SECTION III

HEAT TRANSFER AND ITS APPLICATIONS

Practically all the operations that are carried out by the chemical engineer involve the production or absorption of energy in the form of heat. The laws governing the transfer of heat and the types of apparatus that have for their main object the control of heat flow are therefore of great importance. This section of the book deals with heat transfer and its applications in process engineering.

NATURE OF HEAT FLOW. When two objects at different temperatures are brought into thermal contact, heat flows from the object at the higher temperature to that at the lower temperature. The net flow is always in the direction of the temperature decrease. The mechanisms by which the heat may flow are three: conduction, convection, and radiation.

Conduction. If a temperature gradient exists in a continuous substance, heat can flow unaccompanied by any observable motion of matter. Heat flow of this kind is called *conduction.* In metallic solids, thermal conduction results from the motion of unbound electrons, and there is close correspondence between thermal conductivity and electrical conductivity. In solids that are poor conductors of electricity and in most liquids, thermal conduction results from the transport of momentum of individual molecules along the temperature gradient. In gases conduction occurs by the random motion of molecules, so that heat is "diffused" from hotter regions to colder ones. The most common example of conduction is heat flow in opaque solids, as in the brick wall of a furnace or the metal wall of a tube.

Convection. When a current or macroscopic particle of fluid crosses a specific surface, such as the boundary of a control volume, it carries with it a definite quantity of enthalpy. Such a flow of enthalpy is called a *convective flow of heat* or simply *convection.* Since convection is a macroscopic phenomenon, it can occur only when forces act on the particle or stream of fluid and maintain its motion against the forces of friction. Convection is closely associated with fluid mechanics. In fact, thermodynamically, convection is not considered as heat flow but as flux of enthalpy. The identification of convection with heat flow is a matter of convenience, because in practice it is difficult to separate convection from true conduction when both are lumped together under the name convection. Examples of convection are the transfer of enthalpy by the eddies of turbulent flow and by the current of warm air from a household furnace flowing across a room.

Natural and forced convection. The forces used to create convection currents in fluids are of two types. If the currents are the result of buoyancy forces generated by differences in density and the differences in density are in turn caused by temperature gradients in the fluid mass, the action is called *natural convection.* The flow of air across a heated radiator is an example of natural convection. If the currents are set in motion by the action of a mechanical device such as a pump or agitator, the flow is independent of density gradients and is called *forced convection.* Heat flow to a fluid pumped through a heated pipe is an example of forced convection. The two kinds of force may be active simultaneously in the same fluid, and natural and forced convection then occur together.

Radiation. Radiation is a term given to the transfer of energy through space by electromagnetic waves. If radiation is passing through empty space, it is not transformed into heat or any other form of energy nor is it diverted from its path. If, however, matter appears in its path, the radiation will be transmitted, reflected, or absorbed. It is only the absorbed energy that appears as heat, and this transformation is quantitative. For example, fused quartz transmits practically all the radiation that strikes it; a polished opaque surface or mirror will reflect most of the radiation impinging on it; a black or matte surface will absorb most of the radiation received by it and will transform such absorbed energy quantitatively into heat.

Monatomic and most diatomic gases are transparent to thermal radiation, and it is quite common to find that heat is flowing through masses of such gases both by radiation and by conduction-convection. Examples are the loss of heat from a radiator or uninsulated steam pipe to the air of a room and heat transfer in furnaces and other high-temperature gas-heating equipment. The two mechanisms are mutually independent and occur in parallel, so that one type of heat flow can be controlled or varied independently of the other. Conduction-convection and radiation can be studied separately and their separate effects added together in cases where both are important. In very general terms, radiation becomes important at high temperatures and is independent of the circumstances of the flow of the fluid. Conduction-convection is sensitive to flow conditions and is relatively unaffected by temperature level.

Chapter 10 deals with conduction in solids, Chaps. 11 to 13 with heat transfer to fluids by conduction and convection, and Chap. 14 with heat transfer by radiation. In Chaps. 15 and 16 the principles developed in the preceding chapters are applied to the design of equipment for heating, cooling, condensing, and evaporating.

CHAPTER 10

HEAT TRANSFER BY CONDUCTION

Conduction is most easily understood by considering heat flow in homogeneous isotropic solids because in these there is no convection and the effect of radiation is negligible unless the solid is translucent to electromagnetic waves. First, the general law of conduction is discussed; second, situations of steady-state heat conduction, where the temperature distribution within the solid does not change with time, are treated; third, some simple cases of unsteady conduction, where the temperature distribution does change with time, are considered.

FOURIER'S LAW. The basic relation of heat flow by conduction is the proportionality between the rate of heat flow across an isothermal surface and the temperature gradient at the surface. This generalization, which applies at any location in a body and at any time, is called *Fourier's law*.[3] It can be written

$$\frac{dq}{dA} = -k\frac{\partial T}{\partial n} \tag{10.1}$$

where A = area of isothermal surface
n = distance measured normally to surface
q = rate of heat flow across surface in direction normal to surface
T = temperature
k = proportionality constant

The partial derivative in Eq. (10.1) calls attention to the fact that the temperature may vary with both location and time. The negative sign reflects the physical fact that heat flow occurs from hot to cold and the sign of the gradient is opposite that of the heat flow.

In using Eq. (10.1) it must be clearly understood that the area A is that of a surface perpendicular to the flow of heat and distance n is the length of path measured perpendicularly to area A.

Although Eq. (10.1) applies specifically across an isothermal surface, the same equation can be used for heat flow across any surface, not necessarily isothermal, provided the area A is the area of the surface and the length of the path is measured normally to the surface.[2a] This extension of Fourier's law is vital in the study of two- or three-dimensional flows, where heat flows along curves instead of straight lines. In one-dimensional flow, which is the only situation considered in this text, the normals representing the direction of heat flow are straight. One-dimensional heat flow is analogous to one-dimensional fluid flow, and only one linear coordinate is necessary to measure the length of the path.

An example of one-dimensional heat flow is shown in Fig. 10.1, which represents a flat water-cooled furnace wall. Initially the wall is all at 25°C, in equilibrium with cooling water at the same temperature. The temperature distribution in the wall is represented by line I. At temperature equilibrium, T is independent of both time and position. Assume now that one side of the wall is suddenly exposed to furnace gas at 700°C. Compared with the thermal resistance of the wall, the resistances to heat flow between the hot gas and the wall and between the wall and the cooling water may be considered negligible. The

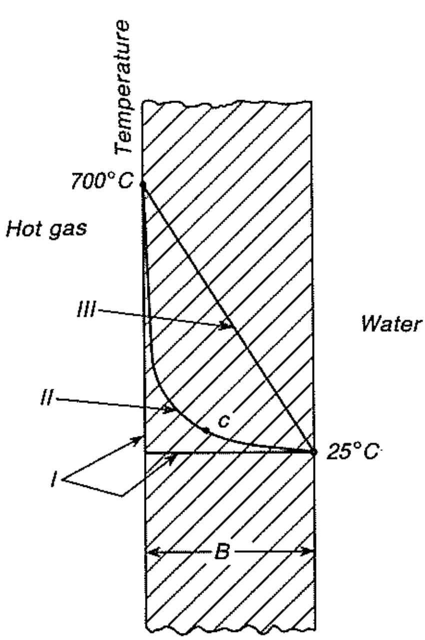

FIGURE 10.1
Temperature distributions, unsteady-state heating of furnace wall: I, at instant of exposure of wall to high temperature; II, during heating at time t; III, at steady state.

temperature at the gas side of the wall immediately rises to 700°C; that at the other side remains at 25°C. Heat flow begins, and after the elapse of some time, the temperature distribution can be represented by a curve like that of curve II. The temperature at a given distance, e.g., that at point c, is increasing; and T depends upon both time and location. The process is called *unsteady-state conduction*, and Eq. (10.1) applies at each point at each time in the slab. Finally, if the wall is kept in contact with hot gas and cool air for a sufficiently long time, the temperature distribution shown by line III will be obtained, and this distribution will remain unchanged with further elapse of time. Conduction under the condition of constant temperature distribution is called *steady-state conduction*. In the steady state T is a function of position only, and the rate of heat flow at any one point is a constant. For steady one-dimensional flow, Eq. (10.1) may be written

$$\frac{q}{A} = -k\frac{dT}{dn} \tag{10.2}$$

THERMAL CONDUCTIVITY. The proportionality constant k is a physical property of the substance called the *thermal conductivity*. It, like the newtonian viscosity μ, is one of the so-called transport properties of the material. This terminology is based on the analogy between Eqs. (3.4) and (10.2). In Eq. (3.4) the quantity τg_c is a rate of momentum flow per unit area, the quantity du/dy is the velocity gradient, and μ is the required proportionality factor. In Eq. (10.2), q/A is the rate of heat flow per unit area, dT/dn is the temperature gradient, and k is the proportionality factor. The minus sign is omitted in Eq. (3.4) because of convention in choosing the direction of the force vector.

In engineering units, q is measured in watts or Btu/h and dT/dn in °C/m or °F/ft. Then the units of k are W/m-°C or Btu/ft^2-h-(°F/ft), which may be written Btu-ft-h-°F.

Fourier's law states that k is independent of the temperature gradient but not necessarily of temperature itself. Experiment does confirm the independence of k for a wide range of temperature gradients, except for porous solids, where radiation between particles, which does not follow a linear temperature law, becomes an important part of the total heat flow. On the other hand, k is a function of temperature, but not a strong one. For small ranges of temperature, k may be considered constant. For larger temperature ranges, the thermal conductivity can usually be approximated by an equation of the form

$$k = a + bT \tag{10.3}$$

where a and b are empirical constants. Line III in Fig. 10.1 applies to a solid of constant k, where $b = 0$; the line would show some curvature if k were dependent on temperature.

Thermal conductivities of metals cover a wide range of values, from about 10 Btu/ft-h-°F (17 W/m-°C) for stainless steel to 240 Btu/ft-h-°F (415 W/m-°C) for silver. For glass and most nonporous minerals the thermal conductivities are much lower, from about 0.2 to 2 Btu/ft-h-°F (0.35 to 3.5 W/m-°C). Water has a thermal conductivity of 0.3 to 0.4 Btu/ft-h-°F (0.5 to 0.7 W/m-°C), about three times that of most organic liquids. Gases have the lowest conductivities; for air k is 0.014 Btu/ft-h-°F (0.024 W/m-°C) at 32°F (0°C). The thermal conductivity of gases is nearly independent of pressure but increases with temperature because of the increase in molecular velocity.

Solids having low k values are used as heat insulators to minimize the rate of heat flow. Porous insulating materials such as polystyrene foam act by entrapping air and thus eliminating convection. Their k values are about equal to that of air itself. Data showing typical thermal conductivities are given in Appendixes 10 to 14.

STEADY-STATE CONDUCTION

For the simplest case of steady-state conduction, consider a flat slab like that shown in Fig. 10.1. Assume that k is independent of temperature and that the area of the wall is very large in comparison with its thickness, so that heat losses from the edges are negligible. The external surfaces are at right angles to the plane of the illustration, and both are isothermal surfaces. The direction of heat flow is perpendicular to the wall. Also, since in steady state there can be neither accumulation nor depletion of heat within the slab, q is constant along the path of heat flow. If x is the distance from the hot side, Eq. (10.2) can be written

$$\frac{q}{A} = -k\frac{dT}{dx}$$

or

$$dT = -\frac{q}{kA}\,dx \tag{10.4}$$

Since the only variables in Eq. (10.4) are x and T, direct integration gives

$$\frac{q}{A} = k\frac{T_1 - T_2}{x_2 - x_1} = k\frac{\Delta T}{B} \tag{10.5}$$

where $x_2 - x_1 = B$ = thickness of slab
$T_1 - T_2 = \Delta T$ = temperature drop across slab

When the thermal conductivity k varies linearly with temperature, in accordance with Eq. (10.3), Eq. (10.5) still can be used rigorously by taking an average value $\bar{k}$ for k, which may be found either by using the arithmetic average of the individual values of k for the two surface temperatures, T_1 and T_2, or by calculating the arithmetic average of the temperatures and using the value of k at that temperature.

Equation (10.5) can be written in the form

$$\frac{q}{A} = \frac{\Delta T}{R} \tag{10.6}$$

where R is the thermal resistance of the solid between points 1 and 2. Equation (10.6) is an instance of the general rate principle, which equates a rate to the ratio of a driving force to a resistance. In heat conduction, q is the rate and ΔT the driving force. The resistance R, as shown by Eq. (10.6) and using $\bar{k}$ for k to account for a linear variation of k with temperature, is $B/\bar{k}$. The reciprocal of a resistance is called a *conductance*, which for heat conduction is $\bar{k}/B$. Both resistance and the conductance depend upon the dimensions of the solid as well as on the conductivity k, which is a property of the material.

Example 10.1. A layer of pulverized cork 6 in. (152 mm) thick is used as a layer of thermal insulation in a flat wall. The temperature the cold side of the cork is 40°F (4.4°C), and that of the warm side is 180°F (82.2°C). The thermal conductivity of the cork at 32°F (0°C) is 0.021 Btu/ft-h-°F (0.036 W/m-°C), and that at 200°F (93.3°C) is 0.032 (0.055). The area of the wall is 25 ft² (2.32 m²). What is the rate of heat flow through the wall in Btu per hour (watts)?

Solution

The arithmetic average temperature of the cork layer is (40 + 180)/2 = 110°F. By linear interpolation the thermal conductivity at 110°F is

$$\bar{k} = 0.021 + \frac{(110 - 32)(0.032 - 0.021)}{200 - 32}$$

$$= 0.021 + 0.005 = 0.026 \text{ Btu/ft-h-°F}$$

Also,

$$A = 25 \text{ ft}^2 \qquad \Delta T = 180 - 40 = 140°F \qquad B = \tfrac{6}{12} = 0.5 \text{ ft}$$

Substituting in Eq. (10.5) gives

$$q = \frac{0.026 \times 25 \times 140}{0.5} = 182 \text{ Btu/h (53.3 W)}$$

COMPOUND RESISTANCES IN SERIES. Consider a flat wall constructed of a series of layers, as shown in Fig. 10.2. Let the thicknesses of the layers be B_A, B_B, and B_C and the average conductivities of the materials of which the layers are made be $\bar{k}_A$, $\bar{k}_B$, and $\bar{k}_C$, respectively. Also, let the area of the compound wall, at right angles to the plane of the illustration, be A. Let ΔT_A, ΔT_B, and ΔT_C be the temperature drops across layers A, B, and C, respectively. Assume, further, that the layers are in excellent thermal contact, so that no temperature difference exists across the interfaces between the layers. Then, if ΔT is the total temperature drop across the entire wall,

$$\Delta T = \Delta T_A + \Delta T_B + \Delta T_C \qquad (10.7)$$

It is desired, first, to derive an equation for calculating the rate of heat flow through the series of resistances and, second, to show how the rate can be calculated as the ratio of the overall temperature drop ΔT to the overall resistance of the wall.

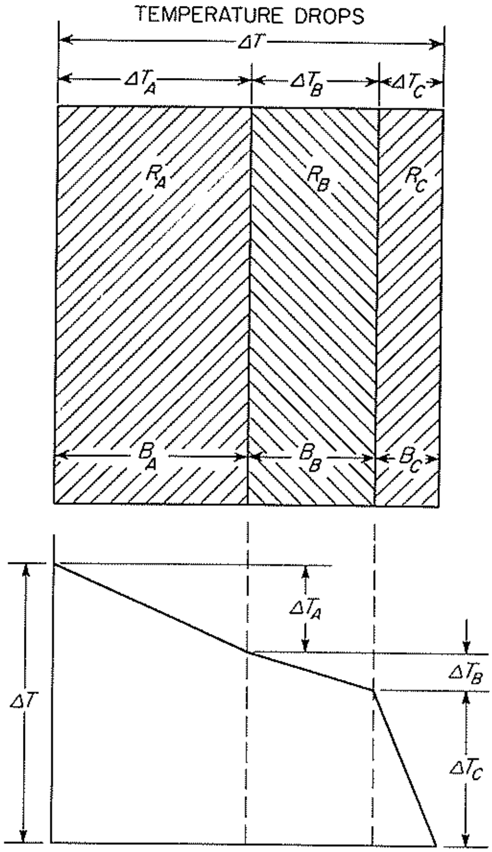

FIGURE 10.2
Thermal resistances in series.

Equation (10.5) can be written for each layer, using $\bar{k}$ in place of k,

$$\Delta T_A = q_A \frac{B_A}{\bar{k}_A A} \qquad \Delta T_B = q_B \frac{B_B}{\bar{k}_B A} \qquad \Delta T_C = q_C \frac{B_C}{\bar{k}_C A} \tag{10.8}$$

Adding Eqs. (10.8) gives

$$\Delta T_A + \Delta T_B + \Delta T_C = \frac{q_A B_A}{A\bar{k}_A} + \frac{q_B B_B}{A\bar{k}_B} + \frac{q_C B_C}{A\bar{k}_C} = \Delta T$$

Since, in steady heat flow, all the heat that passes through the first resistance must pass through the second and in turn pass through the third, q_A, q_B, and q_C are equal and all can be denoted by q. Using this fact and solving for q/A gives

$$\frac{q}{A} = \frac{\Delta T}{B_A/\bar{k}_A + B_B/\bar{k}_B + B_C/\bar{k}_C} = \frac{\Delta T}{R_A + R_B + R_C} = \frac{\Delta T}{R} \tag{10.9}$$

where R_A, R_B, R_C = resistance of individual layers
R = overall resistance

Equation (10.9) shows that in heat flow through a series of layers the overall thermal resistance equals the sum of the individual resistances.

The rate of flow of heat through several resistances in series clearly is analogous to the current flowing through several electric resistances in series. In an electric circuit the potential drop over any one of several resistances is to the total potential drop in the circuit as the individual resistances are to the total resistance. In the same way the potential drops in a thermal circuit, which are the temperature differences, are to the total temperature drop as the individual thermal resistances are to the total thermal resistance. This can be expressed mathematically as

$$\frac{\Delta T}{R} = \frac{\Delta T_A}{R_A} = \frac{\Delta T_B}{R_B} = \frac{\Delta T_C}{R_C} \tag{10.10}$$

Figure 10.2 also shows the pattern of temperatures and the temperature gradients. Depending on the thickness and thermal conductivity of the layer, the temperature drop in that layer may be a large or small fraction of the total temperature drop; a thin layer of low conductivity may well cause a much larger temperature drop and a steeper thermal gradient than a thick layer of high conductivity.

Example 10.2. A flat furnace wall is constructed of a 4.5-in. (114-mm) layer of Sil-o-cel brick, with a thermal conductivity of 0.08 Btu/ft-h-°F (0.138 W/m-°C) backed by a 9-in. (229-mm) layer of common brick, of conductivity 0.8 Btu/ft-h-°F (1.38 W/m-°C). The temperature of the inner face of the wall is 1400°F (760°C), and that of the outer face is 170°F (76.6°C). (*a*) What is the heat loss through the wall? (*b*) What is the temperature of the interface between the refractory brick and the common brick? (*c*) Supposing that the contact between the two brick layers is poor and that a "contact resistance" of 0.50°F-h-ft²/Btu (0.088°C-m²/W) is present, what would be the heat loss?

