## Homework $\mathrm{N}^{o} 3$ : particle separation systems

$a$.
Ash particles (density $\rho_{p}=800 \mathrm{~kg} / \mathrm{m}^{3}$, diameter $D_{p}=$ $100 \mu \mathrm{~m}$ ) must be separated by a gas stream (flow rate $Q=1 \mathrm{~m}^{3} / \mathrm{s}$, density $\rho=1.4 \mathrm{~kg} / \mathrm{m}^{3}$, viscosity $\mu=1.8$. $\left.10^{-5} \mathrm{~Pa} \cdot \mathrm{~s}\right)$.

1. Calculate the length of a settling chamber (maximum width $L=2 \mathrm{~m}$ ) to reach $100 \%$ separation efficiency (assume laminar flow inside the settling chamber);
2. Calculate the length of the settling chamber if turbulent flow in the chamber is considered and target separation efficiency should be $95 \%$.
$b$.
Size a settling chamber to separate coal particles (density $\rho_{p}=2600 \mathrm{~kg} / \mathrm{m}^{3}$ ) from an air stream ( $\rho=$ $\left.1.4 \mathrm{~kg} / \mathrm{m}^{3}, \mu=1.8 \cdot 10^{-5} \mathrm{~Pa} \cdot \mathrm{~s}\right)$. The air flow rate is $Q=4 \mathrm{~m}^{3} / \mathrm{s}$ and dimensions of the settling chamber are $W=2 m, H=2 m$ and $L=15 \mathrm{~m}$.
3. Check if the flow is turbulent.
4. Considering the sizing equations valid for turbulent flow, calculate the number of deposition trays iwhich should be installed inside the settling chamber to separate with at least $90 \%$ efficiency particles having diameter equal to $D_{p}=10 \mu \mathrm{~m}$.
5. Calculate the collection efficiency for the chosen number of trays.
6. Calculate the overall collection efficiency of the settling chamber if the particle size ditribution is given by:

| $D_{p},[\mu m]$ | 1 | 5 | 10 | 15 | 20 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\%$ | 5 | 10 | 50 | 20 | 15 |

$c$.
In an industrial plant, cooling water is drawn from the river (density $\rho$, viscosity $\mu$ ). Sand particles (diameter $D_{p}$ and density $\rho_{p}$ ) which are suspended in the water at the drawing point should be separated before entering in the cooling tower. An elutriator, i.e. a segment of vertical pipe in which the flow carrying the particles moves upward, is used to separate the suspended solids. Assuming that the sand particles move in the Stokes regime $\left(C_{D}=24 / R e_{p}, R e_{p}=\rho D_{p}\left\|v_{p}-v\right\| / \mu\right):$

1. calculate the maximum value of water bulk velocity (moving upward inside the pipe) to allow particles deposition;
2. calculate the minimum vertical length of the pipe to separate the solids when water velocity is $80 \%$ of the maximum value which allows particle separation.
$d$.
A consulting firm suggests to purchase two batteries composed of 450 Starirmand type cyclones of diameter $D_{c}=0.25 \mathrm{~m}$ as the optimal solution for the purification of a particle laden flow of air $\left(\mu=2.48 \cdot 10^{-5} \mathrm{~Pa} \cdot s\right.$, $M M=29 \mathrm{~kg} / \mathrm{m}^{3}$ ). Flow rate characteristics are the following: flow rate $Q=165 \mathrm{~m}^{3} / \mathrm{s}$, temperature $T=450 \mathrm{~K}$ and pressure $p=1 \mathrm{~atm}$, laden by particles with diameter $D_{p}=10 \mu \mathrm{~m}$ and density $\rho_{p}=1600 \mathrm{~kg} / \mathrm{m}^{3}$.
3. Using the equations for the practical design of multiple cyclones (p. 351 Process Engineering and design for air pollution control) calculate the separation efficiency and pressure losses for the proposed configuration.
4. Size a single Stairmand cyclone able to separate the particles with the same separation efficiency and compute the pressure losses in this case.
5. Calculate the cost associated with the two design alternatives (multicyclone and single cyclone) considering investment costs (p.353) and annual operating costs (cost of power for moving the flow through the separation system) if (1) the expected useful life for the plant is 10 years, (2) the device is expected to operate $8000 \mathrm{~h} /$ year, (3) the air compressor efficiency is equal to 0.65 and (4) the operating cost (electric power) is equal to $C_{E}=0.08 \$ / k W h$.
6. Calculate what should be the optimal number of cyclones for minimize the total cost of the system (hint: write the expression of the annual cost as a function of the number of cyclones, use the constraint on the efficiency to derive a relationship between number of cyclones and cyclone diameter, find
the minimum of the total cost imposing the cost derivative equal to zero).

