## Homework $\mathrm{N}^{o}$ 5: mass transfer equipments

a.

In an industrial plant an absorption column is used to wash vapors containing $H_{2} S$ coming from a combustion chamber. Sulphuric acid is absorbed by a liquid film flowing along the column walls. The mass transfer coefficient is given by the following expression:

$$
\begin{equation*}
\frac{k z}{D}=0.69\left(\frac{z V_{0}}{D}\right)^{0.5} \tag{1}
\end{equation*}
$$

where $z$ is the position along the column, $V_{0}$ is the average velocity of the film and $D$ is the diffusion coefficient. Considering that wall film has the following characteristics: thickness $\delta=0.07 \mathrm{~cm}, V_{0}=3 \mathrm{~cm} / \mathrm{s}$, that the diffusion coefficient of $H_{2} S$ in water is $D=1.8 \cdot 10^{-7} \mathrm{~cm}^{2} / \mathrm{s}$ and that the diameter of the column is $d=2 \mathrm{~m}$,

1. write a mass balance to describe the transfer of $\mathrm{H}_{2} \mathrm{~S}$ from the gas phase to the liquid phase;
2. determine the minimum height of the column to reach a concentration value equal to $0.1 \cdot C_{\text {sat }}$ in the liquid phase assuming that the concentration at the gas/film interface remains equal to $C_{\text {sat }}$ all along the column.
b.

A river flows at $u=0.5 \mathrm{~m} / \mathrm{s}$ and depth $h=2 \mathrm{~m}$ downstream of a polluted area where the concentration of dissolved oxygen $O_{2}$ has dropped to $C=2 \mathrm{mg} / \mathrm{l}$ causing a fish kill. Considering that the rate of oxygen exchange through the interface is given by :

$$
\begin{equation*}
K_{L}=\frac{3.9 \cdot 10^{-2} u^{0.5}}{h^{1.5}} \tag{2}
\end{equation*}
$$

and that the concentration of oxygen in air at atmospheric pressure at $25^{\circ} \mathrm{C}$ is equal to $C_{\text {sat }}=$ $8 \mathrm{mg} / \mathrm{l}$,

1. write a mass balance to describe the transfer of $O_{2}$ through the free surface of the water stream;
2. determine the distance downstream of the industrial area at which dissolved oxygen concentration rises back to acceptable values $(6 \mathrm{mg} / L)$ for fish survival.
$c$.
An absorption column is used to reduce the amount of $\mathrm{CO}_{2}$ in the fumes exiting from of a combustor. Flue gases move upward along the column in contact with a liquid film falling at the wall. Considering that the film flowrate (per unit width of the wall) is $\Gamma=0.05 \mathrm{~kg} / \mathrm{m} \mathrm{s}$, the column height is $L=5 \mathrm{~m}$, the diffusion coefficient of $\mathrm{CO}_{2}$ in the liquid is $D=1.96 \cdot 10^{-9} \mathrm{~m}^{2} / \mathrm{s}$, the density and the viscosity of the liquid are $\rho=1000 \mathrm{~kg} / \mathrm{m}^{3}$, and $\mu=10^{-3} \mathrm{~Pa} \cdot s$ and the film thickness is given by:

$$
\begin{equation*}
\delta=\left(\frac{3 \mu \Gamma}{\rho^{2} g}\right)^{1 / 3} \tag{3}
\end{equation*}
$$

1. calculate the value of the global mass transfer coefficient of the equipment, $K_{L}=\dot{m}_{i n t} / A_{\text {int }, L}(C(L)-$ $C(0))$ if the concentration of $\mathrm{CO}_{2}$ in the film at a distance $L$ is equal to $C(L)=0.4 C_{\text {sat }}$ (concentration at the interface $\left.C_{s a t}=0.0336 \mathrm{kmoli} / \mathrm{m}^{3}\right)$;
2. determine the relationship between the local mass transfer coefficient $k$ and the global mass transfer coefficient of the equipment, $K_{L}$. Which process parameters and geometrical characteristics of the column can be modified to change $K_{L}$ ?