Solution

(*a*) Consider 1 ft^2 of wall ($A = 1$ ft^2). The thermal resistance of the Sil-o-cel layer is

$$R_A = \frac{4.5/12}{0.08} = 4.687$$

and that of the common brick is

$$R_B = \frac{9/12}{0.8} = 0.938$$

The total resistance is

$$R = R_A + R_B = 4.687 + 0.938 = 5.625\text{°F-h-ft}^2/\text{Btu}$$

The overall temperature drop is

$$\Delta T = 1400 - 170 = 1230\text{°F}$$

Substitution in Eq. (10.9) gives, for the heat loss from 1 ft^2 of wall,

$$q = \frac{1230}{5.625} = 219 \text{ Btu/h } (64.2 \text{ W})$$

(*b*) The temperature drop in one of a series of resistances is to the individual resistance as the overall temperature drop is to the overall resistance, or

$$\frac{\Delta T_A}{4.687} = \frac{1230}{5.625}$$

from which

$$\Delta T_A = 1025°F$$

The temperature at the interface is 1400 − 1025 = 375°F (190.6°C).

(*c*) The total resistance, which now includes a contact resistance, is

$$R = 5.625 + 0.500 = 6.125$$

The heat loss from 1 ft^2 is

$$q = \frac{1230}{6.125} = 201 \text{ Btu/h } (58.9 \text{ W})$$

HEAT FLOW THROUGH A CYLINDER. Consider the hollow cylinder represented by Fig. 10.3. The inside radius of the cylinder is r_i, the outside radius is r_o, and the length of the cylinder is L. The thermal conductivity of the material of which the cylinder is made is k. The temperature of the outside surface is T_o, and that of the inside surface is T_i. It is desired to calculate the rate of heat flow outward for this case.

Consider a very thin cylinder, concentric with the main cylinder, of radius r, where r is between r_i and r_o. The thickness of the wall of this cylinder is dr; and

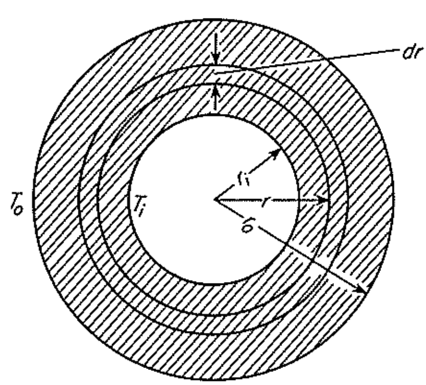

FIGURE 10.3
Flow of heat through thick-walled cylinder.

if dr is small enough with respect to r for the lines of heat flow to be considered parallel, Eq. (10.2) can be applied and written in the form

$$q = -k\frac{dT}{dr}2\pi rL \tag{10.11}$$

since the area perpendicular to the heat flow is equal to $2\pi rL$ and the dn of Eq. (10.2) is equal to dr. Rearranging Eq. (10.11) and integrating between limits gives

$$\int_{r_i}^{r_o}\frac{dr}{r} = \frac{2\pi Lk}{q}\int_{T_o}^{T_i} dT$$

$$\ln r_o - \ln r_i = \frac{2\pi Lk}{q}(T_i - T_o)$$

$$q = \frac{k(2\pi L)(T_i - T_o)}{\ln (r_o/r_i)} \tag{10.12}$$

Equation (10.12) can be used to calculate the flow of heat through a thick-walled cylinder. It can be put in a more convenient form by expressing the rate of flow of heat as

$$q = \frac{k\overline{A}_L(T_i - T_o)}{r_o - r_i} \tag{10.13}$$

This is of the same general form as Eq. (10.5) for heat flow through a flat wall with the exception of $\overline{A}_L$, which must be so chosen that the equation is correct. The term $\overline{A}_L$ can be determined by equating the right-hand sides of Eqs. (10.12) and (10.13) and solving for $\overline{A}_L$:

$$\overline{A}_L = \frac{2\pi L(r_o - r_i)}{\ln (r_o/r_i)} \tag{10.14}$$

Note from Eq. (10.14) that $\overline{A}_L$ is the area of a cylinder of length L and radius $\bar{r}_L$, where

$$\bar{r}_L = \frac{r_o - r_i}{\ln (r_o/r_i)} \tag{10.15}$$

The form of the right-hand side of Eq. (10.15) is important enough to repay memorizing. It is known as the *logarithmic mean*, and in the particular case of Eq. (10.15) $\bar{r}_L$ is called the *logarithmic mean radius*. It is the radius that, when applied to the integrated equation for a flat wall, will give the correct rate of heat flow through a thick-walled cylinder.

The logarithmic mean is less convenient than the arithmetic mean, and the latter can be used without appreciable error for thin-walled tubes, where r_o/r_i is nearly 1. The ratio of the logarithmic mean $\bar{r}_L$ to the arithmetic mean $\bar{r}_a$ is a function of r_o/r_i, as shown in Fig. 10.4. Thus, when $r_o/r_i = 2$, the logarithmic mean is $0.96\bar{r}_a$ and the error in the use of the arithmetic mean is 4 percent. The error is 1 percent where $r_o/r_i = 1.4$.

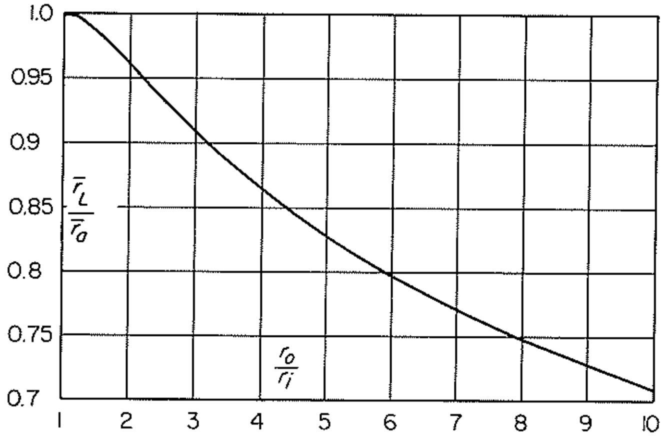

FIGURE 10.4
Relation between logarithmic and arithmetic means.

Example 10.3. A tube 60 mm (2.36 in.) OD is insulated with a 50-mm (1.97-in.) layer of silica foam, for which the conductivity is 0.055 W/m-°C (0.032 Btu/ft-h-°F), followed with a 40-mm (1.57-in.) layer of cork with a conductivity of 0.05 W/m-°C (0.03 Btu/ft-h-°F). If the temperature of the outer surface of the pipe is 150°C (302°F) and the temperature of the outer surface of the cork is 30°C (86°F), calculate the heat loss in watts per meter of pipe.

Solution
These layers are too thick to use the arithmetic mean radius, and the logarithmic mean radius should be used. For the silica layer

$$\bar{r}_L = \frac{80 - 30}{\ln (80/30)} = 50.97 \text{ mm}$$

and for the cork layer

$$\bar{r}_L = \frac{120 - 80}{\ln (120/80)} = 98.64 \text{ mm}$$

Call silica substance A and cork substance B. From Eq. (10.13)

$$q_A = \frac{k_A \bar{A}_A (T_i - T_x)}{x_A} \qquad q_B = \frac{k_B \bar{A}_B (T_x - T_o)}{x_B}$$

where T_x is the temperature at the interface between the silica and the cork. From Eqs. (10.14) and (10.15),

$$\bar{A}_A = 2\pi(0.05097)L = 0.3203L \qquad \bar{A}_B = 2\pi(0.09864)L = 0.6198L$$

Then

$$q_A = \frac{0.055 \times 0.3203L(T_i - T_x)}{0.050} = 0.3522(T_i - T_x)$$

$$q_B = \frac{0.05 \times 0.6198(T_x - T_o)}{0.040} = 0.7748(T_x - T_o)$$

Hence

$$2.839q/L = T_i - T_x \qquad 1.291q/L = T_x - T_o$$

Adding these gives

$$4.13q/L = T_i - T_o = 150 - 30 = 120$$

$$q/L = 29.1 \text{ W/m } (30.3 \text{ Btu/ft-h})$$

UNSTEADY-STATE CONDUCTION

A full treatment of unsteady-state heat conduction is not in the field of this text.[2–4] A derivation of the partial differential equation for one-dimensional heat flow and the results of the integration of the equations for some simple shapes are the only subjects covered in this section. It is assumed throughout that k is independent of temperature.

EQUATION FOR ONE-DIMENSIONAL CONDUCTION. Figure 10.5 represents a section through a large slab of material of thickness $2s$, initially all at a uniform temperature T_a. At the start of heating both surface temperatures are quickly increased to and subsequently held at temperature T_s. The temperature pattern shown in Fig. 10.5 reflects conditions after a relatively short time t_T has elapsed since the start of heating.

Focus attention on the thin layer of thickness dx located at distance x from the left side of the slab. The two sides of the element are isothermal surfaces. The temperature gradient at x is, at a definite instant of time, $\partial T/\partial x$, and the heat input in time interval dt at x is $-kA(\partial T/\partial x)\,dt$, where A is the area of the layer perpendicular to the flow of heat and k is the thermal conductivity of the solid. The gradient at distance $x + dx$ is slightly different from that at x and may be represented as

$$\frac{\partial T}{\partial x} + \frac{\partial}{\partial x}\frac{\partial T}{\partial x}\,dx$$

The heat flow out of the layer at $x + dx$ is, then,

$$-kA\left(\frac{\partial T}{\partial x} + \frac{\partial}{\partial x}\frac{\partial T}{\partial x}\,dx\right)dt$$

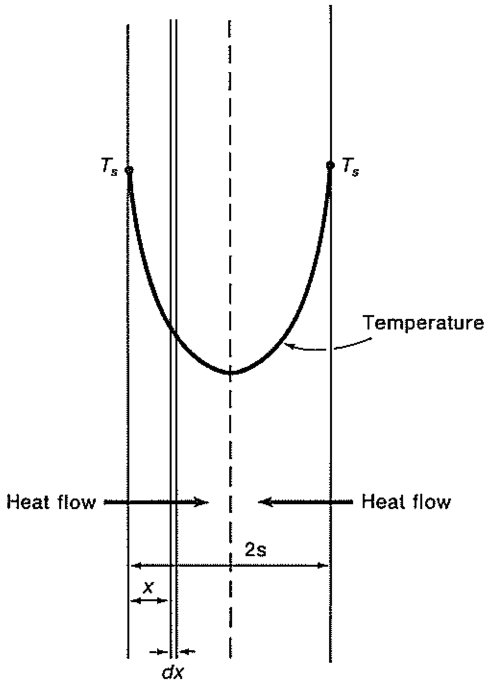

FIGURE 10.5
Unsteady-state conduction in solid slab.

The excess of heat input over heat output, which is the accumulation of heat in layer dx, is

$$-kA\frac{\partial T}{\partial x}dt + kA\left(\frac{\partial T}{\partial x} + \frac{\partial^2 T}{\partial x^2}dx\right)dt = kA\frac{\partial^2 T}{\partial x^2}dx\,dt$$

The accumulation of heat in the layer must increase the temperature of the layer. If c_p and ρ are the specific heat and density, respectively, the accumulation is the product of the mass (volume times density), the specific heat, and the increase in temperature, or $(\rho A\,dx)c_p(\partial T/\partial t)\,dt$. Then, by a heat balance,

$$kA\frac{\partial^2 T}{\partial x^2}dx\,dt = \rho c_p A\,dx\frac{\partial T}{\partial t}dt$$

or, after division by $\rho c_p A\,dx\,dt$,

$$\frac{\partial T}{\partial t} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial x^2} = \alpha\frac{\partial^2 T}{\partial x^2} \tag{10.16}$$

The term α in Eq. (10.16) is called the *thermal diffusivity* of the solid and is a property of the material. It has the dimensions of area divided by time.

General solutions of unsteady-state conduction equations are available for certain simple shapes such as the infinite slab, the infinitely long cylinder, and the sphere. For example, the integration of Eq. (10.16) for the heating or cooling of an infinite slab of known thickness from both sides by a medium at constant surface temperature gives

$$\frac{T_s - \overline{T}_b}{T_s - T_a} = \frac{8}{\pi^2}\left(e^{-a_1 N_{\mathrm{Fo}}} + \tfrac{1}{9}e^{-9a_1 N_{\mathrm{Fo}}} + \tfrac{1}{25}e^{-25a_1 N_{\mathrm{Fo}}} + \cdots\right) \tag{10.17}$$

where T_s = constant average temperature of surface of slab
T_a = initial temperature of slab
$\overline{T}_b$ = average temperature of slab at time t_T
N_{Fo} = Fourier number, defined as $\alpha t_T/s^2$
α = thermal diffusivity
t_T = time of heating or cooling
s = one-half slab thickness
$a_1 = (\pi/2)^2$

Equation (10.17) may also be used for a slab heated from one side only, provided no heat is transferred at the other side and $\partial T/\partial x = 0$ at that surface. Here s is the full slab thickness.

For an infinitely long solid cylinder of radius r_m the average temperature $\overline{T}_b$ is given by the equation[5b]

$$\frac{T_s - \overline{T}_b}{T_s - T_a} = 0.692e^{-5.78N_{\mathrm{Fo}}} + 0.131e^{-30.5N_{\mathrm{Fo}}} + 0.0534e^{-74.9N_{\mathrm{Fo}}} + \cdots \tag{10.18}$$

where $N_{\mathrm{Fo}} = \alpha t_T/r_m^2$. For a sphere of radius r_m the corresponding equation is[2b]

$$\frac{T_s - \overline{T}_b}{T_s - T_a} = 0.608e^{-9.87N_{\mathrm{Fo}}} + 0.152e^{-39.5N_{\mathrm{Fo}}} + 0.0676e^{-88.8N_{\mathrm{Fo}}} + \cdots \tag{10.19}$$

When N_{Fo} is greater than about 0.1, only the first term of the series in Eqs. (10.17) to (10.19) is significant and the other terms may be ignored. Under these conditions the time required to change the temperature from T_a to $\overline{T}_b$ can be found by rearranging Eq. (10.17), with all except the first term of the series omitted, to give for the slab

$$t_T = \frac{1}{\alpha}\left(\frac{2s}{\pi}\right)^2 \ln \frac{8(T_s - T_a)}{\pi^2(T_s - \overline{T}_b)} \tag{10.20}$$

For the infinite cylinder the corresponding equation, found from Eq. (10.18), is

$$t_T = \frac{r_m^2}{5.78\alpha} \ln \frac{0.692(T_s - T_a)}{T_b - \overline{T}_b} \tag{10.21}$$

For a sphere,† from Eq. (10.19),

$$t_T = \frac{r_m^2}{9.87\alpha} \ln \frac{0.608(T_s - T_a)}{T_s - \overline{T}_b} \tag{10.22}$$

† An alternate treatment of this problem in terms of heat-transfer coefficients is given in Chap. 11, p. 327.

Figure 10.6 is a plot of Eqs. (10.17) to (10.19). The ordinate of this figure is known as the *unaccomplished temperature change*, i.e., the fraction of the total possible temperature change that remains to be accomplished at any time. Except at very low values of N_{Fo}, Eqs. (10.20) to (10.22) apply and all three semilogarithmic plots are straight lines.

Equations (10.17) to (10.19) apply only when the surface temperature is constant, so T_s can be equal to the temperature of the heating or cooling medium only when the temperature difference between the medium and the solid surface is negligible. This implies that there is negligible thermal resistance between the

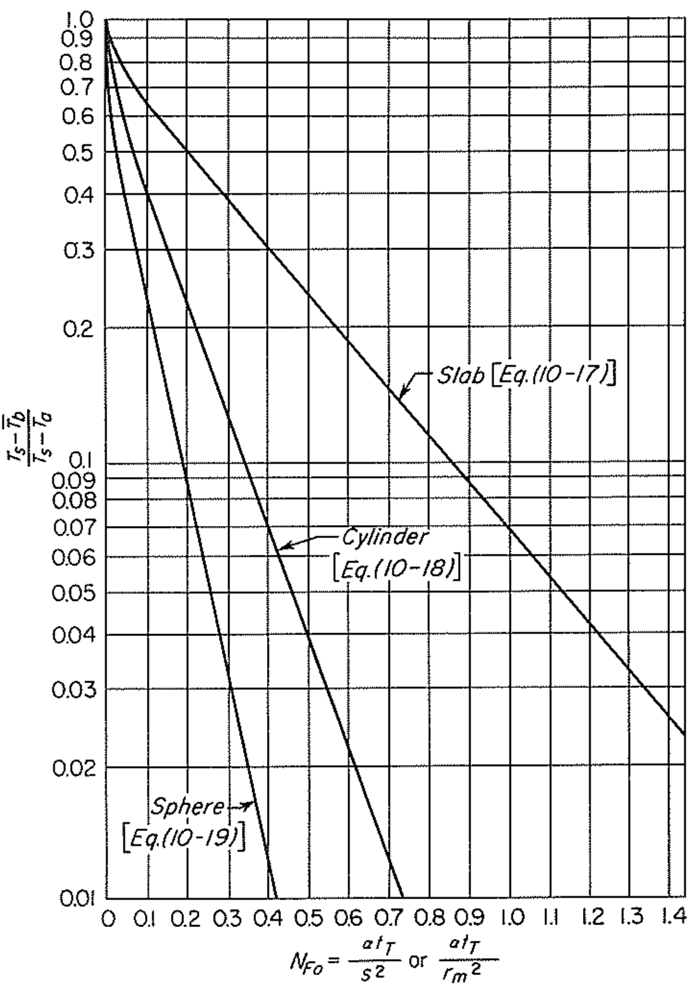

FIGURE 10.6
Average temperatures during unsteady-state heating or cooling of a large slab, an infinitely long cylinder, or a sphere.

surface and the medium. Equations and graphs[5a] similar to Eqs. (10.17) to (10.19) and Fig. 10.6 are available for local temperatures at points inside slabs and cylinders, for temperatures in spheres and other shapes, and for situations in which the thermal resistance at the surface is large enough to cause variations in the surface temperature. Temperature distributions in heterogeneous solids or bodies of complex shape are found by fluid or electrical analogs or numerical approximation methods.[1]

The total heat Q_T transferred to the solid in time t_T through a unit area of surface is often of interest. From the definition of average temperature the heat required to raise the temperature of a unit mass of solid from T_a to $\overline{T}_b$ is $c_p(\overline{T}_b - T_a)$. For a slab of thickness $2s$ and density ρ the total surface area (both sides) of a unit mass is $1/s\rho$. The total heat transferred per unit area is therefore given by

$$\frac{Q_T}{A} = s\rho c_p(\overline{T}_b - T_a) \tag{10.23}$$

The corresponding equation for an infinitely long cylinder is

$$\frac{Q_T}{A} = \frac{r_m \rho c_p(\overline{T}_b - T_a)}{2} \tag{10.24}$$

For a sphere the equation is

$$\frac{Q_T}{A} = \frac{r_m \rho c_p(\overline{T}_b - T_a)}{3} \tag{10.25}$$

Example 10.4. A flat slab of plastic initially at 70°F (21.1°C) is placed between two platens at 250°F (121.1°C). The slab is 1.0 in. (2.54 cm) thick. (*a*) How long will it take to heat the slab to an average temperature of 210°F (98.9°C)? (*b*) How much heat, in Btu, will be transferred to the plastic during this time per square foot of surface? The density of the solid is 56.2 lb/ft³ (900 kg/m³), the thermal conductivity is 0.075 Btu/ft-h-°F (0.13 W/m-°C), and the specific heat is 0.40 Btu/lb-°F (1.67 J/g-°C).

