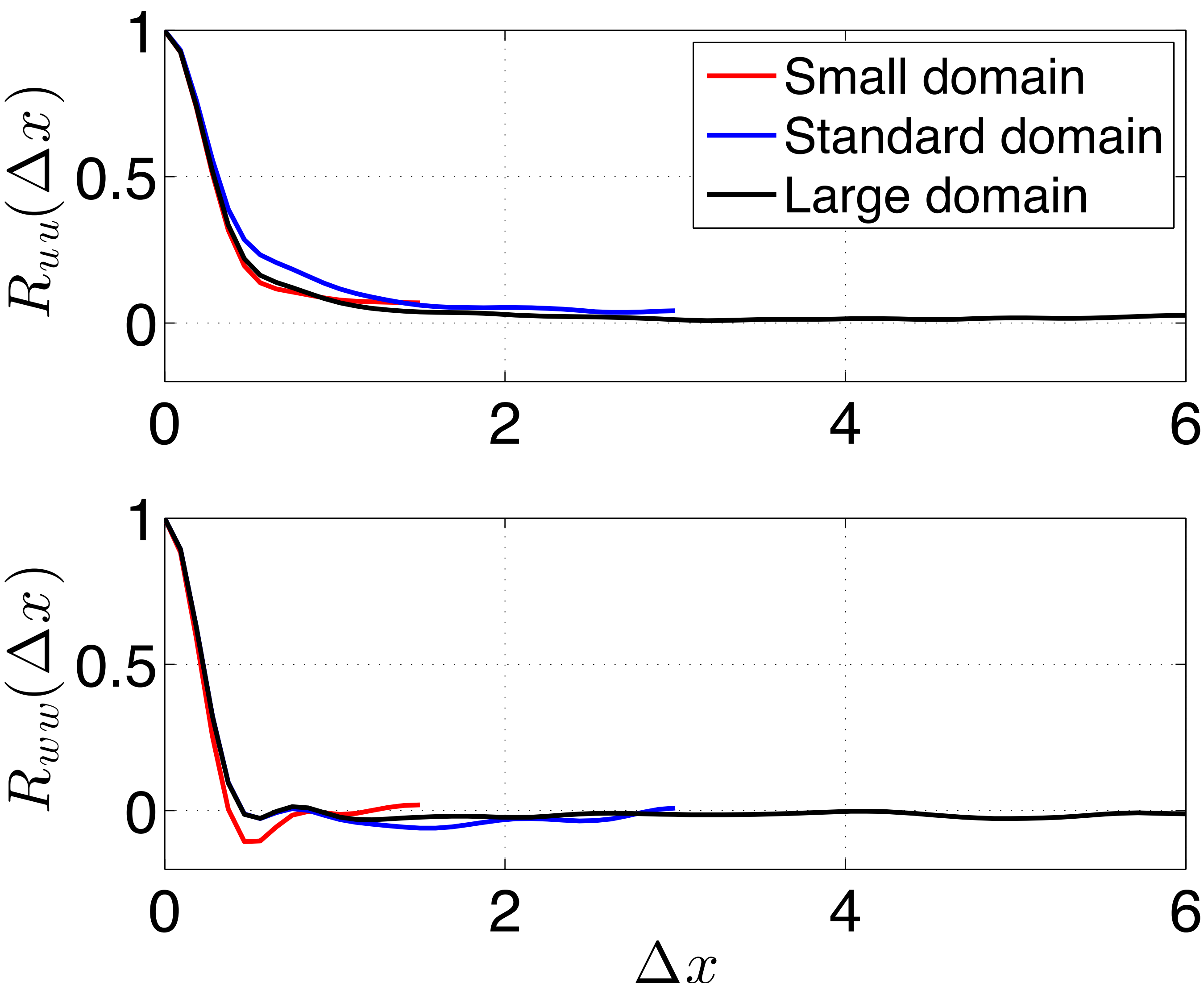


Near-wall (y^+=14):

Elongated low-speed streaks are not captured within the smallest domain, marginally with the standard one

The scales for the spanwise velocity are smaller, and can be captured even by the shortest domain