## $e$.

A power plant producing electricity from coal burning must install an electrostatic precipitator to treat the flue gases emitted from the stack in order to comply with the emissions limits fixed by law. The analysis of a sample of powder taken from the flue gases indicates that particle density and size distribution are the following:

| Particle size distribution | $D_{p}[\mu \mathrm{~m}]$ | $\%$ by weight | $q_{p}[C]$ |
| :--- | :--- | :--- | :--- |
|  | 0.5 | 20 | $1.6 \cdot 10^{-15}$ |
|  | 1.0 | 50 | $6.4 \cdot 10^{-15}$ |
|  | 2.0 | 20 | $25.6 \cdot 10^{-15}$ |
|  | 5.0 | 10 | $160 \cdot 10^{-15}$ |
| density $\rho_{p}\left[\mathrm{~kg} / \mathrm{m}^{3}\right]$ |  | 800 |  |

Assuming that (i) the particles enter the precipitator already charged (as indicated by $q_{p}$ in the Table) (ii) the flow rate to be processed is $4 \mathrm{~m}^{3} / \mathrm{s}$ (viscosity $\mu=$ $1.8 \cdot 10^{-5} \mathrm{~Pa} \cdot s, \rho=1.2 \mathrm{~kg} / \mathrm{m}^{3}$ ), (iii) the voltage available in the plant is $5 \cdot 10^{4} V$, (iv) the maximum dimensions allowed for the precipitator is $V o l=16 \mathrm{~m}^{3}$, and (v) the collection efficiency should be $99 \%$ for $1 \mu m$ particles,

1. size a plate-plate precipitator in order to reach the target collection efficiency;
2. calculate the overall collection efficiency for the dust sample.
f.

Size a two-stage separation system (Swift cyclone Swift + ESP) to separate dust ( $\rho_{p}=2000 \mathrm{~kg} / \mathrm{m}^{3}$ ) with particle size distribution given by:

| Particle size distribution | $D_{p}[\mu \mathrm{~m}]$ | Weight [mg] |
| :--- | :--- | :--- |
|  | 0.5 | 25 |
|  | 1 | 125 |
|  | 5 | 100 |
|  | 10 | 75 |
|  | 20 | 30 |
|  | 50 | 5 |

from an air flow (flow rate $Q=10 \mathrm{~m}^{3} / \mathrm{s}, T=298 \mathrm{~K}$, $p=1 \mathrm{~atm})$.

1. Use the practical design equations to size the Swift cyclone considering that the maximum power available in the plant to handle the flow of air is equal to 15 kW .
2. Calculate the overall collection efficiency of the cyclone.
3. Size a plate-plate electrostatic precipitator ( $L=$ $5 W, H=W$ ) in order to obtain $99.9 \%$ total collection efficiency for $1 \mu \mathrm{~m}$ particles. Consider that the voltage available in the plant is $V=2 \cdot 10^{5} \mathrm{~V}$ and assume that the particles acquire a charge per unit of surface area equal to $q=9.5 \cdot 10^{-5} \mathrm{C} / \mathrm{m}^{2}$.
4. Calculate the overall collection efficiency of the ESP.
$g$.
A Stairmand type cyclone is used to separate wood particles from a gas stream. Particle characteristics are the following: density $\rho_{p}=300 \mathrm{~kg} / \mathrm{m}^{3}$, diameter $D_{p}=$ $100 \mu \mathrm{~m}$; gas flow rate is $\dot{m}=150000 \mathrm{~kg} / \mathrm{h}$, density is $\rho=0.9 \mathrm{~kg} / \mathrm{m}^{3}$, viscosity is $\mu=1.8 \cdot 10^{-5} \mathrm{~Pa} \cdot \mathrm{~s}$, and temperature is $T=20^{\circ} \mathrm{C}$.
5. Using the equations for practical design of cyclones, calculate the cyclone diameter to collect particles larger than $100 \mu \mathrm{~m}$ with an efficiency at least equal to $99 \%$ (assume $m=0.75$ );
6. calculate the power of the compressor feeding the gas stream to the cyclone;
7. calculate the overall separation efficiency if the particle size distribution of dust fed to the cyclone is given by:

| $D_{p}[\mu m]$ | 1 | 5 | 10 | 50 | 100 | 150 |
| :--- | ---: | :---: | :---: | :---: | :---: | :---: |
| $\%$ | 5 | 10 | 20 | 30 | 20 | 15 |

$h$.
Size a two stage separation system to sample the fraction of fine and ultrafine particles ( $P M 10$ and $P M 2.5$, $\rho_{p}=800 \mathrm{~kg} / \mathrm{m}^{3}$ ) from a flow of air $\left(\rho=1.2 \mathrm{~kg} / \mathrm{m}^{3}\right.$, $\mu=1.8 \cdot 10^{-5} \mathrm{~Pa} \cdot s$ ) exiting from a biomass burner. The designer proposes to use a system composed of two Lapple type cyclones in series, suitably sized to separate 50 \% particles having a diameter greater than or equal to the diameter of interest ( $10 \mu \mathrm{~m}$ and $2.5 \mu \mathrm{~m}$ ).

1. Using the practical design equation for cyclones, size the first and the second cyclone of the separation chain (assume $m=0.38$ );
2. Which of the two cyclones should be the first of the sampling chain?
3. Calculate the total pressure drop of the separation system.
$i$.
A company producing oil-water separation systems has to design a system able to separate sand particles oand oil droplets carried by rainwater into the treatment system ( $Q=3 \mathrm{l} / \mathrm{s}$ ). The treatment system consists of tanks (length $L=3 \mathrm{~m}$, width $W=1 \mathrm{~m}$ and $H=1.5 \mathrm{~m}$ deep) which can be put in series and in parallel to treat the effluent.
4. Calculate how many tanks should be installed in parallel to separate with $90 \%$ efficiency oil droplets ( $\rho_{\text {oil }}=800 \mathrm{~kg} / \mathrm{m}^{3}$ ) of diameter $100 \mu \mathrm{~m}$;
5. Calculate the separation efficiency of the system if the size distribution of sands and droplets entering into the system is that reported in Table.

| Sand  <br> $\rho_{s}=2000 \mathrm{~kg} / \mathrm{m}^{3}$  |  |  | Oil <br> $\rho_{o}=800 \mathrm{~kg} / \mathrm{m}^{3}$ |  |
| :--- | :--- | :--- | :--- | :--- |
| $D_{p}$ | $m_{p}$ | $D_{o}$ |  | $m_{o}$ |
| $[\mu \mathrm{~m}]$ | $[\mathrm{mg}]$ |  | $[\mu \mathrm{m}]$ | $[\mathrm{mg}]$ |
| 50 | 325 | 50 | 0.50 |  |
| 100 | 450 | 100 | 1.0 |  |
| 150 | 225 | 200 | 0.20 |  |

$j$.
Flue gas exiting from an industrial burner ( $Q=$ $2 \mathrm{~m}^{3} / \mathrm{s}, M M=29 \mathrm{~kg} / \mathrm{kmol}, P=1 \mathrm{~atm}, T=100^{\circ} \mathrm{C}$, $\mu=1.8 \cdot 10^{-5} \mathrm{~Pa} \cdot s$ ) contains ash particles ( $\rho_{p}=$ $\left.500 \mathrm{~kg} / \mathrm{m}^{3}\right)$. The operating licence of the plant indicates that a separation system should be installed to collect particles having diameter $D_{p}=100 \mu m$ with $98 \%$ separation efficiency. Two projects have been submitted for review: the first involves the construction of a settling chamber ( $L=5 \mathrm{~m}, H=2 \mathrm{~m}, W=2 \mathrm{~m}$ ) and a Swift type cyclone; the second involves the installation of a multiple cyclone system made of 20 cyclones (Swift type).