Solution

(*a*) The quantities for use with Fig. 10.6 are

$$k = 0.075 \text{ Btu/ft-h-°F} \qquad \rho = 56.2 \text{ lb/ft}^3 \qquad c_p = 0.40 \text{ Btu/lb-°F}$$

$$s = \frac{0.5}{12} = 0.0417 \text{ ft} \qquad T_s = 250\text{°F} \qquad T_a = 70\text{°F} \qquad \overline{T}_b = 210\text{°F}$$

Then

$$\frac{T_s - \overline{T}_b}{T_s - T_a} = \frac{250 - 210}{250 - 70} = 0.222 \qquad \alpha = \frac{k}{\rho c_p} = \frac{0.075}{56.2 \times 0.40} = 0.00335$$

From Fig. 10.6, for a temperature-difference ratio of 0.222,

$$N_{\text{Fo}} = 0.52 = \frac{0.00335 t_T}{0.0417^2} \qquad t_T = 0.27 \text{ h} = 16 \text{ min}$$

(*b*) Substitution in Eq. (10.23) gives heat flow per total surface area

$$\frac{Q_T}{A} = 0.0417 \times 56.2 \times 0.40(210 - 70) = 131 \text{ Btu/ft}^2 \text{ (1487 kJ/m}^2\text{)}$$

SEMI-INFINITE SOLID. Sometimes solids are heated or cooled in such a way that the temperature changes in the solid are confined to the region near one surface. Consider, for example, a very thick flat wall of a chimney, initially all at a uniform temperature T_a. Suppose that the inner surface of the wall is suddenly heated to, and held at, a high temperature T_s, perhaps by suddenly admitting hot flue gas to the chimney. Temperatures inside the chimney wall will change with time, rapidly near the hot surface and more slowly farther away. If the wall is thick enough, there will be no measurable change in the temperature of the outer surface for a considerable time. Under these conditions the heat may be considered to be "penetrating" a solid of essentially infinite thickness. Figure 10.7 shows the temperature patterns in such a wall at various times after exposure to the hot gas, indicating the sharp discontinuity in temperature at the hot surface immediately after exposure and the progressive changes at interior points at later times.

For this situation integration of Eq. (10.16) with the appropriate boundary conditions, gives, for temperature T at any point a distance x from the hot surface, the equation

$$\frac{T_s - T}{T_s - T_a} = \frac{2}{\sqrt{\pi}} \int_0^Z e^{-Z^2}\, dZ \tag{10.26}$$

where $Z = x/2\sqrt{\alpha t}$, dimensionless
α = thermal diffusivity
x = distance from surface
t = time after change in surface temperature, h

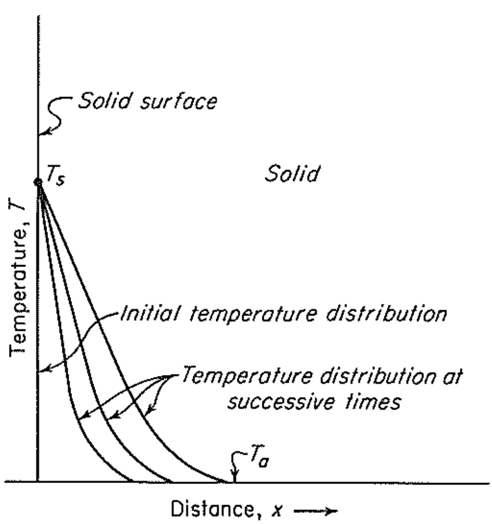

FIGURE 10.7
Temperature distributions in unsteady-state heating of semi-infinite solid.

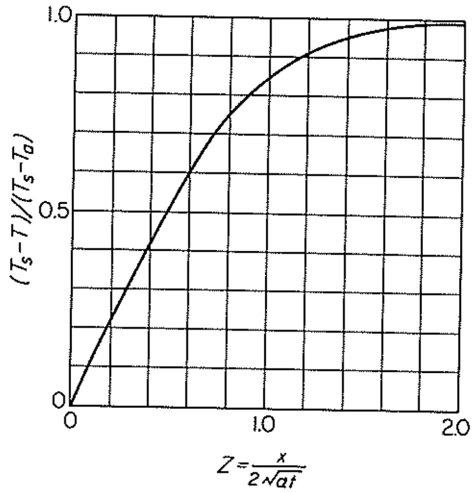

FIGURE 10.8
Unsteady-state heating or cooling of semi-infinite solid.

The function in Eq. (10.26) is known as the *Gauss error integral* or *probability integral.* Equation (10.26) is plotted in Fig. 10.8.

Equation (10.26) indicates that at any time after the surface temperature is changed there will be some change in temperature at all points in the solid, even points far removed from the hot surface. The actual change at such distant points, however, is negligibly small. Beyond a certain distance from the hot surface not enough heat has penetrated to affect the temperature significantly. This *penetration distance* x_p is arbitrarily defined as that distance from the surface at which the temperature change is 1 percent of the initial change in surface temperature. That is to say, $(T - T_a)/(T_s - T_a) = 0.01$ or $(T_s - T)/(T_s - T_a) = 0.99$. Figure 10.8 shows that the probability integral reaches a value of 0.99 when $Z = 1.82$, from which

$$x_p = 3.64\sqrt{\alpha t} \tag{10.27}$$

Example 10.5. A sudden cold wave drops the atmospheric temperature to −20°C (−4°F) for 12 h. (*a*) If the ground was initially all at 5°C (41°F), how deep would a water pipeline have to be buried to be in no danger of freezing? (*b*) What is the penetration distance under these conditions? The thermal diffusivity of soil is 0.0011 m²/h (0.0118 ft²/h).

Solution

(*a*) Assume that the surface of the ground quickly reaches and remains at −20°C. Unless the temperature at the location of the pipe is below 0°C, there is no danger of freezing. The quantities required for use with Fig. 10.8 are therefore

$$T_s = -20°\text{C} \qquad T_a = 5°\text{C} \qquad T = 0°\text{C}$$

$$t = 12\ \text{h} \qquad \alpha = 0.0011\ \text{m}^2/\text{h}$$

$$\frac{T_s - T}{T_s - T_a} = \frac{-20 - 0}{-20 - 5} = 0.80$$

From Fig. 10.8, $Z = 0.91$. The depth x is therefore

$$x = 0.91 \times 2\sqrt{\alpha t} = 0.91 \times 2\sqrt{0.0011 \times 12} = 0.21 \text{ m } (0.69 \text{ ft})$$

(*b*) From Eq. (10.27) the penetration distance is

$$x_p = 3.64\sqrt{0.0011 \times 12} = 0.419 \text{ m } (1.37 \text{ ft})$$

To find the total heat transferred to a semi-infinite solid in a given time it is necessary to find the temperature gradient and heat flux at the hot surface as a function of time. The temperature gradient at the surface is found by differentiating Eq. (10.26) to give

$$\left(\frac{\partial T}{\partial x}\right)_{x=0} = -\frac{T_s - T_a}{\sqrt{\pi \alpha t}} \qquad (10.28)$$

The heat flow rate at the surface is therefore

$$\left(\frac{q}{A}\right)_{x=0} = -k\left(\frac{\partial T}{\partial x}\right)_{x=0} = \frac{k(T_s - T_a)}{\sqrt{\pi \alpha t}} \qquad (10.29)$$

After substitution of dQ/dt for q, Eq. (10.29) can be integrated to give the total quantity of heat transferred per unit area, Q_T/A, in time t_T, as follows:

$$\frac{Q_T}{A} = \frac{k(T_s - T_a)}{\sqrt{\pi \alpha}} \int_0^{t_T} \frac{dt}{\sqrt{t}} = 2k(T_s - T_a)\sqrt{\frac{t_T}{\pi \alpha}} \qquad (10.30)$$

SYMBOLS

A	Area, m^2 or ft^2; $\overline{A}_L$, logarithmic mean
a	Constant in Eq. (10.3)
a_1	$(\pi/2)^2$
B	Thickness of slab, m or ft; B_A, B_B, B_C, of layers A, B, C, respectively
b	Constant in Eq. (10.3)
c_p	Specific heat at constant pressure, J/g-°C or Btu/lb-°F
e	Base of Naperian logarithms, 2.71828 ···
g_c	Newton's-law proportionality factor, 32.174 ft-lb/lb$_f$-s^2
k	Thermal conductivity, W/m-°C or Btu/ft-h-°F; k_A, k_B, k_C, of layers A, B, C, respectively; $\overline{k}$, average value
L	Length of cylinder, m or ft
N_{Fo}	Fourier number, dimensionless; $\alpha t_T/s^2$ for slab; $\alpha t_T/r_m^2$ for cylinder or sphere
n	Distance measured normally to surface, m or ft
Q	Quantity of heat, J or Btu; Q_T, total quantity transferred
q	Heat flow rate, W or Btu/h; q_A, q_B, q_C, in layers A, B, C, respectively
R	Thermal resistance, m^2-°C/W or ft^2-°F-h/Btu; R_A, R_B, R_C, of layers A, B, C, respectively

r	Radial distance or radius, m or ft; r_i, inside radius; r_m, radius of solid cylinder or sphere; r_o, outside radius; $\bar{r}_L$, logarithmic mean; $\bar{r}_a$, arithmetic mean
s	Half-thickness of slab, m or ft
T	Temperature, °C or °F; T_a, initial temperature; $\overline{T}_b$, average temperature at end of time t_T; T_i, of inside surface; T_o, of outside surface; T_s, surface temperature; T_1, T_2, at locations 1, 2, respectively
t	Time, s or h; t_T, time required to heat or cool
u	Velocity, m/s or ft/s
x	Distance from surface, m or ft; x_1, x_2, at locations 1, 2, respectively; x_p, penetration distance in semi-infinite solid
y	Distance, m or ft
Z	$x/2\sqrt{\alpha t}$, dimensionless

Greek letters

α	Thermal diffusivity, $k/\rho c_p$, m^2/s or ft^2/h
ΔT	Overall temperature drop; ΔT_A, ΔT_B, ΔT_C, in layers A, B, C, respectively
μ	Absolute viscosity, Pa · s or lb/ft-h
ρ	Density, kg/m^3 or lb/ft^3
τ	Shear stress, N/m^2 or lb_f/ft^2

PROBLEMS

10.1. A furnace wall consists of 200 mm of refractory fireclay brick, 100 mm of kaolin brick, and 6 mm of steel plate. The fire side of the refractory is at 1150°C, and the outside of the steel is at 30°C. An accurate heat balance over the furnace shows the heat loss from the wall to be 300 W/m^2. It is known that there may be thin layers of air between the layers of brick and steel. To how many millimeters of kaolin are these air layers equivalent? See Appendix 11 for thermal conductivities.

10.2. A standard 1-in. Schedule 40 steel pipe carries saturated steam at 250°F. The pipe is lagged (insulated) with a 2-in. layer of 85 percent magnesia pipe covering, and outside this magnesia there is a $\frac{1}{2}$-in. layer of cork. The inside temperature of the pipe wall is 249°F, and the outside temperature of the cork is 90°F. Thermal conductivities, in Btu/ft-h-°F, are: for steel, 26; for magnesia, 0.034; for cork, 0.03. Calculate (*a*) the heat loss from 100 ft of pipe in Btu per hour; (*b*) the temperatures at the boundaries between metal and magnesia and between magnesia and cork.

10.3. Derive the equation for steady-state heat transfer through a spherical shell of inner radius r_1 and outer radius r_2. Arrange the result for easy comparison with the solution for a thick-walled cylinder.

10.4. A very long, wide sheet of plastic 4 mm thick and initially at 20°C is suddenly exposed on both sides to an atmosphere of steam at 102°C. (*a*) If there is negligible thermal resistance between the steam and the surfaces of the plastic, how long will it take for the temperature at the centerline of the sheet to change significantly? (*b*) What would be the bulk average temperature of the plastic at this time? For the plastic, $k = 0.138$ W/m-°C and $\alpha = 0.00035$ m²/h.

10.5. A long steel rod 1 in. in diameter is initially at a uniform temperature of 1200°F. It is suddenly immersed in a quenching bath of oil at 150°F. In 4 min its average temperature drops to 250°F. How long would it take to lower the temperature from 1200 to 250°F (*a*) if the rod were $2\frac{1}{2}$ in. in diameter? (*b*) If it were 5 in. in diameter? For steel, $k = 26$ Btu/ft-h-°F; $\rho = 486$ lb/ft³: $c_p = 0.11$ Btu/lb-°F.

10.6. Steel spheres 3 in. in diameter heated to 700°F are to be cooled by immersion in an oil bath at 125°F. If there is negligible thermal resistance between the oil and the steel surfaces, (*a*) calculate the average temperature of the spheres 10 s and 1 and 6 min after immersion. (*b*) How long would it take for the unaccomplished temperature change to be reduced to 1 percent of the initial temperature difference? The steel has the same thermal properties as in Prob. 10.5.

10.7. Under the conditions described in Example 10.5, what is the average rate of heat loss per unit area from the ground to the air during the 12-h period? The thermal conductivity of soil is 0.7 W/m-°C.

10.8. For the same initial temperatures T_s and T_a and at the same Fourier number, would the average temperature of a sphere be higher or lower than that of a cylinder or a slab? (See Fig. 10.6.) What are the physical reasons that would lead you to expect this?

10.9. The heat transfer rate to the jacket of an agitated polymerization kettle is 7.4 kW/m² when the polymerization temperature is 50°C and the water in the jacket is at 20°C. The kettle is made of stainless steel with a wall 12 mm thick, and there is a thin layer of polymer ($k = 0.16$ W/m-°C) left on the wall from previous runs. (*a*) What is the temperature drop across the metal wall? (*b*) How thick would the polymer deposit have to be to account for the rest of the temperature difference? (*c*) By what factor could the heat flux be increased by using a stainless-clad reactor with a 3-mm stainless-steel layer bonded to a 9-mm mild-steel shell?

10.10. (a) Compare the thermal conductivities and thermal diffusivities of air and water at 100°F. (*b*) Calculate the penetration distances in a stagnant mass of air and one of water, at 50°F and 1 atm, each of which is exposed for 10 s to a hot metal surface at 100°F. Comment on the difference.

REFERENCES

1. Carnahan, B., H. A. Luther, and J. O. Wilkes: *Applied Numerical Methods*, Wiley, New York, 1969.
2. Carslaw, H. S., and J. C. Jaeger: *Conduction of Heat in Solids*, 2nd ed., Oxford University Press, Fair Lawn, N.J., 1959; (*a*) pp. 6–8, (*b*) p. 234.
3. Fourier, J.: *The Analytical Theory of Heat*, trans. by A. Freeman, Dover, New York, 1955.
4. Gebhart, B.: *Heat Transfer*, 2nd ed., McGraw-Hill, New York, 1971.
5. McAdams, W. H.: *Heat Transmission*, 3rd ed., McGraw-Hill, New York, 1954; (*a*) pp. 35–43, (*b*) p. 232.

CHAPTER 11

PRINCIPLES OF HEAT FLOW IN FLUIDS

Heat transfer from a warmer fluid to a cooler fluid, usually through a solid wall separating the two fluids, is common in chemical engineering practice. The heat transferred may be latent heat accompanying a phase change such as condensation or vaporization or it may be sensible heat from the rise or fall in the temperature of a fluid without any phase change. Typical examples are reducing the temperature of a fluid by transfer of sensible heat to a cooler fluid, the temperature of which is increased thereby; condensing steam by cooling water; and vaporizing water from a solution at a given pressure by condensing steam at a higher pressure. All such cases require that heat be transferred by conduction and convection.

TYPICAL HEAT-EXCHANGE EQUIPMENT. To establish a basis for specific discussion of heat transfer to and from flowing fluids, consider the simple tubular condenser of Fig. 11.1. It consists essentially of a bundle of parallel tubes A, the ends of which are expanded into tube sheets B_1 and B_2. The tube bundle is inside a cylindrical shell C and is provided with two channels D_1 and D_2, one at each end, and two channel covers E_1 and E_2. Steam or other vapor is introduced through nozzle F into the shell-side space surrounding the tubes, condensate is withdrawn through connection G, and any noncondensable gas that might enter with the inlet vapor is removed through vent K. Connection G leads to a trap, which is a device that allows flow of liquid but holds back vapor. The fluid to be heated is pumped through connection H into channel D_2. It flows through the

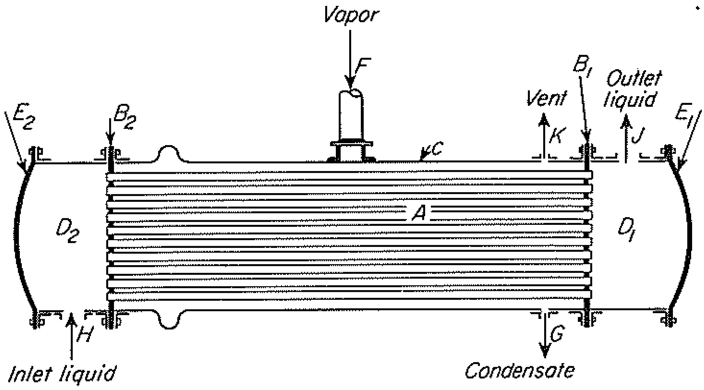

FIGURE 11.1
Single-pass tubular condenser: A, tubes; B_1, B_2, tube sheets; C, shell; D_1, D_2, channels; E_1, E_2, channel covers; F, vapor inlet; G, condensate outlet; H, cold-liquid inlet; J, warm-liquid outlet; K, non-condensed-gas vent.

tubes into the other channel D_1 and is discharged through connection J. The two fluids are physically separated but are in thermal contact with the thin metal tube walls separating them. Heat flows through the tube walls from the condensing vapor to the cooler fluid in the tubes.

If the vapor entering the condenser is not superheated and the condensate is not subcooled below its boiling temperature, the temperature throughout the shell side of the condenser is constant. The reason for this is that the temperature of the condensing vapor is fixed by the pressure of the shell-side space, and the pressure in that space is constant. The temperature of the fluid in the tubes increases continuously as the fluid flows through the tubes.

The temperatures of the condensing vapor and of the liquid are plotted against the tube length in Fig. 11.2. The horizontal line represents the temperature

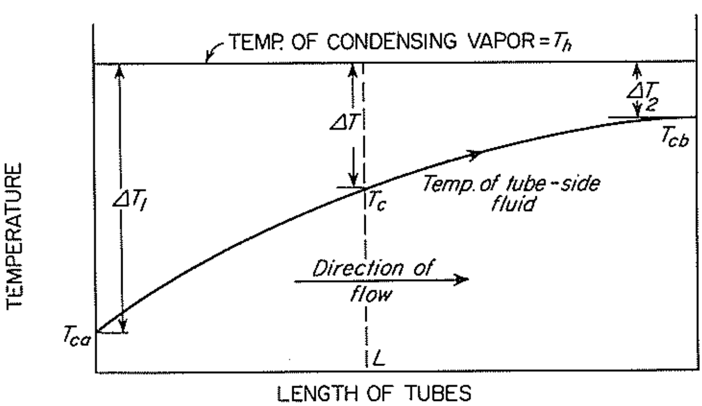

FIGURE 11.2
Temperature-length curves for condenser.

of the condensing vapor, and the curved line below it represents the rising temperature of the tube-side fluid. In Fig. 11.2, the inlet and outlet fluid temperatures are T_{ca} and T_{cb}, respectively, and the constant temperature of the vapor is T_h. At a length L from the entrance end of the tubes, the fluid temperature is T_c, and the local difference between the temperatures of vapor and fluid is $T_h - T_c$. This temperature difference is called a *point temperature difference* and is denoted by ΔT. The point temperature difference at the inlet of the tubes is $T_h - T_{ca}$, denoted by ΔT_1, and that at the exit end is $T_h - T_{cb}$, denoted by ΔT_2. The terminal point temperature differences ΔT_1 and ΔT_2 are called the *approaches.*

The change in temperature of the fluid, $T_{cb} - T_{ca}$, is called the *temperature range* or, simply, the *range.* In a condenser there is but one range, that of the cold fluid being heated.

