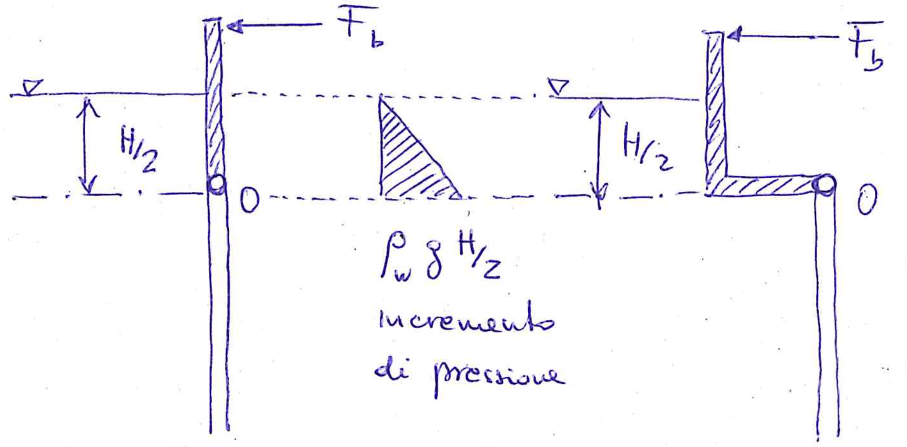
EXE 1

CONFIG. A :

CONFIG. B :



1.1) Calcolo di F_b per configurazione A :

Momento generato dall'acqua sul lato sx della paratia :

$$M_w = F_w \cdot b$$

con $F_w = \rho_w g \frac{H}{2} \cdot \frac{H}{2} \cdot \frac{1}{2} W = \frac{1}{8} \rho_w g H^2 \cdot W$

$b = \frac{1}{3} \cdot \frac{H}{2} = \frac{H}{6}$

$$\left.\right\} \underline{\underline{M_w = \frac{1}{48} \rho_w g H^3 w}}$$

[5%]

Momento generato dalla forza F_b sul lato dx della paratia:

$M_b = F_b \cdot b'$ [2%]

con $b' = H$

Dovendo risultare $M_w = M_b$ si ottiene :

[3%] $F_b = \frac{M_w}{b'} = \underline{\underline{\frac{1}{48} \rho_w g H^2 \cdot W}}$ [10%]

1.2) Calcolo di F_b per configurazione B:

Momento generato dall'acqua sul lato sx della pastra:

$$M_w = F_w^{or} \cdot b' + F_w^{ver} \cdot b''$$

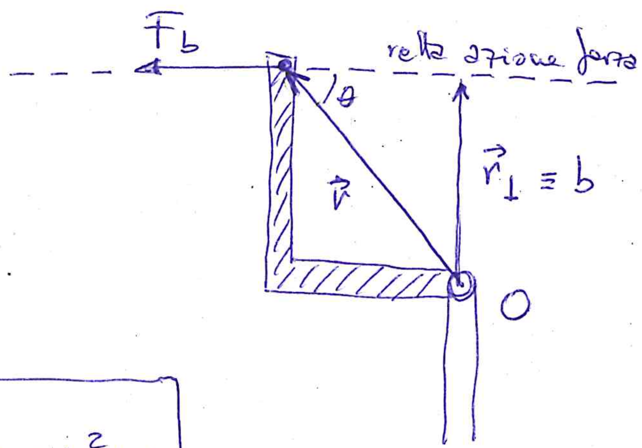
Con $F_w^{or} = \frac{1}{8} \rho_w g H^2 \cdot W$; $b' = \frac{H}{6}$ [2%]

$F_w^{ver} = \rho_w g \frac{H}{2} \cdot L W$; $b'' = \frac{L}{2}$ [6%]

$$M_w = \frac{1}{48} \rho_w g H^3 \cdot W + \frac{1}{4} \rho_w g H L^2 \cdot W$$

Momento generato dalla forza F_b sul lato dx:

$M_b = F_b \cdot H$ [3%]



