CLASSIFICATION OF STRATIFIED FLOWS

A general classification of stratified flows can be given based on the ratio $\Delta\rho/\rho_0$ where $\Delta\rho = \rho - \rho_0$ is the density difference driving stratification and buoyancy, and ρ_0 is the reference density. Based on the % value of $\Delta\rho/\rho_0$, the following types of flow can be obtained:

- $0\% < \dfrac{\Delta\rho}{\rho_0} \leq 0{,}2\%$: DENSITY CURRENTS (−)
- $0.2\% < \dfrac{\Delta\rho}{\rho_0} \leq 2\%$: GRAVITY CURRENTS — Ri ↓
- $2\% < \dfrac{\Delta\rho}{\rho_0} \leq 20\%$: TURBIDITY CURRENTS (+)

Recalling that $Ri = \dfrac{\Delta\rho}{\rho_0} \cdot \dfrac{g\,\ell_{ref}}{U_{ref}^2}$, it follows that density currents are characterized by values of the Richardson number that are lower than those typical of gravity currents. In turn, gravity currents are characterized by values of the Richardson number that are lower that those typical of turbidity currents.

- **DENSITY CURRENTS** are typically originated by temperature gradients, but can also be generated by salinity gradients or differences in the concentration of sediments suspended within the fluid.
  In the environment, density currents are often found along the bottom of oceans or lakes in the form of subacqueous currents originated by the presence of colder or saltier or more concentrated (in terms of suspended sediments) volumes of water surrounded by other volumes of water with lower density (e.g. lighter).
  Denser water volumes thus sink and flow along the bottom under the effect of gravity. The difference in density, moreover, reduces the mixing of the heavier water with the overlying lighter water, enabling the current to mantain itself for relatively-long distances

- **GRAVITY CURRENTS** are typically originated by salinity gradients, which induce moderate levels of stratification (higher than those producing density currents). Gravity currents are primarily horizontal and occur as either top or bottom boundary currents or as intrusions at some intermediate level. These currents appear in estuaries, where the fresh water of the river meets the salt water of the ocean/sea: Due to the density difference between the two water streams, a gravity current of salt water moving upstream along the river bed can form under certain conditions of tide, wind and/or temperature. Such a current is known as <u>SALT WEDGE</u> (cuneo salino) and can extend up to several kilometers without significant mixing between the salt water and the fresh water due to the stability of the stratification. This reduced mixing can represent a source of environmental contamination, especially if the water of the river is used for irrigation purposes: The salt water has no means of getting oxygen from the surface (because of the fresh water layer above) and, over time, biological activity within the wedge will consume all the available oxygen. The wedge will thus become anaerobic and, therefore, unhealthy and unproductive.

The boundary between the fresh water of the river and the salt water within the wedge is called HALOCLINE and is characterized by an abrupt change of salinity.

NOTE 1: In gravity currents, density differences alone do not cause the intrusion of the heavier (denser) fluid into the lighter one. It is the difference in specific weights between the fluids that really causes the intrusion and, in turn, the current. To recognize the dominant role played by gravity in such currents, they have been named gravity currents (whereas density currents refer to currents caused by density differences alone).

NOTE 2: Gravity currents represent a ubiquitous phenomenon in nature and technology. They can be generated not only in estuaries (in the form of salt wedges) but also in the atmosphere, when a cold air current advances in a warm atmosphere, or in water treatment facilities (when a muddy stream enters a reservoir of clear water) or in the case of oil spilling into the ocean (as the oil slicks on the ocean's surface).

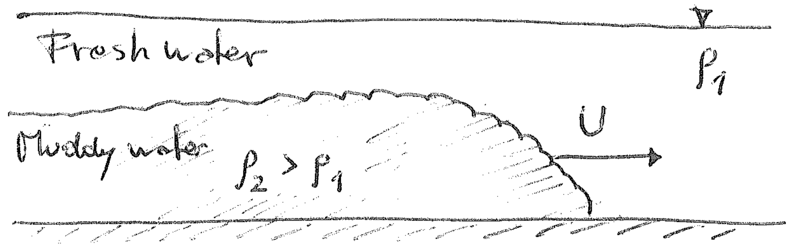
- TURBIDITY CURRENTS : Turbidity currents are gravity currents in which the density difference or the density in the specific weights between the fluids (or within the same fluid) that provide the driving buoyancy forces is due to the presence of sediments held in suspension by a turbulent flow.

  For example, in lakes and oceans, turbidity currents can be generated by the density difference between turbid water containing sand and/or clay and clear ambient water.