In this text the symbol ΔT is used exclusively to signify a temperature *difference* between two objects or two fluids. It does *not* denote the temperature change in a given fluid.

A second example of simple heat-transfer equipment is the double-pipe exchanger shown in Fig. 11.3. It is assembled of standard metal pipe and standardized return bends and return heads, the latter equipped with stuffing boxes. One fluid flows through the inside pipe and the second fluid through the annular space between the outside and the inside pipe. The function of a heat exchanger is to increase the temperature of a cooler fluid and decrease that of a hotter fluid. In a typical exchanger, the inner pipe may be $1\frac{1}{4}$ in. and the outer pipe $2\frac{1}{2}$ in., both IPS. Such an exchanger may consist of several passes arranged in a vertical stack. Double-pipe exchangers are useful when not more than 100 to 150 ft^2 of surface is required. For larger capacities, more elaborate shell-and-tube exchangers, containing up to thousands of square feet of area, and described on pages 428 to 433, are used.

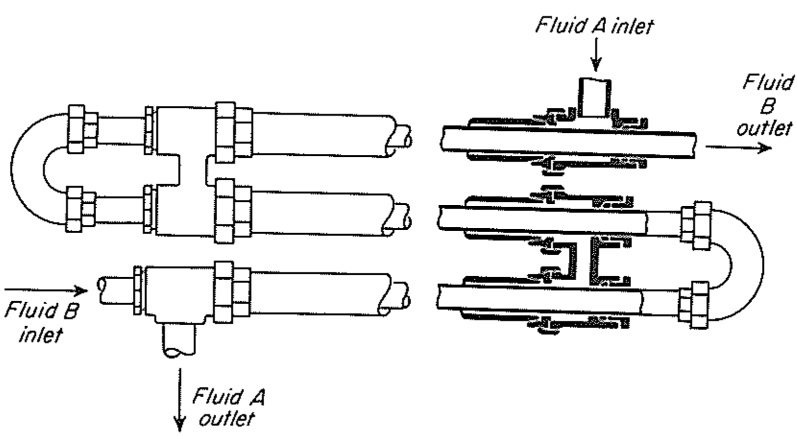

FIGURE 11.3
Double-pipe heat exchanger.

COUNTERCURRENT AND PARALLEL-CURRENT FLOWS. The two fluids enter at different ends of the exchanger shown in Fig. 11.3 and pass in opposite directions through the unit. This type of flow is that commonly used and is called *counterflow* or *countercurrent flow.* The temperature-length curves for this case are shown in Fig. 11.4*a*. The four terminal temperatures are denoted as follows:

Temperature of entering hot fluid, T_{ha}
Temperature of leaving hot fluid, T_{hb}
Temperature of entering cold fluid, T_{ca}
Temperature of leaving cold fluid, T_{cb}

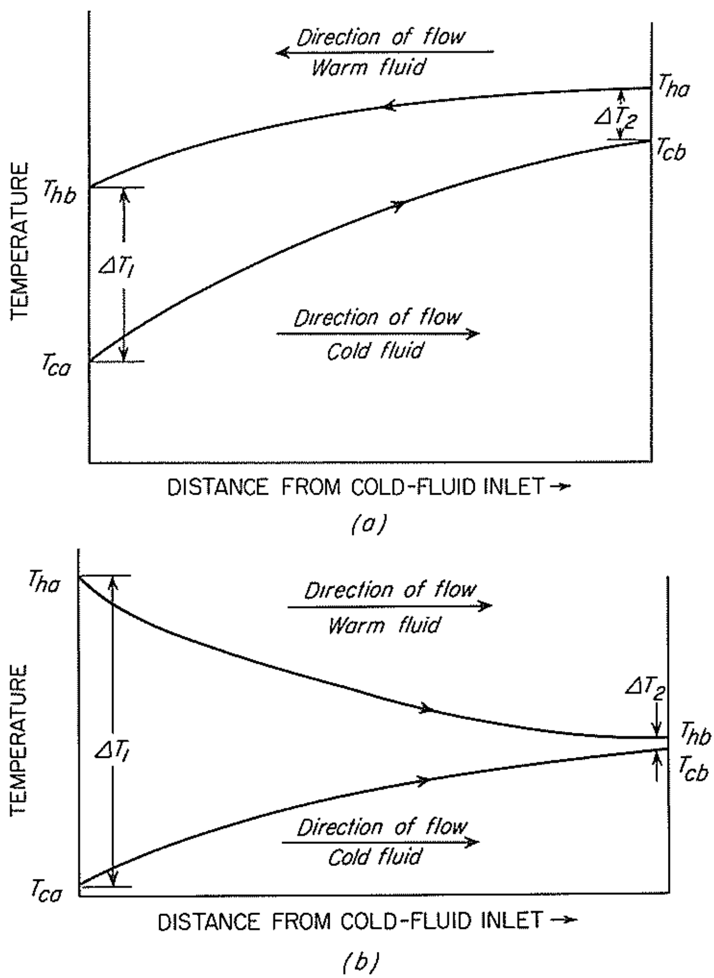

FIGURE 11.4
Temperatures in (*a*) countercurrent and (*b*) parallel flow.

The approaches are

$$T_{ha} - T_{cb} = \Delta T_2 \qquad \text{and} \qquad T_{hb} - T_{ca} = \Delta T_1 \tag{11.1}$$

The warm-fluid and cold-fluid ranges are $T_{ha} - T_{hb}$ and $T_{cb} - T_{ca}$, respectively.

If the two fluids enter at the same end of the exchanger and flow in the same direction to the other end, the flow is called *parallel.* The temperature-length curves for parallel flow are shown in Fig. 11.4*b*. Again, the subscript *a* refers to the entering fluids and subscript *b* to the leaving fluids. The approaches are $\Delta T_1 = T_{ha} - T_{ca}$ and $\Delta T_2 = T_{hb} - T_{cb}$.

Parallel flow is rarely used in a single-pass exchanger such as that shown in Fig. 11.3 because, as inspection of Fig. 11.4*a* and *b* will show, it is not possible with this method of flow to bring the exit temperature of one fluid nearly to the entrance temperature of the other and the heat that can be transferred is less than that possible in countercurrent flow. In the multipass exchangers, described on pages 430 and 431, parallel flow is used in some passes, largely for mechanical reasons, and the capacity and approaches obtainable are thereby affected. Parallel flow is used in special situations where it is necessary to limit the maximum temperature of the cooler fluid or where it is important to change the temperature of at least one fluid rapidly.

In some exchangers one fluid flows across banks of tubes at right angles to the axis of the tubes. This is known as *crossflow.*

ENERGY BALANCES

Quantitative attack on heat-transfer problems is based on energy balances and estimations of rates of heat transfer. Rates of transfer are discussed later in this chapter. Many, perhaps most, heat-transfer devices operate under steady-state conditions, and only this type of operation will be considered here.

ENTHALPY BALANCES IN HEAT EXCHANGERS. In heat exchangers there is no shaft work, and mechanical, potential, and kinetic energies are small in comparison with the other terms in the energy-balance equation. Thus, for one stream through the exchanger

$$\dot{m}(H_b - H_a) = q \tag{11.2}$$

where $\dot{m}$ = flow rate of stream

$q = Q/t$ = rate of heat transfer into stream

H_a, H_b = enthalpies per unit mass of stream at entrance and exit, respectively

Equation (11.2) can be written for each stream flowing through the exchanger.

A further simplification in the use of the heat-transfer rate q is justified. One of the two fluid streams, that outside the tubes, can gain or lose heat by transfer with the ambient air if the fluid is colder or hotter than the ambient. Heat transfer to or from the ambient is not usually desired in practice, and it is usually reduced to a small magnitude by suitable insulation. It is customary to neglect it in comparison with the heat transfer through the walls of the tubes from the warm fluid to the cold fluid, and q is interpreted accordingly.

Accepting the above assumptions, Eq. (11.2) can be written for the warm fluid as

$$\dot{m}_h(H_{hb} - H_{ha}) = q_h \tag{11.3}$$

and for the cold fluid as

$$\dot{m}_c(H_{cb} - H_{ca}) = q_c \tag{11.4}$$

where $\dot{m}_c, \dot{m}_h$ = mass flow rates of cold fluid and warm fluid, respectively
H_{ca}, H_{ha} = enthalpy per unit mass of entering cold fluid and entering warm fluid, respectively
H_{cb}, H_{hb} = enthalpy per unit mass of leaving cold fluid and leaving hot fluid, respectively
q_c, q_h = rates of heat addition to cold fluid and warm fluid, respectively

The sign of q_c is positive, but that of q_h is negative because the warm fluid loses, rather than gains, heat. The heat lost by the warm fluid is gained by the cold fluid, and

$$q_c = -q_h$$

Therefore, from Eqs. (11.3) and (11.4),

$$\dot{m}_h(H_{ha} - H_{hb}) = \dot{m}_c(H_{cb} - H_{ca}) = q \tag{11.5}$$

Equation (11.5) is called the *overall enthalpy balance.*

If only sensible heat is transferred and constant specific heats are assumed, the overall enthalpy balance for a heat exchanger becomes

$$\dot{m}_h c_{ph}(T_{ha} - T_{hb}) = \dot{m}_c c_{pc}(T_{cb} - T_{ca}) = q \tag{11.6}$$

where c_{pc} = specific heat of cold fluid
c_{ph} = specific heat of warm fluid

ENTHALPY BALANCES IN TOTAL CONDENSERS. For a condenser

$$\dot{m}_h \lambda = \dot{m}_c c_{pc}(T_{cb} - T_{ca}) = q \tag{11.7}$$

where $\dot{m}_h$ = rate of condensation of vapor
λ = latent heat of vaporization of vapor

Equation (11.7) is based on the assumption that the vapor enters the condenser as saturated vapor (no superheat) and the condensate leaves at condensing temperature without being further cooled. If either of these sensible-heat effects is important, it must be accounted for by an added term in the left-hand side of Eq. (11.7). For example, if the condensate leaves at a temperature T_{hb} that is less than T_h, the condensing temperature of the vapor, Eq. (11.7) must be written

$$\dot{m}_h[\lambda + c_{ph}(T_h - T_{hb})] = \dot{m}_c c_{pc}(T_{cb} - T_{ca}) \tag{11.8}$$

where c_{ph} is now the specific heat of the condensate.

RATE OF HEAT TRANSFER

HEAT FLUX. Heat-transfer calculations are based on the area of the heating surface and are expressed in watts per square meter or Btu per hour per square foot of surface through which the heat flows. The rate of heat transfer per unit area is called the *heat flux*. In many types of heat-transfer equipment the transfer surfaces are constructed from tubes or pipe. Heat fluxes may then be based on either the inside area or the outside area of the tubes. Although the choice is arbitrary, it must be clearly stated, because the numerical magnitude of the heat fluxes will not be the same for both.

AVERAGE TEMPERATURE OF FLUID STREAM. When a fluid is being heated or cooled, the temperature will vary throughout the cross section of the stream. If the fluid is being heated, the temperature of the fluid is a maximum at the wall of the heating surface and decreases toward the center of the stream. If the fluid is being cooled, the temperature is a minimum at the wall and increases toward the center. Because of these temperature gradients throughout the cross section of the stream, it is necessary, for definiteness, to state what is meant by the temperature of the stream. It is agreed that it is the temperature that would be attained if the entire fluid stream flowing across the section in question were withdrawn and mixed adiabatically to a uniform temperature. The temperature so defined is called the *average* or *mixing-cup stream temperature*. The temperatures plotted in Fig. 11.4 are all average stream temperatures.

Overall Heat-Transfer Coefficient

As shown in Chap. 10, Eqs. (10.5) and (10.9), the heat flux through layers of solids in series is proportional to a driving force, the overall temperature difference ΔT. This also applies to heat flow through liquid layers and solids in series. In a heat exchanger the driving force is taken as $T_h - T_c$, where T_h is the average temperature of the hot fluid and T_c is that of the cold fluid. The quantity $T_h - T_c$ is the *overall local temperature difference* ΔT. It is clear from Fig. 11.4 that ΔT can vary considerably from point to point along the tube, and therefore, since the heat flux is proportional to ΔT, the flux also varies with tube length. It is necessary to start with a differential equation by focusing attention on a differential area dA through which a differential heat flow dq occurs under the driving force of a local value of ΔT. The local flux is then dq/dA and is related to the local value of ΔT by the equation

$$\frac{dq}{dA} = U\,\Delta T = U(T_h - T_c) \tag{11.9}$$

The quantity U, defined by Eq. (11.9) as a proportionality factor between dq/dA and ΔT, is called the *local overall heat-transfer coefficient.*

To complete the definition of U in a given case, it is necessary to specify the area. If A is taken as the outside tube area A_o, U becomes a coefficient based on that area and is written U_o. Likewise, if the inside area A_i is chosen, the coefficient is also based on that area and is denoted by U_i. Since ΔT and dq are independent of the choice of area, it follows that

$$\frac{U_o}{U_i} = \frac{dA_i}{dA_o} = \frac{D_i}{D_o} \tag{11.10}$$

where D_i and D_o are the inside and outside tube diameters, respectively.

INTEGRATION OVER TOTAL SURFACE; LOGARITHMIC MEAN TEMPERATURE DIFFERENCE. To apply Eq. (11.9) to the entire area of a heat exchanger, the equation must be integrated. This can be done formally where certain simplifying assumptions are accepted. The assumptions are (1) the overall coefficient U is constant, (2) the specific heats of the hot and cold fluids are constant, (3) heat exchange with the ambient is negligible, and (4) the flow is steady and either parallel or countercurrent, as shown in Fig. 11.4.

The most questionable of these assumptions is that of a constant overall coefficient. The coefficient does in fact vary with the temperatures of the fluids, but its change with temperature is gradual, so that when the temperature ranges are moderate, the assumption of constant U is not seriously in error.

Assumptions 2 and 4 imply that if T_c and T_h are plotted against q, as shown in Fig. 11.5, straight lines are obtained. Since T_c and T_h vary linearly with q, ΔT

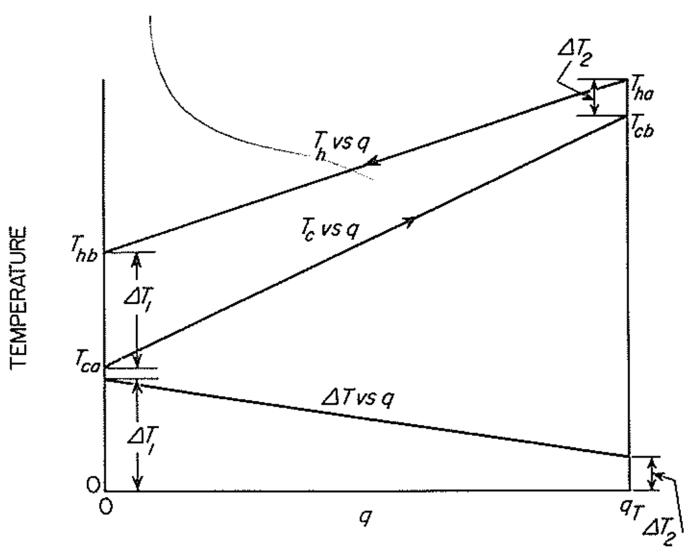

FIGURE 11.5
Temperature vs. heat flow rate in countercurrent flow.

does likewise and $d(\Delta T)/dq$, the slope of the graph of ΔT vs. q, is constant. Therefore

$$\frac{d(\Delta T)}{dq} = \frac{\Delta T_2 - \Delta T_1}{q_T} \tag{11.11}$$

where $\Delta T_1, \Delta T_2$ = approaches
q_T = rate of heat transfer in entire exchanger

Elimination of dq from Eqs. (11.9) and (11.11) gives

$$\frac{d(\Delta T)}{U\,\Delta T\,dA} = \frac{\Delta T_2 - \Delta T_1}{q_T} \tag{11.12}$$

The variables ΔT and A can be separated, and if U is constant, the equation can be integrated over the limits A_T and 0 for A and ΔT_2 and ΔT_1, where A_T is the total area of the heat-transfer surface. Thus

$$\int_{\Delta T_1}^{\Delta T_2} \frac{d(\Delta T)}{\Delta T} = \frac{U(\Delta T_2 - \Delta T_1)}{q_T} \int_0^{A_T} dA$$

or

$$\ln \frac{\Delta T_2}{\Delta T_1} = \frac{U(\Delta T_2 - \Delta T_1)}{q_T} A_T \tag{11.13}$$

Equation (11.13) can be written

$$q_T = UA_T \frac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2/\Delta T_1)} = UA_T \overline{\Delta T_L} \tag{11.14}$$

where

$$\overline{\Delta T_L} = \frac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2/\Delta T_1)} \tag{11.15}$$

Equation (11.15) defines the *logarithmic mean temperature difference* (LMTD). It is of the same form as that of Eq. (10.15) for the logarithmic mean radius of a thick-walled tube. When ΔT_1 and ΔT_2 are nearly equal, their arithmetic average can be used for $\overline{\Delta T_L}$ within the same limits of accuracy given for Eq. (10.15), as shown in Fig. 10.4.

If one of the fluids is at constant temperature, as in a condenser, no difference exists among countercurrent flow, parallel flow, or multipass flow, and Eq. (11.15) applies to all of them. In countercurrent flow, ΔT_2, the warm-end approach, may be less than ΔT_1, the cold-end approach. In this case, for convenience and to eliminate negative numbers and logarithms, the subscripts in Eq. (11.15) may be interchanged. The LMTD is not always the correct mean temperature difference to use. It should *not* be used when U changes appreciably or when ΔT is not a linear function of q. As an example, consider an exchanger used to cool and condense a superheated vapor, with the temperature diagram shown in Fig. 11.6.

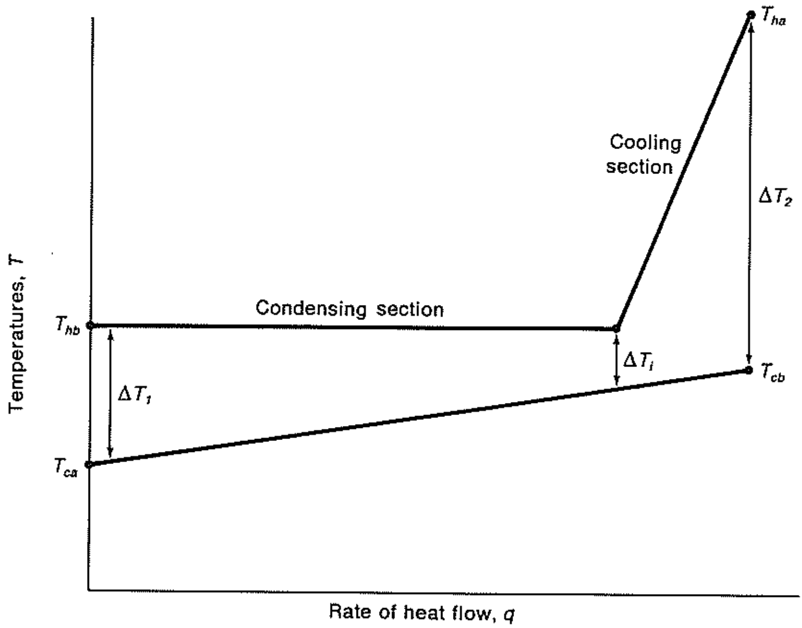

FIGURE 11.6
Temperature profiles in cooling and condensing superheated vapor.

The ΔT driving force is a linear function of q while the vapor is being cooled, but ΔT is a different linear function of q in the condensing section of the exchanger. Furthermore, U is not the same in the two parts of the exchanger. The cooling and condensing sections must be sized separately using the appropriate values of q, U, and LMTD rather than some kind of average U and an overall LMTD.