Dalla condizione $M_w = M_b$

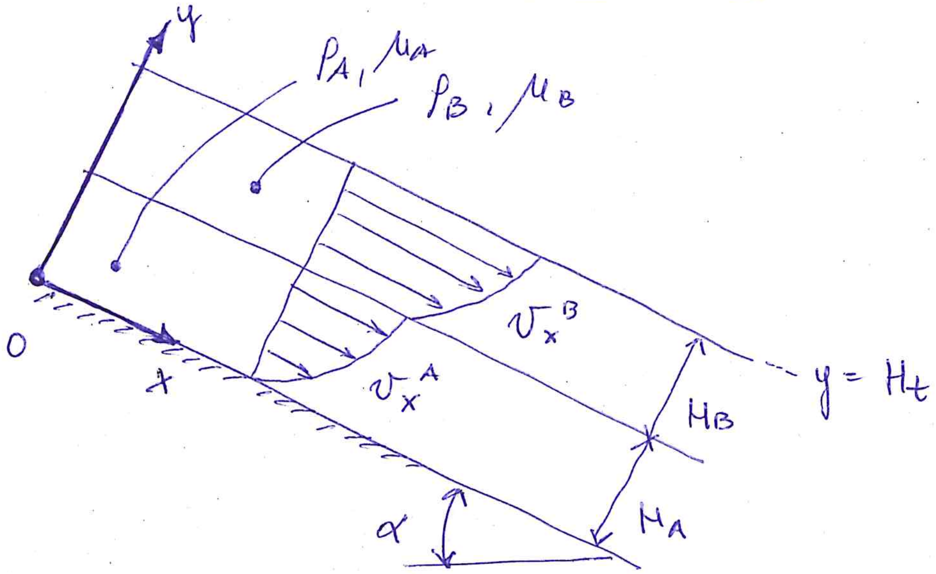
si ricava:

[4%]

$$\boxed{F_b = \frac{1}{48} \rho_w g H^2 W + \frac{1}{4} \rho_w g \cdot L^2 W = \frac{1}{4} \rho_w g \cdot W \left(\frac{H^2}{12} + L^2 \right)}$$

[15%]

EXE 2



$$\frac{\partial P}{\partial x} = \frac{dP}{dx} = -\rho g \operatorname{sen}\alpha$$

[4%]

$\rho_A = \rho_B$

$\mu_A > \mu_B$

−4% se non calcolato

2.1) Calcolo dei profili di velocità

Eq. di conservazione semplificate:

$$\frac{\partial v_x}{\partial x} = 0 \quad [1\%]$$

$$0 = -\frac{\partial P}{\partial x} + \mu \frac{\partial^2 v_x}{\partial y^2} \Longrightarrow \begin{cases} v_x^A = \dfrac{1}{2\mu_A}\left(\dfrac{dP}{dx}\right) y^2 + C_1 y + C_2 \\ v_x^B = \dfrac{1}{2\mu_B}\left(\dfrac{dP}{dx}\right) y^2 + C_3 y + C_4 \end{cases}$$

[2%]

C.C. #1: $v_x^A (y=0) = 0 \longrightarrow \boxed{C_2 = 0}$ [2%]

C.C. #2: $\tau_{yx}^B (y = H_t) = 0$; $\mu_B \left.\dfrac{\partial v_x^B}{\partial y}\right|_{y=H_t} = 0$

$$\mu_B \left[\frac{1}{\mu_B}\left(\frac{dP}{dx}\right) y + C_3 \right]\Bigg|_{y=H_t} = 0$$

$$\boxed{C_3 = -\left(\frac{dP}{dx}\right)\cdot \frac{H_t}{\mu_B}} \quad [4\%]$$

C.C. #3 : $\tau_{yx}^{A}(y=H_A) = \tau_{yx}^{B}(y=H_A)$

$$\mu_A \left.\frac{\partial v_x^A}{\partial y}\right|_{y=H_A} = \mu_B \left.\frac{\partial v_x^B}{\partial y}\right|_{y=H_A}$$

$$\mu_A\left[\frac{1}{\mu_A}\left(\frac{dP}{dx}\right)\cdot H_A + C_1\right] = \mu_B\left[\frac{1}{\mu_B}\left(\frac{dP}{dx}\right)H_A + C_3\right]$$

$$\mu_A\cdot C_1 = \mu_B\cdot C_3 \rightarrow \boxed{C_1 = \frac{\mu_B}{\mu_A}C_3}$$