  Based on the classification provided before, turbidity currents are characterized by the largest $\Delta\rho$ and, therefore, by the largest stratification levels. Such currents require strong turbulence levels to be maintained: Turbulence is needed to sustain the processes of <u>sedimentation</u> and <u>resuspension</u> of the particulate matter within the turbid water. These processes make the study of turbidity currents much more complex than the study of density currents and gravity currents.

From a practical viewpoint, turbidity currents are important for the formation of submarine sediment waves, dunes and canyons (which are generated by the interaction between the current and the seafloor via erosion - namely resuspension - and deposition - namely sedimentation - of sediments); for oil and gas exploration (under certain conditions, the organic matter contained in the sediment may form hydrocarbons, which in turn forms sedimentary rock from turbidity current deposits: Sedimentary rock plays an important role in oil and gas exploration). From an engineering perspective, turbity currents pose a significant hazard to submarine oil pipelines, well heads and telecommunication cables (when the current is produced by a landslide).

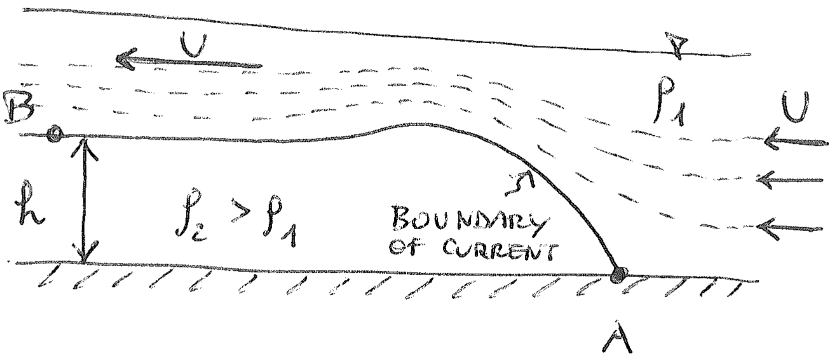
CONCEPTUAL MODEL OF IDEALIZED GRAVITY CURRENT



Example of a real gravity current

U = front velocity of the gravity current

An idealization of the gravity current, introduced for the first time by von Karman, is the following:



h = height of the current

where the flow is considered in the reference frame moving with the current front (such that the front velocity U becomes the velocity of the incoming - from the point of view of the current - flow). The following simplifying assumptions are also made:

1. the flow is steady in the moving frame
2. the flow is inviscid (viscous effects can be neglected)
3. the fluid inside the current is at rest

Based on these assumptions, application of the inviscid Bernoulli equation (in which energy loss due to viscous effects is neglected) between points A and B along the boundary of the current yields:

(1)

$$\frac{P_A}{\rho_1} + g h_A + \frac{1}{2} \cancelto{=0}{v_A^2} = \frac{P_B}{\rho_1} + g h_B + \frac{1}{2} v_B^2$$

$(v_A = 0)$ $(v_B = -U)$

$$P_A = P_B + \rho_1 g \underbrace{(h_B - h_A)}_{h} + \frac{1}{2}\rho_1 U^2$$

$$= P_B + \rho_1 g h + \frac{1}{2}\rho_1 U^2$$

if Bernoulli is applied just outside of the current boundary

(2)

$$\frac{P_A}{\rho_2} + g h_A + \frac{1}{2} \cancelto{=0}{v_A^2} = \frac{P_B}{\rho_2} + g h_B + \frac{1}{2} \cancelto{=0}{v_B^2}$$

$(v_A = 0)$ $(v_B = 0)$

$$P_A = P_B + \rho_2 g h$$

if Bernoulli is applied just inside of the current's boundary

where $v_A = 0$ and $v_B = 0$ because the fluid within the current is assumed to be at rest.

From these two equations one gets:

$$\rho_1 g h + \frac{1}{2}\rho_1 U^2 = \rho_2 g h$$

$$U = \sqrt{2h\left(\frac{\rho_2 - \rho_1}{\rho_1} \cdot g\right)}$$

FRONT VELOCITY OF THE GRAVITY CURRENT

The quantity $g' = \frac{\rho_2 - \rho_1}{\rho_1} \cdot g$ is also referred to as reduced gravity. Note that $[g' \cdot h] = \left[\frac{m^2}{s^2}\right]$ and hence the ratio $U/\sqrt{g'h}$ is a dimensionless quantity. Indeed this ratio represents the well-known FROUDE NUMBER:

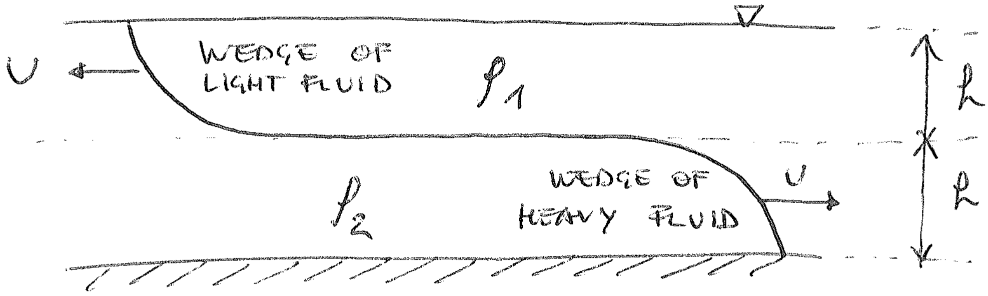
$$Fr = \frac{U}{\sqrt{g' \cdot h}}$$

Physical meaning:

$$Fr = \frac{\text{inertial forces}}{\text{gravitational forces}}$$

Note that $Fr^2 = 1/Ri$!! Indeed, gravitational forces coincide with buoyancy forces in a stratified flow (subject to gravity).

In the case of <u>mutual intrusion</u> of two fluids of slightly different specific weights but equal depth, the front velocity can be derived in terms of energy balance.



SCHEMATIC OF GRAVITY CURRENT DUE TO MUTUAL INTRUSION

In a small time interval Δt, the two wedges advance a distance $\Delta x = U \cdot \Delta t$ if U is assumed to be constant and the two wedges anti-symmetric (for all practical purposes, this latter assumption is quite reasonable).

The light fluid gains the following amount of potential energy per unit width of the wedge:

$$\frac{1}{2}\rho_1 g h \cdot \underbrace{U\Delta t \cdot h}_{\text{This is a volume per unit width}} = E_{P,1}$$

The heavy fluid loses the following amount of potential energy per unit width of the wedge:

$$\frac{1}{2}\rho_2 g h \cdot U\Delta t \cdot h = E_{P,2} \quad (> E_{P,1})$$

The net amount of potential energy lost is:

$$\boxed{\Delta E_P = E_{P,2} - E_{P,1} = \frac{1}{2}(\rho_2 - \rho_1) g h^2 U \Delta t}$$

In the same interval Δt, the two fluids gain the following amount of kinetic energy per unit width:

$$\frac{1}{2}\rho_1 U^2 \cdot (U\Delta t \cdot h) = E_{k,1}$$

$$\frac{1}{2}\rho_2 U^2 \cdot (U\Delta t \cdot h) = E_{k,2}$$

*per unit width!

where $U\Delta t \cdot h$ is the volume of fluid* that has advanced in the interval Δt. The total kinetic energy gained by the two fluids is:

$$\boxed{\Delta E_k = E_{k,1} + E_{k,2} = \frac{1}{2}(\rho_1 + \rho_2)\, U^3 \Delta t\, h}$$

Invoking the principle of conservation of energy:

$$\Delta E_p = \Delta E_k \Rightarrow \frac{1}{2}(\rho_2 - \rho_1)\, g h^2\, U \Delta t = \frac{1}{2}(\rho_1 + \rho_2)\, U^3 \Delta t\, h$$