The LMTD is also incorrect when heat is transferred to or from a reacting fluid in a jacketed reactor. Figure 11.7 shows the temperature profiles for an exothermic reaction in a water-cooled reactor—the lower line shows the temperature of the coolant, the upper line that of the reacting mixture. Because of heat generated by the reaction, the reactant temperature rises rapidly near the reactor inlet, and then, as the reaction slows, the reactant temperature drops. The ΔT's

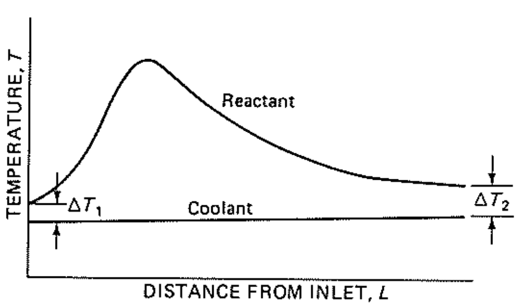

FIGURE 11.7
Temperature patterns in jacketed tubular reactor.

at both the reactor inlet and outlet are relatively small. Clearly the average temperature drop is much greater than the drop at either end of the reactor and cannot be found from the logarithmic mean of the terminal ΔT's.

VARIABLE OVERALL COEFFICIENT. When the overall coefficient varies regularly, the rate of heat transfer may be predicted from Eq. (11.16), which is based on the assumption that U varies linearly with the temperature drop over the entire heating surface[1]:

$$q_T = A_T \frac{U_2 \, \Delta T_1 - U_1 \, \Delta T_2}{\ln (U_2 \, \Delta T_1 / U_1 \, \Delta T_2)} \tag{11.16}$$

where U_1, U_2 = local overall coefficients at ends of exchanger
$\Delta T_1, \Delta T_2$ = temperature approaches at corresponding ends of exchanger

Equation (11.16) calls for use of a logarithmic mean value of the $U \Delta T$ cross product, where the overall coefficient at one end of the exchanger is multiplied by the temperature approach at the other. The derivation of this equation requires that assumptions 2 to 4 above be accepted.

In the completely general case, where none of the assumptions is valid and U varies markedly from point to point, Eq (11.9) can be integrated by evaluating local values of U, ΔT, and q at several intermediate points in the exchanger. Graphical or numerical evaluation of the area under a plot of $1/U \Delta T$ vs. q, between the limits of zero and q_T, will then give the area A_T of the heat-transfer surface required.

MULTIPASS EXCHANGERS. In multipass shell-and-tube exchangers the flow pattern is complex, with parallel, countercurrent, and crossflow all present. Under these conditions, even when the overall coefficient U is constant, the LMTD cannot be used. Calculation procedures for multipass exchangers are given in Chap. 15.

Individual Heat-Transfer Coefficients

The overall coefficient depends upon so many variables that it is necessary to break it into its parts. The reason for this becomes apparent if a typical case is examined. Consider the local overall coefficient at a specific point in the double-pipe exchanger shown in Fig. 11.3. For definiteness, assume that the warm fluid is flowing through the inside pipe and that the cold fluid is flowing through the annular space. Assume also that the Reynolds numbers of the two fluids are sufficiently large to ensure turbulent flow and that both surfaces of the inside tube are clear of dirt or scale. If, now, a plot is prepared, as shown in Fig. 11.8, with temperature as the ordinate and distance perpendicular to the wall as the abscissa, several important facts become evident. In the figure, the metal wall of the tube separates the warm fluid on the right from the cold fluid on the left. The change in temperature with distance is shown by the line $T_a T_b T_{wh} T_{wc} T_e T_g$. The temperature profile is thus divided into three separate parts, one through each of the two

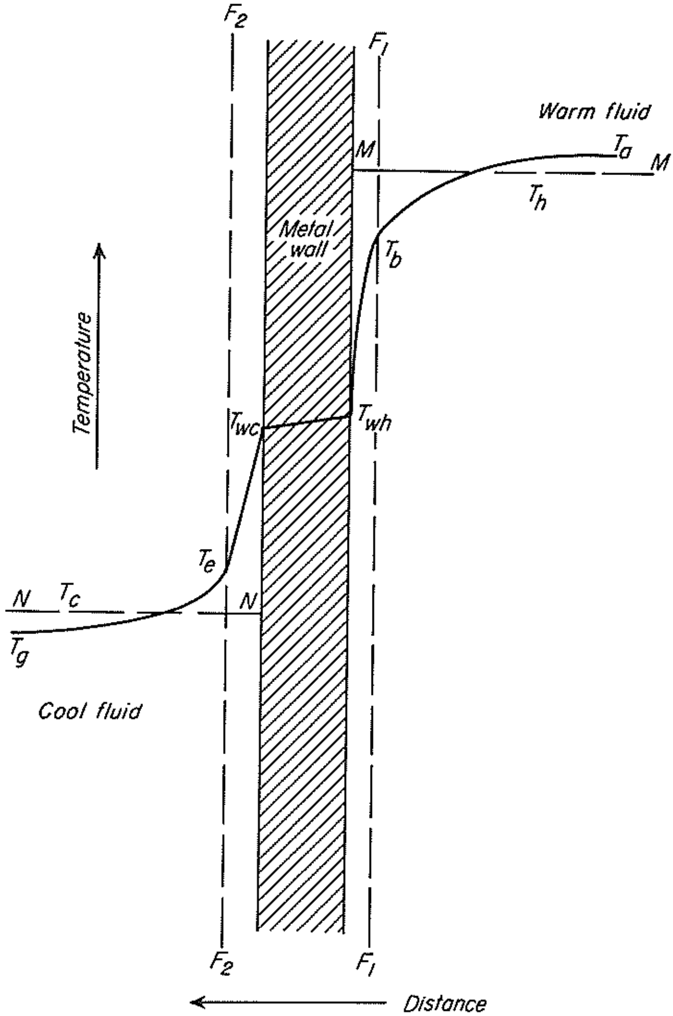

FIGURE 11.8
Temperature gradients in forced convection.

fluids and the other through the metal wall. The overall effect, therefore, should be studied in terms of these individual parts.

It was shown in Chap. 5 that in turbulent flow through conduits three zones exist, even in a single fluid, so that the study of one fluid is, itself, complicated. In each fluid shown in Fig. 11.8 there is a thin sublayer at the wall, a turbulent core occupying most of the cross section of the stream, and a buffer zone between them. The velocity gradients were described in Chap. 5. The velocity gradient is large near the wall, small in the turbulent core, and in rapid change in the buffer zone.

It has been found that the temperature gradient in a fluid being heated or cooled when flowing in turbulent flow follows much the same course. The temperature gradient is large at the wall and through the viscous sublayer, small in the turbulent core, and in rapid change in the buffer zone. Basically, the reason for this is that heat must flow through the viscous sublayer by conduction, which calls for a steep temperature gradient in most fluids because of the low thermal conductivity, whereas the rapidly moving eddies in the core are effective in equalizing the temperature in the turbulent zone. In Fig. 11.8 the dashed lines F_1F_1 and F_2F_2 represent the boundaries of the viscous sublayers.

The average temperature of the warm stream is somewhat less than the maximum temperature T_a and is represented by the horizontal line MM, which is drawn at temperature T_h. Likewise, line NN, drawn at temperature T_c, represents the average temperature of the cold fluid.

The overall resistance to the flow of heat from the warm fluid to the cold fluid is a result of three separate resistances operating in series. Two resistances are those offered by the individual fluids, and the third is that of the solid wall. In general, also, as shown in Fig. 11.8, the wall resistance is small in comparison with that of the fluids. The overall coefficient is best studied by analyzing it in terms of the separate resistances and treating each separately. The separate resistances can then be combined to form the overall coefficient. This approach requires the use of individual heat-transfer coefficients for the two fluid streams.

The individual, or surface, heat-transfer coefficient h is defined generally by the equation

$$h = \frac{dq/dA}{T - T_w} \tag{11.17}$$

where dq/dA = local heat flux, based on the area in contact with fluid
T = local average temperature of fluid
T_w = temperature of wall in contact with fluid

The reciprocal of this coefficient, $1/h$, is called a thermal resistance. For conduction through a solid, such as a metal wall of thickness x_w and thermal conductivity k, the thermal resistance equals x_w/k. Appropriately corrected for changes in area, the individual resistances may be added to give the overall resistance $1/U$.

A second expression for h is derived from the assumption that there are no velocity fluctuations normal to the wall at the surface of the wall itself. The mechanism of heat transfer at the wall is then by conduction, and the heat flux is given by Eq. (10.2), noting that the normal distance n may be replaced by y, the normal distance measured into the fluid from the wall in the direction of the flow of heat. Thus

$$\frac{dq}{dA} = -k\left(\frac{dT}{dy}\right)_w \tag{11.18}$$

The subscript w calls attention to the fact that the gradient must be evaluated at the wall. Eliminating dq/dA from Eqs. (11.17) and (11.18) gives

$$h = -k \frac{(dT/dy)_w}{T - T_w} \tag{11.19}$$

Note that h must always be positive. Equation (11.19) can be put into a dimensionless form by multiplying by the ratio of an arbitrary length to the thermal conductivity. The choice of length depends on the situation. For heat transfer at the inner surface of a tube, the tube diameter D is the usual choice. Multiplying Eq. (11.19) by D/k gives

$$\frac{hD}{k} = -D \frac{(dT/dy)_w}{T - T_w} \tag{11.20}$$

On the cold-fluid side of the tube wall $T < T_w$, and the denominator in Eqs. (11.19) and (11.20) becomes $T_w - T$. The dimensionless group hD/k is called a Nusselt number N_{Nu}. That shown in Eq. (11.20) is a local Nusselt number based on diameter. The physical meaning of the Nusselt number can be seen by inspection of the right-hand side of Eq. (11.20). The numerator $(dT/dy)_w$ is, of course, the gradient at the wall. The factor $(T - T_w)/D$ can be considered the average temperature gradient across the entire pipe, and the Nusselt number is the ratio of these two gradients.

Another interpretation of the Nusselt number can be obtained by considering the gradient that would exist if all the resistance to heat transfer were in a laminar layer of thickness x in which heat transfer was only by conduction. The heat-transfer rate and coefficient follow from Eqs. (10.1) and (11.17):

$$\frac{dq}{dA} = \frac{k(T - T_w)}{x} \tag{11.21}$$

$$h = \frac{k}{x} \tag{11.22}$$

From the definition of the Nusselt number,

$$\frac{hD}{k} = N_{\mathrm{Nu}} = \frac{k}{x}\frac{D}{k} = \frac{D}{x} \tag{11.23}$$

The Nusselt number is the ratio of the tube diameter to the equivalent thickness of the laminar layer. Sometimes x is called the film thickness, and it is generally slightly greater than the thickness of the laminar boundary layer because there is some resistance to heat transfer in the buffer zone.

Equation (11.17), when applied to the two fluids of Fig. 11.8, becomes, for the inside of the tube (the warm side in Fig. 11.8),

$$h_i = \frac{dq/dA_i}{T_h - T_{wh}} \tag{11.24}$$

and for the outside of the tube (the cold side)

$$h_o = \frac{dq/dA_o}{T_{wc} - T_c} \tag{11.25}$$

where A_i and A_o are the inside and outside areas of the tube, respectively.

The cold fluid could, of course, be inside the tubes and the warm fluid outside. Coefficients h_i and h_o refer to the *inside* and the *outside* of the tube, respectively, and not to a specific fluid.

CALCULATION OF OVERALL COEFFICIENTS FROM INDIVIDUAL COEFFICIENTS. The overall coefficient is constructed from the individual coefficients and the resistance of the tube wall in the following manner.

The rate of heat transfer through the tube wall is given by the differential form of Eq. (10.13),

$$\frac{dq}{d\overline{A}_L} = \frac{k_m(T_{wh} - T_{wc})}{x_w} \tag{11.26}$$

where $T_{wh} - T_{wc}$ = temperature difference through tube wall
k_m = thermal conductivity of wall
x_w = tube-wall thickness
$dq/d\overline{A}_L$ = local heat flux, based on logarithmic mean of inside and outside areas of tube

If Eqs. (11.24) to (11.26) are solved for the temperature differences and the temperature differences added, the result is

$$\begin{aligned}(T_h - T_{wh}) + (T_{wh} - T_{wc}) + (T_{wc} - T_c) &= T_h - T_c = \Delta T \\ &= dq\left(\frac{1}{dA_i h_i} + \frac{x_w}{d\overline{A}_L k_m} + \frac{1}{dA_o h_o}\right)\end{aligned} \tag{11.27}$$

Assume that the heat-transfer rate is arbitrarily based on the outside area. If Eq. (11.27) is solved for dq, and if both sides of the resulting equation are divided by dA_o, the result is

$$\frac{dq}{dA_o} = \frac{T_h - T_c}{\dfrac{1}{h_i}\left(\dfrac{dA_o}{dA_i}\right) + \dfrac{x_w}{k_m}\left(\dfrac{dA_o}{d\overline{A}_L}\right) + \dfrac{1}{h_o}} \tag{11.28}$$

Now

$$\frac{dA_o}{dA_i} = \frac{D_o}{D_i} \qquad \text{and} \qquad \frac{dA_o}{d\overline{A}_L} = \frac{D_o}{\overline{D}_L}$$

where D_o, D_i, and $\overline{D}_L$ are the outside, inside, and logarithmic mean diameters of the tube, respectively. Therefore

$$\frac{dq}{dA_o} = \frac{T_h - T_c}{\dfrac{1}{h_i}\left(\dfrac{D_o}{D_i}\right) + \dfrac{x_w}{k_m}\left(\dfrac{D_o}{\overline{D}_L}\right) + \dfrac{1}{h_o}} \tag{11.29}$$

Comparing Eq. (11.9) with Eq. (11.29) shows that

$$U_o = \frac{1}{\dfrac{1}{h_i}\left(\dfrac{D_o}{D_i}\right) + \dfrac{x_w}{k_m}\left(\dfrac{D_o}{\overline{D}_L}\right) + \dfrac{1}{h_o}} \tag{11.30}$$

If the inside area A_i is chosen as the base area, division of Eq. (11.27) by dA_i gives for the overall coefficient

$$U_i = \frac{1}{\dfrac{1}{h_i} + \dfrac{x_w}{k_m}\left(\dfrac{D_i}{\overline{D}_L}\right) + \dfrac{1}{h_o}\left(\dfrac{D_i}{D_o}\right)} \tag{11.31}$$

RESISTANCE FORM OF OVERALL COEFFICIENT. A comparison of Eqs. (10.9) and (11.30) suggests that the reciprocal of an overall coefficient can be considered to be an overall resistance composed of three resistances in series. The total, or overall, resistance is given by the equation

$$\frac{1}{U_o} = \frac{D_o}{D_i h_i} + \frac{x_w}{k_m}\frac{D_o}{\overline{D}_L} + \frac{1}{h_o} \tag{11.32}$$

The individual terms on the right-hand side of Eq. (11.32) represent the individual resistances of the two fluids and of the metal wall. The overall temperature drop is proportional to $1/U$, and the temperature drops in the two fluids and the wall are proportional to the individual resistances, or, for the case of Eq. (11.32),

$$\frac{\Delta T}{1/U_o} = \frac{\Delta T_i}{D_o/D_i h_i} = \frac{\Delta T_w}{(x_w/k_m)(D_o/\overline{D}_L)} = \frac{\Delta T_o}{1/h_o} \tag{11.33}$$

where ΔT = overall temperature drop
ΔT_i = temperature drop through inside fluid
ΔT_w = temperature drop through metal wall
ΔT_o = temperature drop through outside fluid

FOULING FACTORS. In actual service, heat-transfer surfaces do not remain clean. Scale, dirt, and other solid deposits form on one or both sides of the tubes, provide additional resistances to heat flow, and reduce the overall coefficient. The effect of such deposits is taken into account by adding a term $1/dA\, h_d$ to the term in parentheses in Eq. (11.27) for each scale deposit. Thus, assuming that scale is deposited on both the inside and the outside surface of the tubes, Eq. (11.27) becomes, after correction for the effects of scale,

$$\Delta T = dq\left(\frac{1}{dA_i h_{di}} + \frac{1}{dA_i h_i} + \frac{x_w}{d\overline{A}_L k_m} + \frac{1}{dA_o h_o} + \frac{1}{dA_o h_{do}}\right) \tag{11.34}$$

where h_{di} and h_{do} are the *fouling factors* for the scale deposits on the inside and outside tube surfaces, respectively. The following equations for the overall coefficients based on outside and inside areas, respectively, follow from Eq. (11.34):

$$U_o = \frac{1}{(D_o/D_i h_{di}) + (D_o/D_i h_i) + (x_w/k_w)(D_o/\overline{D}_L) + (1/h_o) + (1/h_{do})} \tag{11.35}$$

and

$$U_i = \frac{1}{(1/h_{di}) + (1/h_i) + (x_w/k_m)(D_i/\overline{D}_L) + (D_i/D_o h_o) + (D_i/D_o h_{do})} \tag{11.36}$$

The actual thicknesses of the deposits are neglected in Eqs. (11.35) and (11.36).

Numerical values of fouling factors are given in Ref. 3 corresponding to satisfactory performance in normal operation, with reasonable service time between cleanings. They cover a range of approximately 600 to 11,000 W/m^2-°C (100 to 2000 Btu/ft^2-h-°F). Fouling factors for ordinary industrial liquids fall in the range 1700 to 6000 W/m^2-°C (300 to 1000 Btu/ft^2-h-°F). Fouling factors are usually set at values that also provide a safety factor for design.

Example 11.1. Methyl alcohol flowing in the inner pipe of a double-pipe exchanger is cooled with water flowing in the jacket. The inner pipe is made from 1-in. (25-mm) Schedule 40 steel pipe. The thermal conductivity of steel is 26 Btu/ft-h-°F (45 W/m-°C). The individual coefficients and fouling factors are given in Table 11.1. What is the overall coefficient, based on the outside area of the inner pipe?

Solution

The diameters and wall thickness of 1-in. Schedule 40 pipe, from Appendix 5, are

$$D_i = \frac{1.049}{12} = 0.0874 \text{ ft} \qquad D_o = \frac{1.315}{12} = 0.1096 \text{ ft} \qquad x_w = \frac{0.133}{12} = 0.0111 \text{ ft}$$

The logarithmic mean diameter $\overline{D}_L$ is calculated as in Eq. (10.15) using diameter in place of radius:

$$\overline{D}_L = \frac{D_o - D_i}{\ln (D_o/D_i)} = \frac{0.1096 - 0.0874}{\ln (0.1096/0.0874)} = 0.0983 \text{ ft}$$

TABLE 11.1
Data for Example 11.1

	Coefficient	
	Btu/ft^2-h-°F	**W/m^2-°C**
Alcohol coefficient h_i	180	1020
Water coefficient h_o	300	1700
Inside fouling factor h_{di}	1000	5680
Outside fouling factor h_{do}	500	2840

The overall coefficient is found from Eq. (11.35):

$$U_o = \frac{1}{\dfrac{0.1096}{0.0874 \times 1000} + \dfrac{0.1096}{0.0874 \times 180} + \dfrac{0.0111 \times 0.1096}{26 \times 0.0983} + \dfrac{1}{300} + \dfrac{1}{500}}$$

$= 71.3$ Btu/ft²-h-°F (405 W/m²-°C)

SPECIAL CASES OF THE OVERALL COEFFICIENT. Although the choice of area to be used as the basis of an overall coefficient is arbitrary, sometimes one particular area is more convenient than others. Suppose, for example, that one individual coefficient, h_i, is large numerically in comparison with the other, h_o, and that fouling effects are negligible. Also, assuming the term representing the resistance of the metal wall is small in comparison with $1/h_o$, the ratios D_o/D_i and $D_o/\overline{D}_L$ have so little significance that they can be disregarded, and Eq. (11.30) can be replaced by the simpler form

$$U_o = \frac{1}{1/h_o + x_w/k_m + 1/h_i} \tag{11.37}$$

In such a case it is advantageous to base the overall coefficient on that area that corresponds to the largest resistance, or the lowest value of h.