[3%] $$= \boxed{-\left(\frac{dP}{dx}\right)\cdot\frac{H_t}{\mu_A}}$$

C.C. #4 : $v_x^{A}(y=H_A) = v_x^{B}(y=H_A)$

$$\frac{1}{2\mu_A}\left(\frac{dP}{dx}\right)\cdot H_A^2 + C_1\cdot H_A = \frac{1}{2\mu_B}\left(\frac{dP}{dx}\right)\cdot H_A^2 + C_3\cdot H_A + C_4$$

$$\boxed{C_4 = \frac{1}{2}\left(\frac{dP}{dx}\right)H_A^2\cdot\left(\frac{1}{\mu_A}-\frac{1}{\mu_B}\right) + (C_1 - C_3)\cdot H_A}$$

$$= \frac{1}{2}\left(\frac{dP}{dx}\right)H_A^2\cdot\left(\frac{1}{\mu_A}-\frac{1}{\mu_B}\right) + H_A\left[-\left(\frac{dP}{dx}\right)\cdot\frac{H_t}{\mu_A} + \left(\frac{dP}{dx}\right)\cdot\frac{H_t}{\mu_B}\right]$$

$$= \frac{1}{2}\left(\frac{dP}{dx}\right)H_A^2\left(\frac{1}{\mu_A}-\frac{1}{\mu_B}\right) - \left(\frac{dP}{dx}\right)\cdot H_t\cdot\left(\frac{1}{\mu_A}-\frac{1}{\mu_B}\right)H_A$$

$$= \boxed{\left(\frac{dP}{dx}\right)\left(\frac{1}{\mu_A}-\frac{1}{\mu_B}\right)\cdot\left(\frac{H_A^2}{2} - H_t\cdot H_A\right)}$$ [1] [6%]

o anche:

$$C_4 = \left(-\frac{dP}{dx}\right)\left(\frac{1}{\mu_A} - \frac{1}{\mu_B}\right) H_A \left(\frac{1}{2} H_A + H_B\right) \quad [2]$$

Le espressioni [1] e [2] sono equivalenti.

Pertanto risulta:

$$\boxed{v_x^A = \frac{1}{2\mu_A} \underbrace{\left(\frac{dP}{dx}\right)}_{-\rho g \operatorname{sen}\alpha} (y^2 - 2H_t \cdot y)}$$

[3%]

$$\boxed{v_x^B = \frac{1}{2\mu_B}\left(\frac{dP}{dx}\right)(y^2 - 2H_t \cdot y) + \left(\frac{dP}{dx}\right)\left(\frac{1}{\mu_A} - \frac{1}{\mu_B}\right)\left(\frac{H_A^2}{2} - H_t \cdot H_A\right)}$$

[5%]

CONTROLLO (NON RICHIESTO):

$$v_x^A(y = H_A) = \frac{1}{2\mu_A}\left(\frac{dP}{dx}\right)(H_A^2 - 2H_t \cdot H_A)$$

$$v_x^B(y = H_A) = \frac{1}{2\mu_B}\left(\frac{dP}{dx}\right)(H_A^2 - 2H_t \cdot H_A) + \left(\frac{dP}{dx}\right)\left(\frac{1}{\mu_A} - \frac{1}{\mu_B}\right)\frac{1}{2}(H_A^2 - 2H_t \cdot H_A)$$

$$= \frac{1}{2\mu_A}\left(\frac{dP}{dx}\right)(H_A^2 - 2H_t \cdot H_A) \quad \checkmark$$