## $d$.

In an industrial plant, a scrubber is used to wash vapours containing VOCs produced from a resin coating process. VOCs are absorbed by liquid droplets sprayed inside an absorption tower in which the gas flows upward while the drops fall by gravity. The gas flowrate to be treated is $Q=2 \mathrm{~m}^{3} / \mathrm{s}$, the concentration of VOC is $C_{\text {sat }}=$ $5 \mathrm{mg} / \mathrm{m}^{3}$, the section of the scrubber is $S=5 \mathrm{~m}^{2}$, the diffusion coefficient of VOCs into the liquid phase is equal to $D=2 . \cdot 10^{-5} \mathrm{~cm}^{2} / \mathrm{s}$ and the viscosity of gas is $\mu=$ $1.8 \cdot 10^{-5} \mathrm{~Pa} \cdot \mathrm{~s}$. Assuming that the mass transfer of VOC between from the gas to the liquid can be written as:

$$
\begin{equation*}
J(z)=D \frac{C_{s a t}-C(z)}{D_{p} / 2} \tag{4}
\end{equation*}
$$

where $C(z)$ in the VOC concentration in the drop at a generic position along the column and $D_{p}$ is the drop diameter,

1. write a mass balance to describe the transfer of VOC from the gas to the liquid phase;
2. calculate the size of the drop which allows to reach VOC concentration up to $0.8 \cdot C_{\text {sat }}$ in the liquid phase when the drop reaches the base of the tower ( $L=5 \mathrm{~m}$ ).
$e$.
To contain odor emissions from a wastewater treatment plant, a bubble column is used to feed oxygen. Column diameter and height are $D=0.5 \mathrm{~m}$ and $L=2 \mathrm{~m}$. The liquid to be oxygenated is fed from the top of the column ( $Q=3 l / s$ ) while oxygen is injected from below as small diameter bubble. The oxygen contained in the bubbles partially diffuse into the liquid while the bubbles rise along the column.
3. Write a mass balance to describe the transfer of $\mathrm{O}_{2}$ from the gas to the liquid to be oxygenated; assume that oxygen concentration in the bubbles remains equal to $C_{s a t}$.
4. Considering a mass transfer coefficient equal to $K=1.6 \cdot 10^{-3} \mathrm{~cm} / \mathrm{s}$ and a specific (i.e. per unit volume of column) gas/liquid surface area equal to $a=40 \mathrm{~m}^{2} / \mathrm{m}^{3}$, calculate the rate of oxygenation produced $\left(C_{O_{2}}(z=L)\right.$.
5. Considering that $a$ is a function of oxygen flow rate and size of bubbles, discuss which are the best conditions for bubble injection to maximize oxygen transfer to the liquid.
$f$.
In a wastewater treatment plant, a biofilter is used to abate odor emissions due the presence of sulfur compounds in the gas extracted from a sludge digestor. The biofilter is $L=15 \mathrm{~m}$ long, $W=20 \mathrm{~m}$ wide, and $H=1 \mathrm{~m}$ high: the gas is fed through the bottom section of the biofilter, flowing countecurrently to a liquid film, which falls along the solid substrate. Sulfur compounds are absorbed into the liquid phase being completely degraded $\left(C_{l i q}=0\right)$ by the biomass adhering to the solid support. Considering that the specific surface area of the film flowing along the solid packing is $a=50 \mathrm{~m}^{2} / \mathrm{m}^{3}$, the flow of gas to be treated is $\dot{m}=2.4 \mathrm{~kg} / \mathrm{s}$, the gas density is $1.2 \mathrm{~kg} / \mathrm{m}^{3}$ and the mass transfer coefficient of sulfur compounds from the gas to the liquid phase is equal to $K=4 \cdot 10^{-3} m / h$ :
6. Write a mass balance to describe the transfer of sulfur compounds from the gas to the liquid and calculate the removal efficiency of the biofilter;
7. Calculate the surface area of the biofilter required to achieve a removal efficiency equal to $90 \%$.
$g$.
In an industrial plant, water is drawn from a reservoir to feed the top of an absorption column (height $H=20 \mathrm{~m}$, diameter $D_{\text {col }}=2 \mathrm{~m}$ ) to treat a gas stream containing vapors of $H_{2} S$.
8. Calculate the pumping power necessary to circulate the washing liquid (design flow rate $Q=12 \mathrm{l} / \mathrm{s}$ ) along a vertical pipe (diameter $D_{\text {pipe }}=0.1 \mathrm{~m}$ ) extending from the bottom to the top of the absorbing tower.
9. Under continuous operation, the formation of limestone aggregates ( $D_{p}=2 \mathrm{~mm}, \rho_{p}=1500 \mathrm{~kg} / \mathrm{m}^{3}$ ) has been observed along the pipe. Aggegates should be separated from the washing liquid before it is distributed inside the absorption column to avoid accumulation of solids inside the distribution nozzles. The plant manager proposes to use the vertical section of pipe as a "settling" section. Determine the maximum velocity of the fluid inside the vertical pipe to avoid accumulation of limestone particles inside the nozzles.
10. Write a mass balance to describe the transfer of $H_{2} S$ from the gas to the washing liquid. Assume
that the gas concentration at the interface of the film remains equal to $C_{\text {sat }}$ and that the coefficient of mass transfer $K$ is known.
11. Evaluate the impact, in terms of mass of $\mathrm{H}_{2} \mathrm{~S}$ absorbed in the liquid phase, of operating the column at a reduced flow rate of washing liquid. The film thickness at the wall of the column is given by:

$$
\begin{equation*}
\delta=\left(\frac{3 \mu \Gamma}{\rho^{2} g}\right)^{1 / 3} \tag{5}
\end{equation*}
$$

and the local mass transfer coefficient is given by:

$$
\begin{equation*}
K(x)=\left(\frac{6 D \Gamma}{\pi \rho \delta x}\right)^{0.5} \tag{6}
\end{equation*}
$$

where $\Gamma$ is the mass flow rate per unit of column perimeter, $D=1.8 \cdot 10^{-7} \mathrm{~cm}^{2} / \mathrm{s}$ is the diffusion coefficient of $H_{2} S$ in water and $x$ is the position along the column.

## $h$.

VOC contained in the flue gases exhausted from an industrial plant should be removed from the gas stream before it can be discharged into the atmosphere through a stack. An absorption column is used in which VOC contained in the flue gases feeded at the bottom are absorbed into a liquid film falling on structured packing of characteristic size $d$. The mass transfer coefficient is given by:

$$
\begin{equation*}
\frac{k d}{D}=25\left(\frac{d}{V_{O} \nu}\right)^{0.45} \cdot\left(\frac{n u}{D}\right)^{0.5} \tag{7}
\end{equation*}
$$

where $D$ is the diffusion coefficient and $V_{O}$ is the superficial velocity of the liquid inside the column. Assume the following operating conditions: the flow rate of gas to be treated is $Q=4 \mathrm{~m}^{3} / \mathrm{s}$, the concentration of VOCs in the gas phase is $C_{\text {sat }}=10 \mathrm{mg} / \mathrm{m}^{3}$, the cross section of the scrubber is $S=6 \mathrm{~m}^{2}$, the column is $L=5 \mathrm{~m}$ long, the diffusion coefficient of VOC in the liquid phase is equal to $D=2 \cdot \cdot 10^{-12} m^{2} / s$ and the flow rate of the washing liquid is $Q=0.075 \mathrm{~m}^{3} / \mathrm{s}$.

1. write the mass balance on the liquid phase to describe the transfer of VOC from the gas to the liquid phase;
2. valuate which size of the structured packing proposed allows to obtain a concentration of VOC in the liquid phase at the base of the column at least equal to $0.75 \cdot C_{s a t}$.

| Packing type | $\mathrm{d}[\mathrm{mm}]$ | $a\left[\mathrm{~m}^{2} / \mathrm{m}^{3}\right]$ |
| :--- | :---: | :---: |
| Berl Saddles | 50 | 105 |
|  | 38 | 150 |
|  | 25 | 250 |
|  | 13 | 465 |

Table 1: Characteristic size of packings proposed for installation in the absorbing tower.
$i$.
At a livestock farm, a stripping column is used to treat Nitrogen rich wastewaters generated by the food processing industry. Inside a sealed tank (diameter $D=2 m$, height $H=2 m$ ), the clarified effluent ( $\rho=1000 \mathrm{~kg} / \mathrm{m}^{3}$, $\mu=10^{-3} \mathrm{~Pa} \cdot \mathrm{~s}$ ) rich in ammonia comes into contact with hot air bubbles ( $\rho_{b}=1.2 \mathrm{~kg} / \mathrm{m}^{3}$ ) bubbled from the bottom of the tank through appropriate nozzles to promote the mass transfer of ammonia from the liquid phase to the gas, reducing the ammonia content of wastewaters.