For large-diameter thin-walled tubes, flat plates or any other case where a negligible error is caused by using a common area for A_i, $\overline{A}_L$, and A_o, Eq. (11.37) can be used for the overall coefficient, and U_i and U_o are identical.

Sometimes one coefficient, say, h_o, is so very small in comparison with both x_w/k and the other coefficient h_i that the term $1/h_o$ is very large compared with the other terms in the resistance sum. When this is true, the larger resistance is called the *controlling resistance*, and it is sufficiently accurate to equate the overall coefficient to the small individual coefficient, or in this case, $h_o = U_o$.

CLASSIFICATION OF INDIVIDUAL HEAT-TRANSFER COEFFICIENTS. The problem of predicting the rate of heat flow from one fluid to another through a retaining wall reduces to the problem of predicting the numerical values of the individual coefficients of the fluids concerned in the overall process. A wide variety of individual cases is met in practice, and each type of phenomenon must be considered separately. The following classification will be followed in this text:

1. Heat flow to or from fluids inside tubes, without phase change
2. Heat flow to or from fluids outside tubes, without phase change
3. Heat flow from condensing fluids
4. Heat flow to boiling liquids

MAGNITUDE OF HEAT-TRANSFER COEFFICIENTS. The ranges of values covered by the coefficient h vary greatly, depending upon the character of the process.[2] Some typical ranges are shown in Table 11.2.

TABLE 11.2
Magnitudes of heat-transfer coefficients†

Type of processes	Range of values of *h*	
	W/m²-°C	Btu/ft²-h-°F
Steam (dropwise condensation)	30,000–100,000	5000–20,000
Steam (film-type condensation)	6000–20,000	1000–3000
Boiling water	1700–50,000	300–9000
Condensing organic vapors	1000–2000	200–400
Water (heating or cooling)	300–20,000	50–3000
Oils (heating or cooling)	50–1500	10–300
Steam (superheating)	30-100	5–20
Air (heating or cooling)	1–50	0.2–10

† By permission of author and publisher from W. H. McAdams, *Heat Transmission*, 3rd ed., p. 5. Copyright by author, 1954, McGraw-Hill Book Company.

Effective Coefficients for Unsteady-State Heat Transfer

In some cases it is convenient to treat unsteady-state heat transfer using an effective coefficient rather than the exact equations or plots such as Fig. 10.6. For example, the rate of heat transfer to a spherical particle can be approximated using an internal coefficient equal to $5k/r_m$ and the external area $4\pi r_m^2$. The unsteady-state heat balance for a sphere then becomes

$$\rho c_p(\tfrac{4}{3}\pi r_m^3)\frac{d\overline{T}_b}{dt} = h_i A\, \Delta T$$

$$= \frac{5k}{r_m}(4\pi r_m^2)(T_s - \overline{T}_b) \tag{11.38}$$

After rearranging and integrating, with T_a the initial temperature,

$$\ln\left(\frac{T_s - T_a}{T_s - \overline{T}_b}\right) = \frac{15kt}{\rho c_p r_m^2} = \frac{15\alpha t}{r_m^2} \tag{11.39}$$

Equation (11.39) is fairly close to the exact solution, Eq. (10.19), when $\alpha t/r_m^2$ is greater than 0.1. One advantage of using an effective coefficient for the internal heat transfer is that the effect of external resistance is easily accounted for by making use of an overall coefficient. Thus for a sphere being heated with air at temperature T_h,

$$q = UA(T_h - \overline{T}_b) \tag{11.40}$$

where $A = 4\pi r_m^2$
$1/U = (1/h_o) + (r_m/5k)$

SYMBOLS

A	Area m^2 or ft^2; A_T, total area of heat-transfer surface; A_i, of inside of tube; A_o, of outside of tube; $\overline{A}_L$, logarithmic mean
c_p	Specific heat at constant pressure, J/g-°C or Btu/lb-°F; c_{pc}, of cool fluid; c_{ph}, of warm fluid
D	Diameter, m or ft; D_i, inside diameter of tube; D_o, outside diameter of tube; $\overline{D}_L$, logarithmic mean
H	Enthalpy, J/g or Btu/lb; H_a, at entrance; H_b, at exit; H_{ca}, H_{cb}, of cool fluid; H_{ha}, H_{hb}, of warm fluid
h	Individual or surface heat-transfer coefficient, W/m^2-°C or Btu/ft^2-h-°F; h_i, for inside of tube; h_o, for outside of tube
h_d	Fouling factor, W/m^2-°C or Btu/ft^2-h-°F; h_{di}, inside tube; h_{do}, outside tube
k	Thermal conductivity, W/m-°C or Btu/ft-h-°F; k_m, of tube wall
L	Length, m or ft
$\dot{m}$	Mass flow rate, kg/h or lb/h; $\dot{m}_c$, of cool fluid; $\dot{m}_h$, of warm fluid
N_{Nu}	Nusselt number, hD/k, dimensionless
Q	Quantity of heat, J or Btu
q	Heat flow rate, W or Btu/h; q_T, total in exchanger; q_c, to cool fluid; q_h, to warm fluid
r_m	Radius of spherical particle, m or ft
T	Temperature, °F or °C; T_a, at inlet, or initial value; T_b, at outlet; T_c, of cool fluid; T_{ca}, at cool-fluid inlet; T_{cb}, at cool-fluid outlet; T_h, of warm fluid; T_{ha}, at warm-fluid inlet; T_{hb}, at warm-fluid outlet; T_s, of surface; T_w, of tube wall; T_{wc}, on cool-fluid side; T_{wh}, on warm-fluid side; $\overline{T}_b$, bulk average temperature of solid sphere
t	Time, h or s
U	Overall heat-transfer coefficient, W/m^2-°C or Btu/ft^2-h-°F; U_i, based on inside surface area; U_o, based on outside surface area; U_1, U_2, at ends of exchanger
x	Film thickness, m or ft [Eqs. (11.21) to (11.23)]
x_w	Thickness of tube wall, m or ft
y	Distance into fluid normal to tube wall, m or ft, measured in direction of heat flow

Greek letters

α	Thermal diffusivity, $k/\rho c_p$, m^2/s or ft^2/h
ΔT	Overall temperature difference, $T_h - T_c$, °C or °F; ΔT_i, between tube wall and fluid inside tube; ΔT_o, between tube wall and fluid outside tube; ΔT_w, through the tube wall; ΔT_1, ΔT_2, at ends of exchanger; $\overline{\Delta T}_L$, logarithmic mean
λ	Latent heat of vaporization, J/g or Btu/lb
ρ	Density of spherical particle, kg/m^3 or lb/ft^3

PROBLEMS

11.1. Calculate the overall heat-transfer coefficients based on both inside and outside areas for the following cases.

Case 1 Water at 10°C flowing in a $\frac{3}{4}$-in. 16 BWG condenser tube and saturated steam at 105°C condensing on the outside. $h_i = 12$ kW/m²-°C. $h_o = 14$ kW/m²-°C. $k_m =$ 120 W/m-°C.

Case 2 Benzene condensing at atmospheric pressure on the outside of a 25-mm steel pipe and air at 15°C flowing within at 6 m/s. The pipe wall is 3.5 mm thick. $h_i = 20$ W/m²-°C. $h_o = 1200$ W/m²-°C. $k_m = 45$ W/m-°C.

Case 3 Dropwise condensation from steam at a pressure of 50 $\text{lb}_f/\text{in.}^2$ gauge on a 1-in. Schedule 40 steel pipe carrying oil at 100°F. $h_o = 14{,}000$ Btu/ft²-h-°F. $h_i =$ 130 Btu/ft²-h-°F. $k_m = 26$ Btu/ft-h-°F.

11.2. Calculate the temperatures of the inside and outside surfaces of the metal pipe or tubing in cases 1 to 3 of Prob. 11.1.

11.3. Aniline is to be cooled from 200 to 150°F in a double-pipe heat exchanger having a total outside area of 70 ft². For cooling, a stream of toluene amounting to 8600 lb/h at a temperature of 100°F is available. The exchanger consists of $1\frac{1}{4}$-in. Schedule 40 pipe in 2-in. Schedule 40 pipe. The aniline flow rate is 10,000 lb/h. (*a*) If flow is countercurrent, what are the toluene outlet temperature, the LMTD, and the overall heat-transfer coefficient? (*b*) What are they if flow is parallel?

11.4. In the exchanger described in Prob. 11.3, how much aniline can be cooled if the overall heat-transfer coefficient is 70 Btu/ft²-h-°F?

11.5. Carbon tetrachloride flowing at 19,000 kg/h is to be cooled from 85 to 40°C using 13,500 kg/h of cooling water at 20°C. The film coefficient for carbon tetrachloride, outside the tubes, is 1700 W/m²-°C. The wall resistance is negligible, but h_i on the water side, including fouling factors, is 11,000 W/m²-°C. (*a*) What area is needed for a counterflow exchanger? (*b*) By what factor would the area be increased if parallel flow were used to get more rapid initial cooling of the carbon tetrachloride?

REFERENCES

1. Colburn, A. P.: *Ind. Eng. Chem.*, **25**:873 (1933).
2. McAdams, W. H.: *Heat Transmission*, 3rd ed., McGraw-Hill, New York, 1954, p. 5.
3. Perry, J. H. (ed.): *Chemical Engineers' Handbook*, 6th ed., McGraw-Hill, New York, 1984, p. **10**-43.

CHAPTER 12

HEAT TRANSFER TO FLUIDS WITHOUT PHASE CHANGE

In a great many applications of heat exchange, heat is transferred between fluid streams without any phase change in the fluids. This is especially important in heat recovery operations, as when the hot effluent from an exothermic reactor is used to preheat the incoming cooler feed. Other examples include the transfer of heat from a stream of hot gas to cooling water, and the cooling of a hot liquid stream by air. In such situations the two streams are separated by a metal wall, which constitutes the heat-transfer surface. The surface may consist of tubes or other channels of constant cross section, of flat plates, or in such devices as jet engines and advanced power machinery, of special shapes designed to pack a maximum area of transfer surface into a small volume.

Most fluid-to-fluid heat transfer is accomplished in steady-state equipment, but thermal regenerators, in which a bed of solid shapes is alternately heated by a hot fluid and the hot shapes then used to warm a colder fluid, are also used, especially in high-temperature heat transfer. Cyclical unsteady-state processes such as these are not considered in this book.

REGIMES OF HEAT TRANSFER IN FLUIDS. A fluid being heated or cooled may be flowing in laminar flow, in turbulent flow, or in the transition range between laminar and turbulent flow. Also, the fluid may be flowing in forced or natural convection. In some instances more than one flow type may occur in the same stream; for instance, in laminar flow at low velocities, natural convection may be superimposed on forced laminar flow.

The direction of flow of the fluid may be parallel to that of the heating surface, so that boundary-layer separation does not occur, or the direction of flow may be perpendicular or at an angle to the heating surface, and then boundary-layer separation does take place.

At ordinary velocities the heat generated from fluid friction is usually negligible in comparison with the heat transferred between the fluids. In most cases frictional heating may be neglected. It may be important, however, in operations involving very viscous fluids such as the injection molding of polymers. In gas flow at high velocities, at Mach numbers above a few tenths, frictional heat becomes appreciable and cannot be ignored. At very high velocities frictional heating may become of controlling importance.

Because the conditions of flow at the entrance to a tube differ from those well downstream from the entrance, the velocity field and the associated temperature field may depend on the distance from the tube entrance. Also, in some situations the fluid flows through a preliminary length of unheated or uncooled pipe so that the fully developed velocity field is established before heat is transferred to the fluid, and the temperature field is created within an existing velocity field.

Finally, the properties of the fluid—viscosity, thermal conductivity, specific heat, and density—are important parameters in heat transfer. Each of these, especially viscosity, is temperature dependent. Since the temperature varies from point to point in a flowing stream undergoing heat transfer, a problem appears in the choice of temperature at which the properties should be evaluated. For small temperature differences between fluid and wall and for fluids with weak dependence of viscosity on temperature, the problem is not acute, but for highly viscous fluids such as heavy petroleum oils or where the temperature difference between the tube wall and the fluid is large, the variations in fluid properties within the stream become large, and the difficulty of calculating the heat-transfer rate is increased.

Because of the various effects noted above, the entire subject of heat transfer to fluids without phase change is complex and in practice is treated as a series of special cases rather than as a general theory. All cases considered in this chapter do, however, have a phenomenon in common: in all of them the formation of a thermal boundary layer, analogous to the hydrodynamic Prandtl boundary layer described in Chap. 3, takes place; it profoundly influences the temperature field and so controls the rate of heat flow.

THERMAL BOUNDARY LAYER. Consider a flat plate immersed in a stream of fluid in steady flow and oriented parallel to the plate, as shown in Fig. 12.1*a*. Assume that the stream approaching the plate does so at velocity u_0 and temperature T_∞ and that the surface of the plate is maintained at a constant

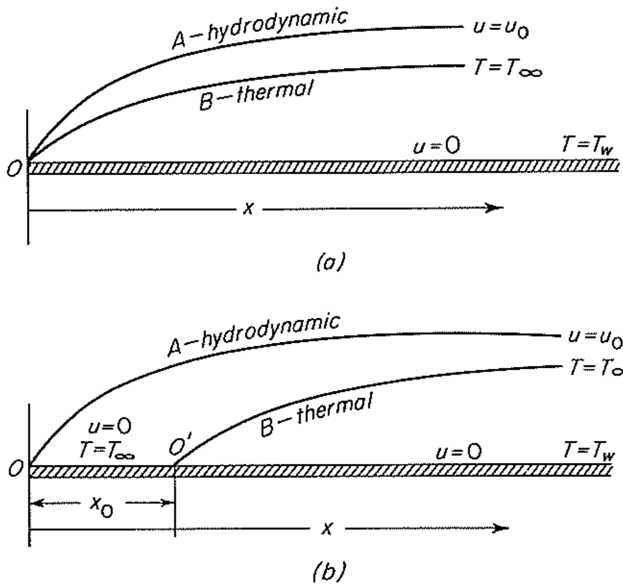

FIGURE 12.1
Thermal and hydrodynamic boundary layers on flat plate: (*a*) entire plate heated; (*b*) unheated length = x_0.

temperature T_w. Assume that T_w is greater than T_∞, so that the fluid is heated by the plate. As described in Chap. 3, a boundary layer develops within which the velocity varies from $u = 0$ at the wall to $u = u_0$ at the outer boundary of the layer. This boundary layer, called the *hydrodynamic boundary layer*, is shown by line *OA* in Fig. 12.1*a*. The penetration of heat by transfer from the plate to the fluid changes the temperature of the fluid near the surface of the plate, and a temperature gradient is generated. The temperature gradient also is confined to a layer next to the wall, and within the layer the temperature varies from T_w at the wall to T_∞ at its outside boundary. This layer, called the *thermal boundary layer*, is shown as line *OB* in Fig. 12.1*a*. As drawn, lines *OA* and *OB* show that the thermal boundary layer is thinner than the hydrodynamic layer at all values of x, where x is the distance from the leading edge of the plate.

The relationship between the thicknesses of the two boundary layers at a given point along the plate depends on the dimensionless Prandtl number, defined as $c_p\mu/k$. When the Prandtl number is greater than unity, which is true for most liquids, the thermal layer is thinner than the hydrodynamic layer, as shown in Fig. 12.1*a*. The Prandtl number of a gas is usually close to 1.0 (0.69 for air, 1.06 for steam), and the two layers are about the same thickness. Only in heat transfer to liquid metals, which have very low Prandtl numbers, is the thermal layer much thicker than the hydrodynamic layer.

Most liquids have higher Prandtl numbers than gases because the viscosity is generally two or more orders of magnitude higher than for gases, which more than offsets the higher thermal conductivity of liquids. With a high-viscosity fluid, the hydrodynamic boundary layer extends further from the surface of the plate, which can perhaps be understood intuitively. Imagine moving a flat plate through a very viscous liquid such as glycerol: fluid at a considerable distance from the plate will be set in motion, which means a thick boundary layer.

The thickness of the thermal boundary layer increases with thermal conductivity, since a high conductivity leads to greater heat flux and to temperature gradients extending further into the fluid. The very high conductivity of liquid metals makes the temperature gradients extend well beyond the hydrodynamic boundary layer.

In Fig. 12.1*a* it is assumed that the entire plate is heated and that both boundary layers start at the leading edge of the plate. If the first section of the plate is not heated and if the heat-transfer area begins at a definite distance x_0 from the leading edge, as shown by line *O'B* in Fig 12.1*b*, a hydrodynamic boundary layer already exists at x_0, where the thermal boundary layer begins to form.

The sketches in Fig. 12.1 exaggerate the thickness of the boundary layers for clarity. The actual thicknesses are usually a few percent of the distance from the leading edge of the plate.

In flow through a tube, it has been shown (Chap. 3) that the hydrodynamic boundary layer thickens as the distance from the tube entrance increases, and finally the layer reaches the center of the tube. The velocity profile so developed, called *fully developed flow*, establishes a velocity distribution that is unchanged with additional pipe length. The thermal boundary layer in a heated or cooled tube also reaches the center of the tube at a definite length from the entrance of the heated length of the tube, and the temperature profile is fully developed at this point. Unlike the velocity profile, however, the temperature profile flattens as the length of the tube increases, and in very long pipes the entire fluid stream reaches the temperature of the tube wall, the temperature gradients disappear, and heat transfer ceases.

HEAT TRANSFER BY FORCED CONVECTION IN LAMINAR FLOW

In laminar flow, heat transfer occurs only by conduction, as there are no eddies to carry heat by convection across an isothermal surface. The problem is amenable to mathematical analysis based on the partial differential equations for continuity, momentum, and energy. Such treatments are beyond the scope of this book and are given in standard treatises on heat transfer.[6a] Mathematical solutions depend on the boundary conditions established to define the conditions of fluid flow and heat transfer. When the fluid approaches the heating surface, it may have an already completed hydrodynamic boundary layer or a partially developed one. Or the fluid may approach the heating surface at a uniform velocity, and both boundary layers may be initiated at the same time. A simple flow situation where the velocity is assumed constant in all cross sections and tube lengths is called *plug* or *rodlike flow*. Independent of the conditions of flow, (1) the heating surface may be isothermal; or (2) the heat flux may be equal at all points on the heating surface, in which case the average temperature of the fluid varies linearly with tube length. Other combinations of boundary conditions are possible.[6a] The basic differential equation for the several special cases is the same, but the final integrated relationships differ.

Most of the simpler mathematical derivations are based on the assumptions that the fluid properties are constant and temperature independent and that flow is truly laminar with no crosscurrents or eddies. These assumptions are valid when temperature changes and gradients are small, but with large temperature changes the simple model is not in accord with physical reality for two reasons. First, variations in viscosity across the tube distort the usual parabolic velocity-distribution profile of laminar flow. Thus, if the fluid is a liquid and is being heated, the layer near the wall has a lower viscosity than the layers near the center and the velocity gradient at the wall increases. A crossflow of liquid toward the wall is generated. If the liquid is being cooled, the reverse effect occurs. Second, since the temperature field generates density gradients, natural convection may set in, which further distorts the flow lines of the fluid. The effect of natural convection may be small or large, depending on a number of factors to be discussed in the section on natural convection.