$$\boxed{U = \sqrt{\frac{\rho_2 - \rho_1}{\rho_2 + \rho_1} \cdot g h}}$$

FRONT VELOCITY OF THE GRAVITY CURRENT DUE TO MUTUAL INTRUSION OF TWO FLUID WEDGES

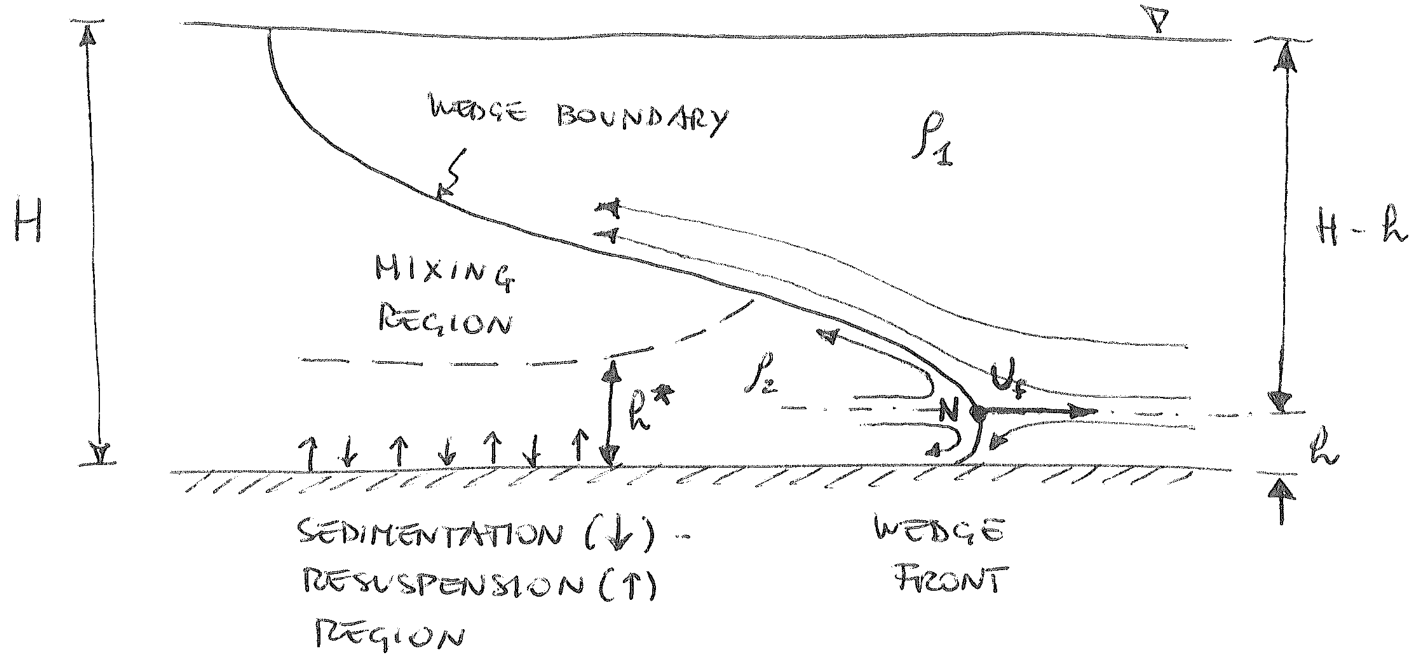
If the velocity is expressed in terms of the total height of the two wedges $H = 2h$, then one gets:

$$\boxed{U \cong 0{,}71 \sqrt{\frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}\, g H}}$$

MODEL PREDICTION

which is very close to the correlation derived from experimental data: $U \cong 0{,}67 \sqrt{\frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}\, g H}$!!

To conclude this section, let us examine the different features of a gravity current (or, more in general, of a current driven by a density difference):



The intrusion of the heavier fluid into the lighter fluid is characterized by a frontal region culminating into the so-called "nose" (point N in the schematic), which advances with velocity U_f. Upstream of the nose, two main regions can be found:

1. The MIXING REGION, where the two fluids can mix (and the density of the mixed suspension is intermediate between ρ_1 and ρ_2)

2. The SEDIMENTATION - RESUSPENSION REGION, where sediments can be deposited or re-entrained by turbulence.

Above the wedge boundary, the lighter fluid flows surrounding the current and producing a little lifting of the current's front close to the nose: as a consequence of such lifting, the lighter fluid can slip through the wedge boundary and enter the current, possibly introducing further sediments.

Below the wedge boundary, the flow is such that the heavier fluid moves against the direction of propagation of the current below the dot-dashed line (i.e. close to the bottom boundary). A similar type of counter-current motion occurs above the dot-dashed line, but only very close to the nose.

The current can be characterized by the following Reynolds number:

$$Re \triangleq \frac{U_f \cdot H}{\nu}$$

where H is the total height of the fluid layer.

For low Reynolds numbers ($Re < 10$) there is practically no mixing zone.

For high-enough Reynolds number ($Re > 10^3$), the mixing zone develops macroscopic features that remain unchanged when the Reynolds number is further increased. At these Reynolds numbers, the mixing zone is generated by the well-known <u>KELVIN - HELMHOLTZ INSTABILITY</u> originating near the current's front.

This instability arises whenever there is a strong-enough velocity shear (read velocity gradient) in a single fluid or a large-enough velocity difference between two fluids.

In the case of gravity currents, the KH instability generates a number of different flow structures, in particular vortical structures known as billows (cavalloni), lobes (lobi) and clefts (fenditure).

LATERAL VIEW