CALCOLO DELLA PRESSIONE:

NS in 2D (ADIM):

$$\frac{\partial p}{\partial x} = -\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) + \frac{1}{Re}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

$$\frac{\partial p}{\partial y} = -\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) + \frac{1}{Re}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

$$\left.\begin{aligned} \frac{\partial}{\partial x}\left(\frac{\partial p}{\partial x}\right) &\equiv \frac{\partial^2 p}{\partial x^2} = \dots \\ \frac{\partial}{\partial y}\left(\frac{\partial p}{\partial y}\right) &\equiv \frac{\partial^2 p}{\partial y^2} = \dots \end{aligned}\right\} \quad \boxed{\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = -2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \frac{\partial v}{\partial x}\cdot\frac{\partial u}{\partial y}\right]}$$

EQUAZIONE DI POISSON PER LA PRESSIONE (discretizzata con schema FTCS)

$$P_{i,j}^{n} = \frac{(P_{i+1,j}^{n} + P_{i-1,j}^{n})(\Delta y)^2 + (P_{i,j+1}^{n} + P_{i,j-1}^{n})(\Delta x)^2}{2\left[(\Delta x)^2 + (\Delta y)^2\right]} + \left[\frac{(\Delta x)^2\cdot(\Delta y)^2}{(\Delta x)^2 + (\Delta y)^2}\right]\cdot\left[\left(\frac{u_{i+1,j}^{n} - u_{i-1,j}^{n}}{2\Delta x}\right)^2 + \right. \quad (A)$$

$$\left. + \frac{u_{i,j+1}^{n} - u_{i,j-1}^{n}}{2\Delta y}\cdot\frac{v_{i+1,j}^{n} - v_{i-1,j}^{n}}{2\Delta x}\right]$$

L'equazione (A) va risolta nei soli punti interni del dominio ($i = 2, N_X - 1$; $j = 2, N_Y - 1$).

Le B.C. si ricavano dalle NS in 2D, discretizzate:

$$\frac{\partial P}{\partial x} = \frac{P^n_{i+1,j} - P^n_{i-1,j}}{2\Delta X} = -\left(u^n_{i,j} \cdot \frac{u^n_{i+1,j} - u^n_{i-1,j}}{2\Delta X} + v^n_{i,j} \frac{u^n_{i,j+1} - u^n_{i,j-1}}{2\Delta y}\right)$$

$$+ \frac{1}{Re}\left(\frac{u^n_{i+1,j} - 2u^n_{i,j} + u^n_{i-1,j}}{\Delta X^2} + \frac{u^n_{i,j+1} - 2u^n_{i,j} + u^n_{i,j-1}}{\Delta y^2}\right)$$

Questa equazione, scritta per $i = 2$, fornisce (omettendo l'apice n):

$$\frac{P_{3,j} - P_{1,j}}{2\Delta X} = -\left(u_{2,j} \cdot \frac{u_{3,j} - u_{1,j}}{2\Delta X} + v_{2,j} \frac{u_{2,j+1} - u_{2,j-1}}{2\Delta y}\right) +$$

$$+ \frac{1}{Re}\left(\frac{u_{3,j} - 2u_{2,j} + u_{1,j}}{\Delta X^2} + \frac{u_{2,j+1} - 2u_{2,j} + u_{2,j-1}}{\Delta y^2}\right)$$

Esplicitando rispetto a $P_{1,j}$ si ottiene:

$$P_{1,j} = P_{3,j} + 2\Delta x \left(u_{2,j} \cdot \frac{u_{3,j} - u_{1,j}}{2\Delta x} + v_{2,j} \cdot \frac{u_{2,j+1} - u_{2,j-1}}{2\Delta y} \right)$$

$$- \frac{2\Delta x}{Re} \left(\frac{u_{3,j} - 2u_{2,j} + u_{1,j}}{(\Delta x)^2} + \frac{u_{2,j+1} - 2u_{2,j} + u_{2,j-1}}{(\Delta y)^2} \right) \quad \text{(B)}$$

Se invece $i = Nx-1$, si ottiene:

$$\frac{P_{Nx,j} - P_{Nx-2,j}}{2\Delta x} = - \left(u_{Nx-1,j} \cdot \frac{u_{Nx,j} - u_{Nx-2,j}}{2\Delta x} + v_{Nx-1,j} \cdot \frac{u_{Nx-1,j+1} - u_{Nx-1,j-1}}{2\Delta y} \right)$$

$$+ \frac{1}{Re} \left(\frac{u_{Nx,j} - 2u_{Nx-1,j} + u_{Nx-2,j}}{(\Delta x)^2} + \frac{u_{Nx-1,j+1} - 2u_{Nx-1,j} + u_{Nx-1,j-1}}{(\Delta y)^2} \right)$$