In this section three types of heat transfer in laminar flow are considered: (1) heat transfer to a fluid flowing along a flat plate, (2) heat transfer in plug flow in tubes, and (3) heat transfer to a fluid stream that is in fully developed flow at the entrance to the tube. In all cases, the temperature of the heated length of the plate or tube is assumed to be constant, and the effect of natural convection is ignored.

LAMINAR-FLOW HEAT TRANSFER TO FLAT PLATE. Consider heat flow to the flat plate shown in Fig. 12.1*b*. The conditions are assumed to be as follows:

Velocity of fluid approaching plate and at and beyond the edge of the boundary layer OA: u_0.

Temperature of fluid approaching plate and at and beyond the edge of the thermal boundary layer $O'B$: T_∞.

Temperature of plate: from $x = 0$ to $x = x_0$, $T = T_\infty$; for $x > x_0$, $T = T_w$, where $T_w > T_\infty$.

The following properties of the fluid are constant and temperature independent: density ρ, conductivity k, specific heat c_p, and viscosity μ.

Detailed analysis of the situation yields the equation[2]

$$\left(\frac{dT}{dy}\right)_w = \frac{0.332(T_w - T_\infty)}{\sqrt[3]{1 - (x_0/x)^{3/4}}} \sqrt[3]{\frac{c_p\mu}{k}} \sqrt{\frac{u_0\rho}{\mu x}} \tag{12.1}$$

where $(dT/dy)_w$ is the temperature gradient at the wall. From Eq. (11.19), the relation between the local heat-transfer coefficient h_x at any distance x from the leading edge and the temperature gradient at the wall is

$$h_x = \frac{k}{T_w - T_\infty}\left(\frac{dT}{dy}\right)_w \tag{12.2}$$

Eliminating $(dT/dy)_w$ gives

$$h_x = \frac{0.332k}{\sqrt[3]{1-(x_0/x)^{3/4}}}\sqrt[3]{\frac{c_p\mu}{k}}\sqrt{\frac{u_0\rho}{\mu x}}$$

This equation can be put into a dimensionless form by multiplying by x/k, giving

$$\frac{h_x x}{k} = \frac{0.332}{\sqrt[3]{1-(x_0/x)^{3/4}}}\sqrt[3]{\frac{c_p\mu}{k}}\sqrt{\frac{u_0 x\rho}{\mu}} \tag{12.3}$$

The left-hand side of this equation is, from Eq. (11.20), a Nusselt number corresponding to the distance x, or $N_{\mathrm{Nu},x}$. The second group is the Prandtl number N_{Pr}, and the third group is a Reynolds number corresponding to distance x, denoted by $N_{\mathrm{Re},x}$. Equation (12.3) then can be written

$$N_{\mathrm{Nu},x} = \frac{0.332}{\sqrt[3]{1-(x_0/x)^{3/4}}}\sqrt[3]{N_{\mathrm{Pr}}}\sqrt{N_{\mathrm{Re},x}} \tag{12.4}$$

The local Nusselt number can be interpreted as the ratio of the distance x to the thickness of the thermal boundary layer, since conduction through a layer of thickness y would give a coefficient k/y. Thus

$$N_{\mathrm{Nu},x} = \frac{h_x x}{k} = \frac{k}{y}\frac{x}{k} = \frac{x}{y} \tag{12.5}$$

When the plate is heated over its entire length, as shown in Fig. 12.1*a*, $x_0 = 0$ and Eq. (12.4) becomes

$$N_{\mathrm{Nu},x} = 0.332\sqrt[3]{N_{\mathrm{Pr}}}\sqrt{N_{\mathrm{Re},x}} \tag{12.6}$$

Equation (12.6) gives the local value of the Nusselt number at distance x from the leading edge. More important in practice is the average value of N_{Nu} over the entire length of the plate x_1, defined as

$$N_{\mathrm{Nu}} = \frac{hx_1}{k} \tag{12.7}$$

where

$$h = \frac{1}{x_1}\int_0^{x_1} h_x\,dx$$

Equation (12.3) can be written for a plate heated over its entire length, since $x_0 = 0$, as

$$h_x = \frac{C}{\sqrt{x}}$$

where C is a constant containing all factors other than h_x and x. Then

$$h = \frac{C}{x_1} \int_0^{x_1} \frac{dx}{\sqrt{x}} = \frac{2C}{x_1} \sqrt{x_1} = \frac{2C}{\sqrt{x_1}} = 2h_{x_1} \tag{12.8}$$

The average coefficient is clearly twice the local coefficient at the end of the plate, and Eq. (12.6) gives

$$N_{\mathrm{Nu}} = 0.664 \sqrt[3]{N_{\mathrm{Pr}}} \sqrt{N_{\mathrm{Re},x_1}} \tag{12.9}$$

These equations are valid only for Prandtl numbers of 1.0 or greater, since the derivation assumes a thermal boundary layer no thicker than the hydrodynamic layer. However, they can be used for gases with $N_{\mathrm{Pr}} \approx 0.7$ with little error. The equations are also restricted to cases where the Nusselt number is fairly large, say, 10 or higher, since axial conduction, which was neglected in the derivation, has a significant effect for thick boundary layers.

LAMINAR-FLOW HEAT TRANSFER IN TUBES. The simplest situation of laminar-flow heat transfer in tubes is defined by the following conditions. The velocity of the fluid throughout the tube and at all points in any cross section of the stream is constant, so that $u = u_0 = \overline{V}$; the wall temperature is constant; and the properties of the fluid are independent of temperature. Mathematically this model is identical to that of heat flow by conduction into a solid rod at constant surface temperature, using as heating time the period of passage of a cross section of the fluid stream at velocity $\overline{V}$ through a tube of length L. This time period is $t_T = L/\overline{V}$. Equation (10.18), then, can be used for plug flow of a fluid by substituting $L/\overline{V}$ for t_T in the Fourier number, which becomes

$$N_{\mathrm{Fo}} = \frac{\alpha t_T}{r_m^2} = \frac{4kt_T}{c_p \rho D^2} = \frac{4kL}{c_p \rho D^2 \overline{V}} \tag{12.10}$$

THE GRAETZ AND PECLET NUMBERS. Two other dimensionless groups are commonly used in place of the Fourier number in treating heat transfer to fluids. The *Graetz number* is defined by the equation

$$N_{\mathrm{Gz}} \equiv \frac{\dot{m} c_p}{kL} \tag{12.11}$$

where $\dot{m}$ is the mass flow rate. Since $\dot{m} = (\pi/4)\rho \overline{V} D^2$,

$$N_{\mathrm{Gz}} = \frac{\pi}{4} \frac{\rho \overline{V} c_p D^2}{kL} \tag{12.12}$$

The *Peclet number* N_{Pe} is defined as the product of the Reynolds number and the Prandtl number, or

$$N_{\mathrm{Pe}} \equiv N_{\mathrm{Re}} N_{\mathrm{Pr}} = \frac{D\overline{V}\rho}{\mu} \frac{c_p \mu}{k} = \frac{\rho \overline{V} c_p D}{k} = \frac{D\overline{V}}{\alpha} \tag{12.13}$$

The choice among these groups is arbitrary. They are related by the equations

$$N_{\mathrm{Gz}} = \frac{\pi D}{4L} N_{\mathrm{Pe}} = \frac{\pi}{N_{\mathrm{Fo}}} \tag{12.14}$$

In the following discussion the Graetz number is used.

PLUG FLOW. Equation (10.18) becomes, for plug flow,

$$\frac{T_w - \overline{T}_b}{T_w - T_a} = 0.692e^{-5.78\pi/N_{\mathrm{Gz}}} + 0.131e^{-30.5\pi/N_{\mathrm{Gz}}} + 0.0534e^{-74.9\pi/N_{\mathrm{Gz}}} + \cdots \tag{12.15}$$

Here T_a and $\overline{T}_b$ are the inlet and average outlet fluid temperatures, respectively.

Plug flow is not a realistic model for newtonian fluids, but it does apply to highly pseudoplastic liquids ($n' \approx 0$) or to plastic liquids having a high value of the yield stress τ_0.

FULLY DEVELOPED FLOW. With a newtonian fluid in fully developed flow, the actual velocity distribution at the entrance to the heated section and the theoretical distribution throughout the tube are both parabolic. For this situation the appropriate boundary conditions lead to the development of another theoretical equation, of the same form as Eq. (12.15). This is[7d]

$$\frac{T_w - \overline{T}_b}{T_w - T_a} = 0.81904e^{-3.657\pi/N_{\mathrm{Gz}}} + 0.09760e^{-22.31\pi/N_{\mathrm{Gz}}} + 0.01896e^{-53\pi/N_{\mathrm{Gz}}} + \cdots \tag{12.16}$$

Because of distortions in the flow field from the effects of temperature on viscosity and density, Eq. (12.16) does not give accurate results. The heat-transfer rates are usually larger than those predicted by Eq. (12.16), and empirical correlations have been developed for design purposes. These correlations are based on the Graetz number, but they give the film coefficient or the Nusselt number rather than the change in temperature, since this permits the fluid resistance to be combined with other resistances in determining an overall heat-transfer coefficient.

The Nusselt number for heat transfer to a fluid inside a pipe is the film coefficient multiplied by D/k:

$$N_{\mathrm{Nu}} \equiv \frac{h_i D}{k} \tag{12.17}$$

The film coefficient h_i is the average value over the length of the pipe and is calculated as follows for the case of constant wall temperature:

$$h_i = \frac{\dot{m} c_p (\overline{T}_b - T_a)}{\pi D L \, \overline{\Delta T}_L} \tag{12.18}$$

Since

$$\overline{\Delta T_L} = \frac{(T_w - T_a) - (T_w - \overline{T}_b)}{\ln\left(\dfrac{T_w - T_a}{T_w - \overline{T}_b}\right)} \tag{12.19}$$

$$h_i = \frac{\dot{m}c_p}{\pi DL} \ln\left(\frac{T_w - T_a}{T_w - \overline{T}_b}\right) \tag{12.20}$$

and

$$N_{\mathrm{Nu}} = \frac{\dot{m}c_p}{\pi kL} \ln\left(\frac{T_w - T_a}{T_w - \overline{T}_b}\right) \tag{12.21}$$

or

$$N_{\mathrm{Nu}} = \frac{N_{\mathrm{Gz}}}{\pi} \ln\left(\frac{T_w - T_a}{T_w - \overline{T}_b}\right) \tag{12.22}$$

Using Eqs. (12.22) and (12.16), theoretical values of the Nusselt number can be obtained, and these values are shown in Fig. 12.2. At low Graetz numbers, only the first term of Eq. (12.16) is significant, and the Nusselt number approaches a limiting value of 3.66. It is difficult to get an accurate measurement of the heat-transfer coefficient at low Graetz numbers, since the final temperature difference is very small. For example, at $N_{\mathrm{Gz}} = 1.0$, the ratio of exit to inlet driving forces is only 8.3×10^{-6}.

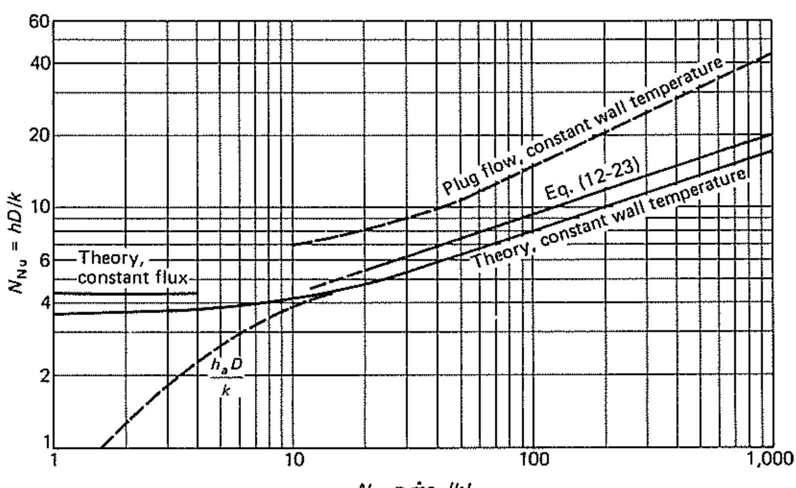

FIGURE 12.2
Heat transfer for laminar flow in tubes with a parabolic velocity profile. (Does not include effects of natural convection or viscosity gradients.)

For Graetz numbers greater than 20, the theoretical Nusselt number increases with about the one-third power of N_{Gz}. Data for air and for moderate-viscosity liquids follow a similar trend, but the coefficients are about 15 percent greater than predicted from theory. An empirical equation for moderate Graetz numbers (greater than 20) is

$$N_{Nu} \approx 2.0 N_{Gz}^{1/3} \tag{12.23}$$

The increase in film coefficient with increasing Graetz number or decreasing length is a result of the change in shape of the temperature profile. For short lengths, the thermal boundary layer is very thin, and the steep temperature gradient gives a high local coefficient. With increasing distance from the entrance, the boundary layer becomes thicker and eventually reaches the center of the pipe, giving a nearly parabolic temperature profile. The local coefficient is approximately constant from that point on, but the average coefficient continues to decrease with increasing length until the effect of the high initial coefficient is negligible. In practice, the change in local coefficient with length is usually not calculated, and the length-average film coefficient is used in obtaining the overall coefficient.

CORRECTION FOR HEATING OR COOLING. For very viscous liquids with large temperature drops, a modification of Eq. (12.23) is required to account for differences between heating and cooling. When a liquid is being heated, the lower viscosity near the wall makes the velocity profile more like that for plug flow, with a very steep gradient near the wall and little gradient near the center. This leads to a higher rate of heat transfer, as can be shown by comparing temperature approaches calculated from Eqs. (12.15) and (12.16). When a viscous liquid is cooled, the velocity gradient at the wall is decreased, giving a lower rate of heat transfer. A dimensionless, but empirical, correction factor ϕ_v accounts for the difference between heating and cooling:

$$\phi_v \equiv \left(\frac{\mu}{\mu_w}\right)^{0.14} \tag{12.24}$$

This factor is added to Eq. (12.23) to give the final equation for laminar-flow heat transfer:

$$N_{Nu} = 2\left(\frac{\dot{m}c_p}{kL}\right)^{1/3}\left(\frac{\mu}{\mu_w}\right)^{0.14} = 2N_{Gz}^{1/3}\phi_v \tag{12.25}$$

In Eqs. (12.24) and (12.25), μ is the viscosity at the arithmetic mean temperature of the fluid, $(\overline{T}_a + \overline{T}_b)/2$, and μ_w is the viscosity at the wall temperature T_w. For liquids $\mu_w < \mu$ and $\phi_v > 1.0$ when the liquid is being heated, and $\mu_w > \mu$ and $\phi_v < 1.0$ when the liquid is being cooled. The viscosity of a gas increases with temperature, so these inequalities are reversed for a gas. The change in viscosity of a gas, however, is relatively small, and the term ϕ_v is usually omitted when dealing with gases. The change in gas density with temperature is of more importance, and this will be discussed when dealing with natural convection.

The coefficients presented in Eqs. (12.17) to (12.25) are based on a logarithmic mean driving force $\overline{\Delta T_L}$. Some workers have presented correlations for a coefficient h_a based on the arithmetic mean driving force, $\overline{\Delta T_a}$. When the Graetz number is 10 or larger, the ratio of inlet to exit driving forces is less than 2.0, and there is little difference between $\overline{\Delta T_L}$ and $\overline{\Delta T_a}$ or between h and h_a. However, at low Graetz numbers the temperature approach becomes very small, and the coefficient h_a varies inversely with the Graetz number, as shown in Fig. 12.2. There is no apparent advantage to using h_a, and h is recommended for design calculations.

The equations and experimental results discussed up to this point are for constant wall temperature and would apply for heating a fluid with a condensing vapor. If a counterflow exchanger is used, the wall temperature will change along the length of the exchanger, and this will affect the film coefficient for laminar flow. If the two streams have about the same flow rate and heat capacity, the temperature driving force and the heat flux will be nearly constant. The theoretical equation for constant heat flux and parabolic flow gives a limiting Nusselt number of 4.37, compared to 3.66 for constant T_w. At high Graetz numbers, the predicted coefficient for constant flux is also higher than for constant wall temperature, but there are not enough experimental results to develop a separate equation for the case of constant flux.

HEAT TRANSFER TO NON-NEWTONIAN LIQUIDS IN LAMINAR FLOW. For heat transfer to and from liquids that follow the power-law relation [Eq. (3.7)], Eq. (12.25) is modified to[10]

$$\frac{h_i D}{k} = 2\delta^{1/3}\left(\frac{\dot{m}c_p}{kL}\right)^{1/3}\left(\frac{m}{m_w}\right)^{0.14} \tag{12.26}$$

where $\delta = (3n' + 1)/4n'$
$m = K'8^{n'-1}$, at arithmetic mean temperature
m_w = value of m at T_w
K' = flow consistency index
n' = flow behavior index

For shear-thinning fluids ($n' < 1$), the non-newtonian behavior makes the velocity profile more like that for plug flow and increases the heat-transfer coefficient. When $n' = 0.1$, the coefficient for large Graetz numbers is about 1.5 times that for parabolic flow ($n' = 1.0$). The limit of $n' = 0$ corresponds to true plug flow and the coefficients can be as large as twice those for parabolic flow. Figure 12.2 shows the Nusselt number for plug flow as a dashed line, the slope of which approaches 0.5 at high Graetz numbers.

HEAT TRANSFER BY FORCED CONVECTION IN TURBULENT FLOW

Perhaps the most important situation in heat transfer is the heat flow in a stream of fluid in turbulent flow in a closed channel, especially in tubes. Turbulence is encountered at Reynolds numbers greater than about 2100, and since the rate of heat transfer is greater in turbulent flow than in laminar flow, most equipment is operated in the turbulent range.

The earliest approach to this case was based on empirical correlations of test data guided by dimensional analysis. The equations so obtained still are much used in design. Subsequently, theoretical study has been given to the problem. A deeper understanding of the mechanism of turbulent-flow heat transfer has been achieved, and improved equations applicable over wider ranges of conditions have been obtained.

DIMENSIONAL-ANALYSIS METHOD. Dimensional analysis of the heat flow to a fluid in turbulent flow in a long, straight pipe yields the dimensionless relationship

$$\frac{hD}{k} = \Phi\left(\frac{D\overline{V}\rho}{\mu}, \frac{c_p\mu}{k}\right) = \Phi\left(\frac{DG}{\mu}, \frac{c_p\mu}{k}\right) \tag{12.27}$$

Here the mass velocity G is used in place of its equal $\overline{V}\rho$. Dividing both sides of Eq. (12.27) by the product $(DG/\mu)(c_p\mu/k)$ gives an alternate relationship

$$\frac{h}{c_pG} = \Phi_1\left(\frac{DG}{\mu}, \frac{c_p\mu}{k}\right) \tag{12.28}$$

The three groups in Eq. (12.27) are the Nusselt, Reynolds, and Prandtl numbers, respectively. The left-hand group in Eq. (12.28) is called the *Stanton number* N_{St}. The four groups are related by the equation

$$N_{St}N_{Re}N_{Pr} = N_{Nu} \tag{12.29}$$

Thus, only three of the four are independent.