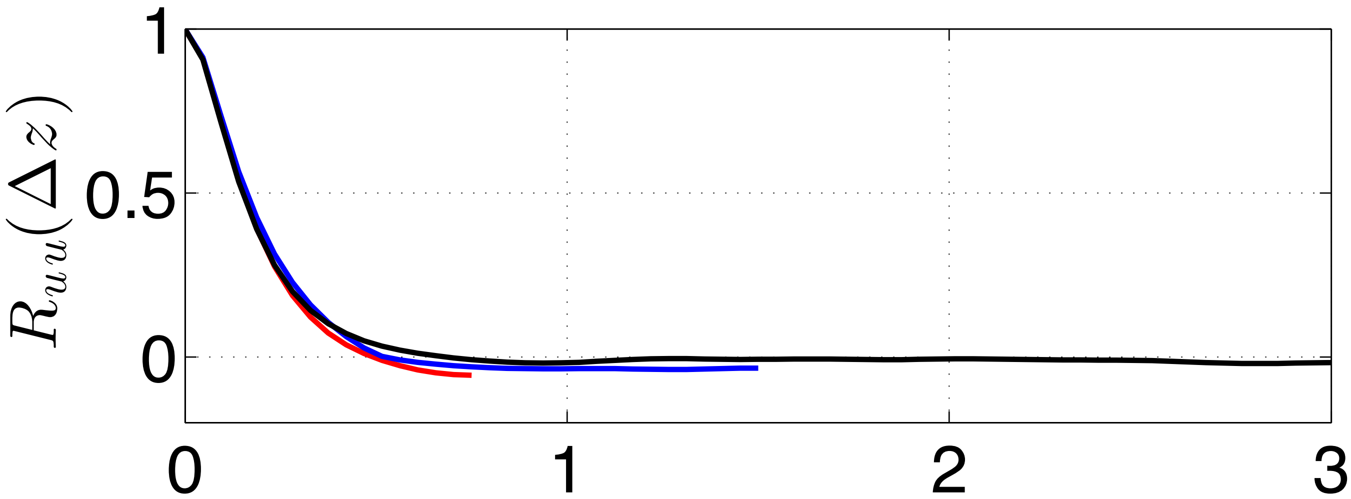
Validation of an LES: Domain size verification



Near-wall (y^+=14):

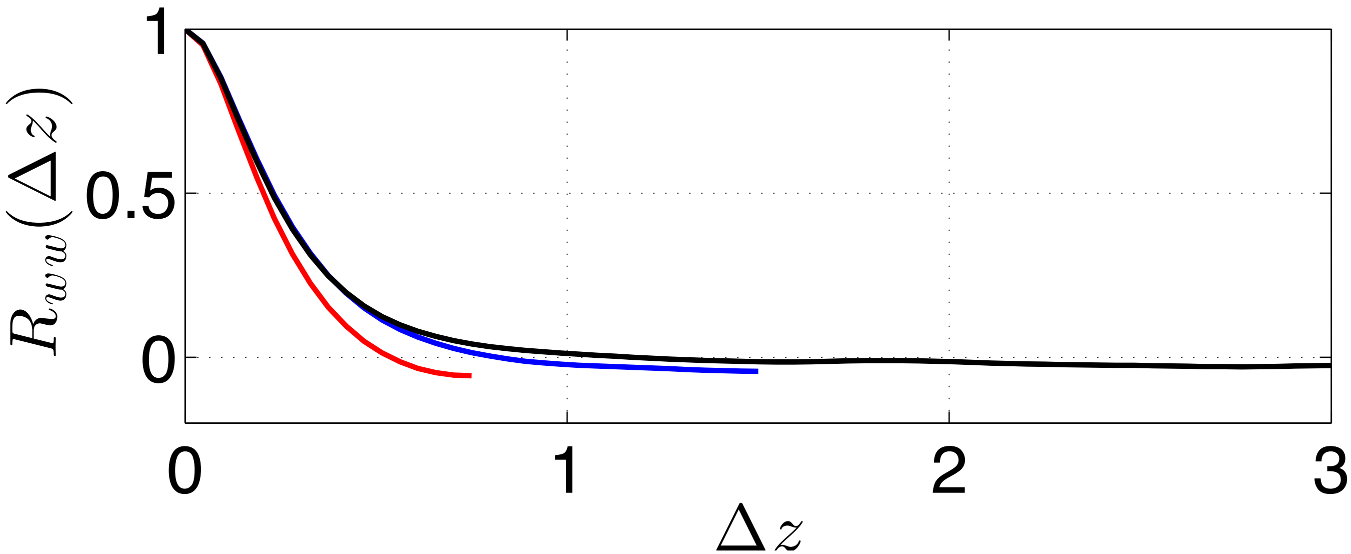
The spanwise scale of the eddies is smaller, and the small domain is sufficient.

Validation of an LES: Domain size verification



Centreline (y^+=390):

The eddies become more isotropic, and the spanwise size of the domain becomes important.



The smallest domain is insufficient, the standard one marginal but OK.

Conclusions

- Validation protocol:
  - *Begin calculation on coarse grids.*
  - *Refine progressively*
  - *Add SFS model*
  - *Achieve grid convergence*
    - Do not trust comparison with (experimental or numerical) data
  - *Check the effect of the model*
    - Does it do anything?
    - Would a coarse DNS give the same results?
  - *Check the boundary conditions*
    - Periodic: is the domain large enough?
    - Outflow: are there reflections?
    - Inflow: is the flow realistic *before* the region of interest?

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- Conclusions