Esplicitando rispetto a $P_{Nx,j}$ si ottiene:

$$P_{Nx,j} = P_{Nx-2,j} - 2\Delta x \left(u_{Nx-1,j} \cdot \frac{u_{Nx,j} - u_{Nx-2,j}}{2\Delta x} + v_{Nx-1,j} \cdot \frac{u_{Nx-1,j+1} - u_{Nx-1,j-1}}{2\Delta y} \right) \quad \text{(C)}$$

$$+ \frac{2\Delta x}{Re} \left[\frac{1}{(\Delta x)^2} \cdot \left(u_{Nx,j} - 2u_{Nx-1,j} + u_{Nx-2,j} \right) + \frac{1}{(\Delta y)^2} \cdot \left(u_{Nx-1,j+1} - 2u_{Nx-1,j} + u_{Nx-1,j-1} \right) \right]$$

Discretizzando invece la NS_y si ha:

$$\frac{\partial P}{\partial y} = \frac{P^n_{i,j+1} - P^n_{i,j-1}}{2\Delta y} = -\left(u^n_{i,j} \frac{v^n_{i+1,j} - v^n_{i-1,j}}{2\Delta x} + v^n_{i,j} \frac{v^n_{i,j+1} - v^n_{i,j-1}}{2\Delta y} \right)$$

$$+ \frac{1}{Re}\left(\frac{v^n_{i+1,j} - 2v^n_{i,j} + v^n_{i-1,j}}{(\Delta x)^2} + \frac{v^n_{i,j+1} - 2v^n_{i,j} + v^n_{i,j-1}}{(\Delta y)^2} \right)$$

Per $j=2$ si ottiene:

$$P_{i,1} = P_{i,3} + 2\Delta y \left(u_{i,2} \frac{v_{i+1,2} - v_{i-1,2}}{2\Delta x} + v_{i,2} \frac{v_{i,3} - v_{i,1}}{2\Delta y} \right)$$

$$- \frac{2\Delta y}{Re}\left(\frac{v_{i+1,2} - 2v_{i,2} + v_{i-1,2}}{(\Delta x)^2} + \frac{v_{i,3} - 2v_{i,2} + v_{i,1}}{(\Delta y)^2} \right)$$

D

Per $j = NY-1$ si ottiene:

$$P_{i,NY} = P_{i,NY-2} - 2\Delta y\left(u_{i,NY-1}\cdot\frac{v_{i+1,NY-1}-v_{i-1,NY-1}}{2\Delta x} + v_{i,NY-1}\cdot\frac{v_{i,NY}-v_{i,NY-2}}{2\Delta y}\right)$$

(E)

$$+\frac{2\Delta y}{Re}\left(\frac{v_{i+1,NY-1}-2v_{i,NY-1}+v_{i-1,NY-1}}{(\Delta x)^2}+\frac{v_{i,NY}-2v_{i,NY-1}+v_{i,NY-2}}{(\Delta y)^2}\right)$$

Le equazioni (B), (C), (D) ed (E) vanno applicate per $i = 2, NX-1$ e per $j = 2, NY-1$ a seconda di quale indice varia nell'equazione. L'equazione di Poisson per la pressione va invece risolta iterativamente imponendo:

$P^{n}_{i,j}$ = valore di pressione a fine iterazione!

$$\frac{(P^{n}_{i+1,j}+P^{n}_{i-1,j})\cdot(\Delta y)^2+(P^{n}_{i,j+1}+P^{n}_{i,j-1})\cdot(\Delta x)^2}{2\left[(\Delta x)^2+(\Delta y)^2\right]} = \text{valori di pressione a inizio iterazione!}$$

Tale equazione va quindi risolta all'interno di un ciclo IF : Le

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IF ( n° punti griglia a convergenza < (NX-2)·(NY-2)  .AND.  n° iteraz. < N)
THEN
    DO i = 2, NX-1
    DO j = 2, NY-1
        P(i,j) = ...
    ENDDO
    ENDDO
ENDIF
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N = max n° di iterazioni da fare (x evitare loop "infiniti")

N.B. Alla parete superiore (lid) : $v = 0$; $\frac{\partial u}{\partial x} = 0$ $\Rightarrow$ $\frac{\partial P}{\partial x} = 0$; $\frac{\partial P}{\partial y} = 0$

lo stesso si ottiene alla parete inferiore

Alle pareti verticali, invece : $u = 0$; $v = 0$ $\Rightarrow$ $\frac{\partial P}{\partial x} = 0$; $\frac{\partial P}{\partial y} = 0$