2.2) Calcolo della portata volumetrica Q/W

per $H_A = H_B = H$ e $\mu_A = 2\mu_B = 2\mu$:

$$v_x^A = \frac{1}{4\mu}\left(\frac{dP}{dx}\right)(y^2 - 4H \cdot y) \quad [1\%]$$

$$v_x^B = \frac{1}{2\mu}\left(\frac{dP}{dx}\right)(y^2 - 4H \cdot y) +$$

$$+ \underbrace{\left(\frac{1}{2\mu} - \frac{1}{\mu}\right)}_{-\frac{1}{2\mu}}\left(\frac{dP}{dx}\right)\underbrace{\left(\frac{H^2}{2} - 2H^2\right)}_{-\frac{3}{2}H^2}$$

$$= \frac{1}{2\mu}\left(\frac{dP}{dx}\right)(y^2 - 4H \cdot y) + \frac{1}{2\mu}\left(\frac{dP}{dx}\right) \cdot \frac{3}{2}H^2 \quad [1\%]$$

$$\frac{Q_A}{W} = \int_0^H v_x^A \, dy = \frac{1}{4\mu}\left(\frac{dP}{dx}\right)\underbrace{\int_0^H (y^2 - 4Hy)\, dy}_{\left.\left(\frac{y^3}{3} - 4H \cdot \frac{y^2}{2}\right)\right|_0^H = \frac{H^3}{3} - 2H^3 = -\frac{5}{3}H^3}$$

$$= \frac{1}{4\mu}\left(-\frac{dP}{dx}\right) \cdot \frac{5}{3}H^3$$

$$\boxed{\frac{Q_A}{W} = \frac{5}{12\mu}\left(-\frac{dP}{dx}\right)H^3} \quad [3\%]$$

$$\frac{Q_B}{W} = \int_H^{2H} v_x^B \, dy = \frac{1}{2\mu}\left(\frac{dP}{dx}\right)\int_H^{2H} (y^2 - 4H \cdot y)\, dy \, +$$

$$+ \frac{1}{2\mu}\left(\frac{dP}{dx}\right) \cdot \frac{3}{2} H^2 \underbrace{\int_H^{2H} dy}_{H}$$

$$= \frac{1}{2\mu}\left(\frac{dP}{dx}\right) \cdot \underbrace{\left(\frac{y^3}{3} - 2H y^2\right)\Bigg|_H^{2H}} + \frac{3}{4\mu}\left(\frac{dP}{dx}\right) H^3$$

$$\frac{8}{3}H^3 - \frac{H^3}{3} - 8H^3 + 2H^3 =$$

$$\frac{7}{3}H^3 - 6H^3 = -\frac{11}{3}H^3$$

$$= \frac{11}{6\mu}\left(-\frac{dP}{dx}\right) H^3 + \frac{3}{4\mu}\left(\frac{dP}{dx}\right) H^3 \qquad \left[\frac{11}{6} - \frac{3}{4} = \frac{\cancel{26}^{13}}{\cancel{24}_{12}}\right]$$

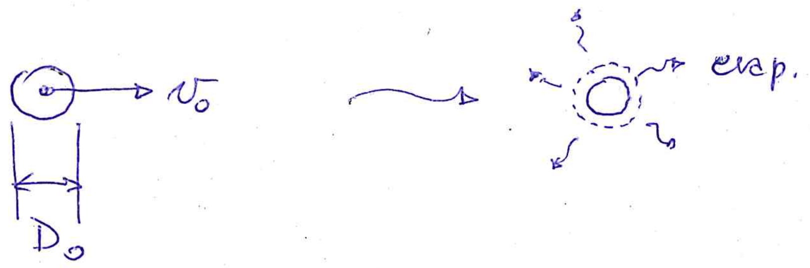
$$= \frac{13}{12\mu}\left(-\frac{dP}{dx}\right) H^3$$

$$\boxed{\frac{Q_B}{W} = \frac{13}{12\mu}\left(-\frac{dP}{dx}\right) H^3} \quad [4\%]$$

$$\frac{Q_{TOT}}{W} = \frac{Q_A}{W} + \frac{Q_B}{W} = \underline{\underline{\frac{3}{2\mu}\left(-\frac{dP}{dx}\right) H^3}} \qquad [1\%]$$

se fanno
la somma

EXE 3



t_0 $\qquad$ t_1

3.1) Calcolo di $D_p(t)$:

$$\frac{dm_p}{dt} = \frac{d}{dt}\left(\rho_p \frac{\pi D_p^3}{6}\right) = \rho_p \frac{\pi}{2} \cdot D_p^2 \frac{dD_p}{dt} \qquad [1\%]$$