1. Write a mass balance to describe the mass transfer of $\mathrm{NH}_{3}$ from the liquid to the gas assuming that the concentration of $\mathrm{NH}_{3}$ at the liquid/gas interface does not change significantly inside the tank $\left(C_{i n t}=4 \mathrm{mg} / \mathrm{m}^{3}\right)$;
2. Assuming that the mass transfer coefficient through the gas/liquid interface is $K=1 \cdot 10^{-4} \mathrm{~m} / \mathrm{s}$, calculate the changes in ammonia concentration in the gas while the bubbles rise through the wastewater as a function diameter of inflated bubbles.
3. Calculate the concentration of ammonia in the gas exititing from the tank if blown air bubbles have a diameter equal to $d_{b}=1 \mathrm{~mm}$.
$j$.
Flue gases exiting from a plant producing wood panels ( $Q=3.5 \mathrm{Nm}^{3} / \mathrm{s}$, normal contition $0^{\circ} \mathrm{C}$ and $1 \mathrm{~atm}, M=$ $29 \mathrm{~kg} / \mathrm{kmol}$ ) contain, in addition to particulate matter, volatile organic compounds (formaldehyde) formed from the resins used for the production of panels and must be washed before being expelled into the atmosphere. Flue gases are fed to the bottom of a structured packing absorbing column in which water is supplied from the top to generate a thin liquid film all above the packing.
4. Write a mass balance on the gas phase to describe the process of VOC transfer from the gas to the liquid phase;
5. assuming that the column height is $H=30 \mathrm{~m}$ and diameter is $D=3 \mathrm{~m}$, that the liquid flow rate is large enough to neglect any variation of VOC concentration in the liquid phase, that the mass transfer coefficient is $k=7 \cdot 10^{-6} \mathrm{~m} / \mathrm{s}$, the temperature of the gas in the column is $50^{\circ} \mathrm{C}$ and that the packing is characterized by a specific surface area $a=5000 \mathrm{~m}^{2} / \mathrm{m}^{3}$, calculate the amount of VOC removed from the gas if the concentration of VOC at the input is $C_{i n}=10 \mu \mathrm{~g} / \mathrm{m}^{3}$;
6. New regulations require more and more stringent limits for the concentration of VOC in the effluent gas. The consultant proposes to revamp the absorption column replacing the existing packing material with second-generation packings ( $a^{\prime}=$ $10000 \mathrm{~m}^{2} / \mathrm{m}^{3}$ ). Check if the proposed solution would keep the outlet concentration below the new limit prescribed by law $\left(0.5 \mu \mathrm{~g} / \mathrm{m}^{3}\right)$.
$k$.
In an lab, leaching tests are carried out to evaluate the possible toxicity of solid materials (sands) to be used as fillers in cement paste. The pulverized material is introduced and compacted in a test column $(D=10 \mathrm{~cm}$, $H=50 \mathrm{~cm}$ ) and sprayed from the top with distilled water which is extracted from the bottom of the column and recirculated to the top of the system for 24 hours. At the end of the test period, the concentration of compounds eventually extracted from the solid is measured in the liquid eluate.
7. Write a mass balance on the liquid phase to describe the mass tranfer of the toxic compounds from the solid to the liquid phase (concentration at the solid/liquid interface equal to $\left.C_{i n t}=5 \mathrm{mg} / L\right)$;
8. Evaluate the rate of release (constant mass transfer, $k$ ) of a species whose concentration in the eluate at end of the test period is $5 \mu \mathrm{~g} / L$ (flow rate of washing liquid equal to $Q=0.1 L / s$, specific area available for mass transfer $a=100 \mathrm{~m}^{2} / \mathrm{m}^{3}$ ). Hint: consider the recirculating flow in the segment of the column equivalent to a non-recirculating flow in a longer column.

## $l$.

In an urban wastewater treatment plant the plant manager wants to recover the ammonia present in the effluent liquid in the gas phase bubbling nitrogen bubbles of small diameter through the liquid $\left(\mathrm{NH}_{3}\right.$ stripping by $\mathrm{N}_{2}$ ). The liquid effluent ( $\rho_{L}=1000 \mathrm{~kg} / \mathrm{m}^{3}, \mu=1 \cdot 10^{-3} \mathrm{~Pa} \cdot \mathrm{~s}$ ) is placed in a tank filled up to $H=2.5 \mathrm{~m}$. At the bottom of the tank, nitrogen is bubbled ( $\rho_{G}=1.2 \mathrm{~kg} / \mathrm{m}^{3}$, $\left.C_{N H_{3}}(0)=0\right)$. Nitrogen is exhausted from the dead volume above the liquid effluent using a fan.

1. Write a mass balance on the nitrogen bubble rising through the liquid effluent to describe the mass transfer of ammonia from the liquid to the gas phase;
2. Assuming that the variation of concentration of $\mathrm{NH}_{3}$ in the liquid is negligible $\left(C_{\mathrm{NH}_{3}, \text { int }}=\right.$ const), calculate the diameter of the bubbles to rise the concentration of ammonia in the gas to $0.6 C_{N H_{3}, \text { int }}$ (consider Stokes regime for the gas bubble and mass transfer rate $k=2 \cdot 10^{-8} \mathrm{~m} / \mathrm{s}$ ).