1. Calculate the collection efficiency of the settling chamber and the efficiency of the cyclone (design alternative 1) ; item using the practical design equations for cyclones, determine the cyclone diameter and the pressure drop for alternative 1 (assume $m=0.7$ );
2. using the prectical design equations for multiple cyclones determine the diameter of the 20 cyclones to be installed and the pressure drop for alternative 2 (assume $m=0.59$ );
3. discuss which of the two design alternatives might be the most effective.
$k$.
A plant producing wood panels is equipped with a boiler fueled by wood waste powders to produce the steam necessary to dry the wood fibers used in the process. Before releasing the flue gases in the atmosphere, the high dust content should be reduced. Characteristics of flue gases are the following: $Q=3.5 \mathrm{Nm} / \mathrm{s}$ (normal conditions $0^{\circ} C$ and 1 atm ), $M=29 \mathrm{~kg} / \mathrm{kmol}$, $\mu=1.8 \cdot 10^{-5} \mathrm{~Pa} \cdot \mathrm{~s}$. Flue gases exit from the boiler at $T=$ $100^{\circ} \mathrm{C}$ and particle concentration $C_{1}=200 \mathrm{mg} / \mathrm{Nm}^{3}$ which should be reduced up to $C_{2}=10 \mathrm{mg} / \mathrm{Nm}^{2}$ before emission from the chimney using any abatement system properly designed. The particle size distribution measured on a powder sample indicates that the average diameter of particles is $20 \mu \mathrm{~m}$. The plant designer proposes to install a two stage separation system including a plate-plate electrostatic precipitator and a fabric filter, in series. Considering that voltage $\Delta V$ available in the plant is 60 kV and the charge acquired by the particles is $q=5 \cdot 10^{-15} C$
4. calculate the collection efficiency of the ESP ( $W=$ $2 m, H=2 m$ and $L=5 \mathrm{~m}$ ) made of 5 plates ( 2 high-voltage and 3 grounded plates);
5. calculate which should be the collection efficiency of the fabric filter to meet the target value of overall collection.
$l$.
Size a fabric filter to reduce the concentration of dust in the floe exiting from a low power biomass combustor ( $Q=5 \mathrm{Nm}^{3} / \mathrm{s}$, normal conditions $0^{\circ} \mathrm{C}$ and $1 \mathrm{~atm}, M=$ $\left.29 \mathrm{~kg} / \mathrm{kmol}, \mu=1.8 \cdot 10^{-5} \mathrm{~Pa} \cdot \mathrm{~s}\right)$. The concentration of powders downstream of the combustion chamber is $500 \mathrm{mg} / \mathrm{Nm}^{3}$ while the emission limit for the plant is $30 \mathrm{mg} / \mathrm{Nm}^{3}$. Considering that the temperature of the gas at the combustor outlet is $250^{\circ} \mathrm{C}$,
6. identify a suitable fabric material for the bag filter, the filtration velocity and the total filtering area (assume reverse airflow filter cleaning system);
7. calculate the cycle time of the filter (i.e. how often the filter should be cleaned) if the resistance of clean filter is $S_{E}=22 \mathrm{kPa} \cdot \mathrm{s} / \mathrm{m}$ and the dust layer resistance is $K_{2}=1.10 \cdot 10^{5} \mathrm{~s}^{-1}$ and the maximum value of allowable pressure drop is $\Delta p_{\max }=3 \mathrm{kPa}$;
8. The particle size distribution of the dust emitted from the combustor indicates that $60 \%$ by mass of the particles has a diameter $\geq 25 \mu m$. Evaluate the increase expected in the cycle time of the system if a cyclone pre-separator is installed before the bag filter to remove the fraction of particles with $D_{p} \geq 25 \mu m$. Size a single cyclone or a multicyclone system to achieve $95 \%$ collection efficiency for $D_{p} \geq 25 \mu m$ particles and calculate the new value of the cycle time of the filter.