$$\frac{dm_p}{dt} = -c \quad \text{dal bilancio di massa} \qquad [1\%]$$

Risulta: $-K\cancel{\pi}\cancel{D_p^2} = \rho_p \frac{\cancel{\pi}}{2} \cancel{D_p^2} \frac{dD_p}{dt}$

$$\frac{dD_p}{dt} = -\frac{2k}{\rho_p} \qquad [1\%]$$

$$\int_{D_0}^{D_p(t)} dD_p = -\frac{2k}{\rho_p}\int_0^t dt$$

$$[2\%] \quad \boxed{D_p(t) = D_0 - \frac{2k}{\rho_p} t} \quad [5\%]$$

3.2) Calcolo di t^* tale che $D_p(t) = D_0/3$:

$$D_p(t^*) = \frac{D_0}{3} = D_0 - \frac{2k}{\rho_p} \cdot t^*$$

$$\boxed{t^* = \frac{1}{3} \cdot \frac{\rho_p \cdot D_0}{k}} \qquad [5\%]$$

Controllo dimensionale : $\left[\frac{\cancel{kg}}{\cancel{m^3}} \cdot \cancel{m} \cdot \frac{\cancel{m^2} \cdot s}{\cancel{kg}} \right]$ ✓

3.3) Calcolo di $v_p(t)$:

$$m_p \frac{dv_p}{dt} + v_p \frac{dm_p}{dt} = - 3\pi \mu v_p D_p \qquad \begin{cases} \vec{u}_f = \vec{0} \\ C_D = 24/Re_p \end{cases} \qquad [3\%]$$

$$\frac{dv_p}{dt} = - \frac{18\mu}{\rho_p D_p^2} v_p + \frac{6k}{\rho_p D_p} \cdot v_p \qquad [3\%]$$

$$\frac{dv_p}{v_p} = \left(- \frac{18\mu}{\rho_p} \cdot \frac{1}{D_p^2} + \frac{6k}{\rho_p} \cdot \frac{1}{D_p} \right) dt$$

$$dt = - \frac{\rho_p}{2k} dD_p$$

$$= \left(+ \frac{9\mu}{k} \cdot \frac{1}{D_p^2} - 3 \cdot \frac{1}{D_p} \right) dD_p \qquad [4\%]$$

Integrando si ottiene :

$$\ln v_p \Big|_{v_0}^{v_p(t)} = \frac{9\mu}{k} \left(- \frac{1}{D_p} \right) \Big|_{D_0}^{D_p(t)} - 3 \ln D_p \Big|_{D_0}^{D_p(t)} \qquad [4\%]$$

$$\ln\left[\frac{v_p(t)}{v_0}\right] = \ln\left[\frac{D_p(t)}{D_0}\right]^{-3} + \frac{9\mu}{k}\left(\frac{1}{D_0} - \frac{1}{D_p(t)}\right) \quad [3\%]$$

Elevando tutto all' esponente:

$$v_p(t) = v_0\left(\frac{D_0}{D_p(t)}\right)^3 \cdot e^{\frac{9\mu}{k}\left(\frac{1}{D_0} - \frac{1}{D_p(t)}\right)} \quad [3\%]$$

con $D_p(t) = D_0 - \frac{2k}{\rho_p} \cdot t$ [20%]

3.4) Calcolo di $v_p(t^*)$:

$$v_p(t^*) = v_0\underbrace{\left(\frac{D_0}{D_p(t^*)}\right)^3}_{\frac{D_0}{(D_0/3)} = 3} \cdot e^{\frac{9\mu}{k}\underbrace{\left(\frac{1}{D_0} - \frac{1}{D_p(t^*)}\right)}_{\frac{1}{D_0} - \frac{3}{D_0} = -\frac{2}{D_0}}}$$

$$= v_0 (3)^3 \cdot e^{\frac{9\mu}{k}\cdot\left(-\frac{2}{D_0}\right)}$$

$$\boxed{v_p(t) = 27 v_0 \cdot e^{-\frac{18\mu}{k D_0}}} \quad [5\%]$$

Controllo dimensionale: $\frac{\mu}{k D_0} = \frac{Kg}{m \cdot s} \cdot \frac{m^2 s}{Kg} \cdot \frac{1}{m}$ ✓