EMPIRICAL EQUATIONS. To use Eq. (12.27) or (12.28), the function Φ or Φ_1 must be known. One empirical correlation for long tubes with sharp-edged entrances is the Dittus-Boelter equation

$$\frac{h_iD}{k} = 0.023\left(\frac{DG}{\mu}\right)^{0.8}\left(\frac{c_p\mu}{k}\right)^{1/3} \tag{12.30}$$

This is the dimensionless form of Eq. (1.42) in Chap. 1, Example 1.2. Equation (12.30) has been modified to allow for heating and cooling, as in Eq. (12.25) for laminar flow. The modified relationship for turbulent flow[8] is known as the Sieder-Tate equation. It is

$$\frac{h_iD}{k} = 0.023\left(\frac{DG}{\mu}\right)^{0.8}\left(\frac{c_p\mu}{k}\right)^{1/3}\left(\frac{\mu}{\mu_w}\right)^{0.14} \tag{12.31}$$

An alternate form of Eq. (12.31) is obtained by dividing both sides by $(DG/\mu)(c_p\mu/k)$ and transposing, to give the Colburn equation:

$$\frac{h_i}{c_pG}\left(\frac{c_p\mu}{k}\right)^{2/3}\left(\frac{\mu_w}{\mu}\right)^{0.14} = \frac{0.023}{(DG/\mu)^{0.2}} \tag{12.32}$$

In using these equations, the physical properties of the fluid, except for μ_w, are evaluated at the bulk temperature T. Equations (12.31) and (12.32) can be written in more compact form as

$$N_{\rm Nu} = 0.023 N_{\rm Re}^{0.8} N_{\rm Pr}^{1/3} \phi_v \tag{12.33}$$

and

$$N_{\rm St} N_{\rm Pr}^{2/3} \phi_v^{-1} = 0.023 N_{\rm Re}^{-0.2} \tag{12.34}$$

Equations (12.31) to (12.34) are not fundamentally different equations; they are merely alternative ways of expressing the same information. They should not be used for Reynolds numbers below 6000 or for molten metals, which have abnormally low Prandtl numbers.

Effect of tube length. Near the tube entrance, where the temperature gradients are still forming, the local coefficient h_x is greater than h_∞ for fully developed flow. At the entrance itself, where there is no previously established temperature gradient, h_x is infinite. Its value drops rapidly toward h_∞ in a comparatively short length of tube. Dimensionally, the effect of tube length is accounted for by another dimensionless group, x/D, where x is the distance from the tube entrance. The local coefficient approaches h_∞ asymptotically with increase in x, but it is practically equal to h_∞ when x/D is about 50. The average value of h_x over the tube length is denoted by h_i. The value of h_i is found by integrating h_x over the length of the tube. Since $h_x \to h_\infty$ as $x \to \infty$, the relation between h_i and h_∞ is of the form[6b]

$$\frac{h_i}{h_\infty} = 1 + \psi\left(\frac{L}{D}\right) \tag{12.35}$$

An equation for short tubes with sharp-edged entrances, where the velocity at entrance is uniform over the cross section, is

$$\frac{h_i}{h_\infty} = 1 + \left(\frac{D}{L}\right)^{0.7} \tag{12.36}$$

The effect of tube length on h_i fades out when L/D becomes greater than about 50.

AVERAGE VALUE OF h_i IN TURBULENT FLOW. Since the temperature of the fluid changes from one end of the tube to the other and the fluid properties μ, k, and c_p are all functions of temperature, the local value of h_i also varies from point to point along the tube. This variation is independent of the effect of tube length.

The effect of fluid properties can be shown by condensing Eq. (12.31) to read, assuming $(\mu/\mu_w) = 1$,

$$h_i = 0.023 \frac{G^{0.8} k^{2/3} c_p^{1/3}}{D^{0.2} \mu^{0.47}} \tag{12.37}$$

For gases the effect of temperature on h_i is small. At constant mass velocity in a given tube, h_i varies with $k^{2/3}c_p^{1/3}\mu^{-0.47}$. The increase in thermal conductivity and heat capacity with temperature offset the rise in viscosity, giving a slight increase in h_i. For example, for air h_i increases about 6 percent when the temperature changes from 50 to 100°C.

For liquids the effect of temperature is much greater than for gases because of the rapid decrease in viscosity with rising temperature. The effects of k, c_p, and μ in Eq. (12.37) all act in the same direction, but the increase in h_i with temperature is due mainly to the effect of temperature on viscosity. For water, for example, h_i increases about 50 percent over a temperature range from 50 to 100°C. For viscous oils the change in h_i may be two- or threefold for a 50°C increase in temperature.

In practice, unless the variation in h_i over the length of the tube is more than about 2:1, an average value of h_i is calculated and used as a constant in calculating the overall coefficient U. This procedure neglects the variation of U over the tube length and allows the use of the LMTD in calculating the area of the heating surface. The average value of h_i is computed by evaluating the fluid properties c_p, k, and μ at the average fluid temperature, defined as the arithmetic mean between the inlet and outlet temperatures. The value of h_i calculated from Eq. (12.31), using these property values, is called the *average coefficient*. For example, assume that the fluid enters at 30°C and leaves at 90°C. The average fluid temperature is $(30 + 90)/2 = 60$°C, and the values of the properties used to calculate the average value of h_i are those at 60°C.

For larger changes in h_i, two procedures can be used: (1) The values of h_i at the inlet and outlet can be calculated, corresponding values of U_1 and U_2 found, and Eq. (11.16) used. Here the effect of L/D on the entrance value of h_i is ignored. (2) For even larger variations in h_i, and therefore in U, the tube can be divided into sections and an average U used for each section. Then the lengths of the individual sections can be added to account for the total length of the tube.

ESTIMATION OF WALL TEMPERATURE T_w. To evaluate μ_w, the viscosity of the fluid at the wall, temperature T_w must be found. The estimation of T_w requires an iterative calculation based on the resistance equation (11.33). If the individual resistances can be estimated, the total temperature drop ΔT can be split into the individual temperature drops by the use of this equation and an approximate value for the wall temperature found. To determine T_w in this way, the wall resistance $(x_w/k_m)(D_o/\overline{D}_L)$ can usually be neglected, and Eq. (11.33) used as follows.

From the first two members of Eq. (11.33)

$$\Delta T_i = \frac{D_o/D_i h_i}{1/U_o}\Delta T \tag{12.38}$$

Substituting $1/U_o$ from Eq. (11.30) and neglecting the wall-resistance term gives

$$\Delta T_i = \frac{1/h_i}{1/h_i + D_i/D_o h_o}\Delta T \tag{12.39}$$

In qualitative terms Eq. (12.39) may be written

$$\Delta T_i = \frac{\text{inside resistance}}{\text{overall resistance}} \Delta T$$

Use of Eq. (12.39) requires preliminary estimates of the coefficients h_i and h_o. To estimate h_i, Eq. (12.31) can be used, neglecting ϕ_v. The calculation of h_o will be described later. The wall temperature T_w is then obtained from the following equations:

For heating: $$T_w = T + \Delta T_i \tag{12.40}$$

For cooling: $$T_w = T - \Delta T_i \tag{12.41}$$

where T is the average fluid temperature.

If the first approximation is not sufficiently accurate, a second calculation of T_w based on the results of the first can be made. Unless the factor ϕ_v is quite different from unity, however, the second approximation is unnecessary.

Example 12.1. Toluene is being condensed at 230°F (110°C) on the outside of $\frac{3}{4}$-in. (19-mm) BWG 16 copper condenser tubes through which cooling water is flowing at an average temperature of 80°F (26.7°C). Individual heat-transfer coefficients are given in Table 12.1. Neglecting the resistance of the tube wall, what is the tube-wall temperature?

Solution

From Appendix 6, $D_i = 0.620$ in.; $D_o = 0.750$ in. Hence, from Eq. (12.39)

$$\Delta T_i = \frac{1/400}{1/400 + 0.620/(0.750 \times 500)}(230 - 80) = 90.3\text{°F}$$

Since the water is being heated by the condensing toluene, the wall temperature is found from Eq. (12.40) as follows:

$$T_w = 80 + 90.3 = 170.3\text{°F } (76.8\text{°C})$$

CROSS SECTIONS OTHER THAN CIRCULAR. To use Eq. (12.31) or (12.32) for cross sections other than circular, it is only necessary to replace the diameter D in both Reynolds and Nusselt numbers by the equivalent diameter D_e, defined as

See Pg 103.

TABLE 12.1
Data for Example 12.1

	Heat-transfer coefficient	
	Btu/ft²-h-°F	**W/m²-°C**
For cooling water h_i	400	2270
For toluene h_o	500	2840

4 times the hydraulic radius r_H. The method is the same as that used in calculating friction loss.

Example 12.2. Benzene is cooled from 141 to 79°F (60.6 to 21.1°C) in the inner pipe of a double-pipe exchanger. Cooling water flows countercurrently to the benzene, entering the jacket at 65°F (18.3°C) and leaving at 75°F (23.9°C). The exchanger consists of an inner pipe of $\frac{7}{8}$-in (22.2-mm) BWG 16 copper tubing jacketed with $1\frac{1}{2}$-in (38.1-mm) Schedule 40 steel pipe. The linear velocity of the benzene is 5 ft/s (1.52 m/s). Neglecting the resistance of the wall and scale films and assuming $L/D > 150$ for both pipes, compute the film coefficients of the benzene and water and the overall coefficient based on the outside area of the inner pipe.

Solution

The average temperature of the benzene is $(141 + 79)/2 = 110$°F; that of the water is $(65 + 75)/2 = 70$°F. The physical properties at these temperatures are given in Table 12.2. The diameters of the inner tube are

$$D_{it} = \frac{0.745}{12} = 0.0621 \text{ ft} \qquad D_{ot} = \frac{0.875}{12} = 0.0729 \text{ ft}$$

The inside diameter of the jacket is, from Appendix 5

$$D_{ij} = \frac{1.610}{12} = 0.1342 \text{ ft}$$

The equivalent diameter of the annular jacket space is found as follows. The cross-sectional area is $(\pi/4)(0.1342^2 - 0.0729^2)$ or 0.00997 ft². The wetted perimeter is $\pi(0.1342 + 0.0729)$. The hydraulic radius is

$$r_H = \frac{(\pi/4)(0.1342^2 - 0.0729^2)}{\pi(0.1342 + 0.0729)} = \tfrac{1}{4}(0.1342 - 0.0729) = \tfrac{1}{4} \times 0.0613 \text{ ft}$$

The equivalent diameter $D_e = 4r_H = 0.0613$ ft.

TABLE 12.2
Data for Example 12.2

	Value at average fluid temperature	
Property	**Benzene**	**Water†**
Density ρ, lb/ft³	53.1	62.3
Viscosity μ, lb/ft-h	1.16‡	2.42 × 0.982 = 2.34
Thermal conductivity k, Btu/ft-h-°F	0.089§	0.346
Specific heat c_p, Btu/lb-°F	0.435¶	1.000

† Appendix 14.
‡ Appendix 9.
§ Appendix 13.
¶ Appendix 16.

First the velocity of the water must be computed from the heat flow and the temperature rise in the water. The heat flow, in turn, is found from the mass flow rate of the benzene, $\dot{m}_b$, given by

$$\dot{m}_b = \overline{V}_b \rho_b S$$

where S is the inside sectional area of the copper tube. From Appendix 6, for a $\frac{7}{8}$-in. BWG 16 tube, $S = 0.00303$ ft^2. Thus

$$\dot{m}_b = 5 \times 53.1 \times 0.00303 = 0.804 \text{ lb/s}$$

The rate of heat flow q, found by multiplying the mass flow, specific heat, and temperature change of the benzene, is

$$q = 0.804 \times 0.435 \times (141 - 79)$$
$$= 21.68 \text{ Btu/s}$$

The mass flow rate of the water $\dot{m}_w$ is then

$$\dot{m}_w = \frac{21.68}{1.000 \times (75 - 65)} = 2.168 \text{ lb/s}$$

The water velocity $\overline{V}_w$ is

$$\overline{V}_w = \frac{2.168}{0.00997 \times 62.3} = 3.49 \text{ ft/s}$$

The Reynolds number and Prandtl number of each stream are next computed:

Benzene: $$N_{\text{Re}} = \frac{D_{it}\overline{V}\rho}{\mu} = \frac{0.0621 \times 5 \times 3600 \times 53.1}{1.16} = 5.12 \times 10^4$$

$$N_{\text{Pr}} = \frac{c_p \mu}{k} = \frac{0.435 \times 1.16}{0.089} = 5.67$$

Water: $$N_{\text{Re}} = \frac{D_e\overline{V}\rho}{\mu} = \frac{0.0613 \times 3.49 \times 3600 \times 62.3}{2.34} = 2.05 \times 10^4$$

$$N_{\text{Pr}} = \frac{1.00 \times 2.34}{0.346} = 6.76$$

Preliminary estimates of the coefficients are obtained from Eq. (12.32) omitting the correction for viscosity ratio:

Benzene: $$h_i = \frac{0.023 \times 5 \times 3600 \times 53.1 \times 0.435}{(5.12 \times 10^4)^{0.2} \times 5.67^{2/3}} = 344 \text{ Btu/ft}^2\text{-h-°F}$$

Water: $$h_o = \frac{0.023 \times 3.49 \times 3600 \times 62.3 \times 1.000}{(2.05 \times 10^4)^{0.2} \times 6.76^{2/3}} = 691 \text{ Btu/ft}^2\text{-h-°F}$$

In these calculations use is made of the fact that $G = \overline{V}\rho$.

The temperature drop over the benzene resistance, from Eq. (12.39), is

$$\Delta T_i = \frac{1/344}{1/344 + 0.0621/(0.0729 \times 691)}(110 - 70) = 28.1°\text{F}$$

$$T_w = 110 - 28.1 = 81.9°\text{F}$$

The viscosities of the liquids at T_w are now found.

$$\mu_w = \begin{cases} 1.45 \text{ lb/ft-h} & \text{for benzene} \\ 0.852 \times 2.42 = 2.06 \text{ lb/ft-h} & \text{for water} \end{cases}$$

The viscosity-correction factors ϕ_v, from Eq. (12.24), are

$$\phi_v = \begin{cases} \left(\dfrac{1.16}{1.45}\right)^{0.14} = 0.969 & \text{for benzene} \\ \left(\dfrac{2.34}{2.06}\right)^{0.14} = 1.018 & \text{for water} \end{cases}$$

The corrected coefficients are

Benzene: $h_i = 344 \times 0.969 = 333$ Btu/ft^2-h-°F (1891 W/m^2-°C)

Water: $h_o = 691 \times 1.018 = 703$ Btu/ft^2-h-°F (3992 W/m^2-°C)

The temperature drop over the benzene resistance and the wall temperature become

$$\Delta T_i = \frac{1/333}{1/333 + 0.0621/(0.0729 \times 703)}(110 - 70) = 28.5°\text{F}$$

$$T_w = 110 - 28.5 = 81.5°\text{F}$$

This is so close to the wall temperature calculated previously that a second approximation is unnecessary.

The overall coefficient is found from Eq. (11.29) neglecting the resistance of the tube wall:

$$\frac{1}{U_o} = \frac{0.0729}{0.0621 \times 333} + \frac{1}{703} = 0.00495$$

$$U_o = \frac{1}{0.00495} = 202 \text{ Btu/ft}^2\text{-h-°F (1147 W/m}^2\text{-°C)}$$

EFFECT OF ROUGHNESS. For equal Reynolds numbers the heat-transfer coefficient in turbulent flow is somewhat greater for a rough tube than for a smooth one. The effect of roughness on heat transfer is much less than on fluid friction, and economically it is usually more important to use a smooth tube for minimum friction loss than to rely on roughness to yield a larger heat-transfer coefficient. The effect of roughness on h_i is neglected in practical calculations.

HEAT TRANSFER AT HIGH VELOCITIES. When a fluid flows through a tube at high velocities, temperature gradients appear even when there is no heat transfer through the wall and $q = 0$. In injection molding, for example, polymer melt flows into the cavity at a very high velocity and the steep velocity gradients in the viscous liquid cause heat to be generated in the fluid by what is called *viscous dissipation*. In the high-velocity flow of compressible gases in pipes, friction at the wall raises the temperature of the fluid at the wall above the average fluid temperature.[4c] The temperature difference between wall and fluid causes a flow of heat from wall to fluid, and a steady state is reached when the rate of heat generation from friction at the wall equals the rate of heat transfer back into the fluid stream. The constant wall temperature thus attained is called the *adiabatic wall temperature*. Further treatment of this subject is beyond the scope of this text. The effect becomes appreciable for Mach numbers above approximately 0.4, and appropriate equations must be used in this range of velocities instead of Eqs. (12.31) and (12.32).

Transfer by Turbulent Eddies and Analogy between Transfer of Momentum and Heat

On pages 92 to 98 the distribution of velocity and its accompanying momentum flux in a flowing stream in turbulent flow through a pipe was described. Three rather ill-defined zones in the cross section of the pipe were identified. In the first, immediately next to the wall, eddies are rare, and momentum flow occurs almost entirely by viscosity; in the second, a mixed regime of combined viscous and turbulent momentum transfer occurs; in the main part of the stream, which occupies the bulk of the cross section of the stream, only the momentum flow generated by the Reynolds stresses of turbulent flow is important. The three zones are called the *viscous sublayer*, the *buffer zone*, and the *turbulent core*, respectively.

In heat transfer at the wall of the tube to or from the fluid stream, the same hydrodynamic distributions of velocity and of momentum fluxes still persist, and in addition, a temperature gradient is superimposed on the turbulent-laminar velocity field. In the following treatment both gradients are assumed to be completely developed and the effect of tube length negligible.

Throughout the stream of fluid, heat flow by conduction occurs in accordance with the equation

$$\frac{q_c}{A} = -k\frac{dT}{dy} \tag{12.42}$$

where q_c = rate of heat flow by conduction
k = thermal conductivity
A = area of isothermal surface
dT/dy = temperature gradient across isothermal surface

The isothermal surface is a cylinder concentric with the axis of the pipe and located a distance y from the wall, or r from the center of the pipe. Here $r + y = r_w$, where r_w is the radius of the pipe.

In addition to conduction, the eddies of turbulent flow carry heat by convection across each isothermal area. Although both mechanisms of heat flow may occur wherever a temperature gradient exists ($dT/dy \neq 0$), their relative importance varies greatly with distance from the wall. At the tube wall itself eddies very rarely exist, and the heat flux is entirely due to conduction. Equation (12.42) written for the wall is

$$\left(\frac{q}{A}\right)_w \doteq -k\left(\frac{dT}{dy}\right)_w \tag{12.43}$$

where $(q/A)_w$ = total heat flux at wall
$(dT/dy)_w$ = temperature gradient at wall

These quantities are identical with those in Eqs. (11.18) and (11.19).

Within the viscous sublayer heat flows mainly by conduction, but eddies are not completely excluded from this zone, and some convection does occur. The relative importance of turbulent heat flux compared with conductive heat flux increases rapidly with distance from the wall. In ordinary fluids, having Prandtl numbers above about 0.6, conduction is entirely negligible in the turbulent core, but it may be significant in the buffer zone when the Prandtl number is in the neighborhood of unity. Conduction is negligible in this zone when the Prandtl number is large.

The situation is analogous to momentum flux, where the relative importance of turbulent shear to viscous shear follows the same general pattern. Under certain ideal conditions, the correspondence between heat flow and momentum flow is exact, and at any specific value of r/r_w the ratio of heat transfer by conduction to that by turbulence equals the ratio of momentum flux by viscous forces to that by Reynolds stresses. In the general case, however, the correspondence is only approximate and may be greatly in error. The study of the relationship between heat and momentum flux for the entire spectrum of fluids leads to the so-called *analogy theory*, and the equations so derived are called *analogy equations*. A detailed treatment of the theory is beyond the scope of this book, but some of the more elementary relationships are considered.

Since eddies continually cross an isothermal surface from both directions, they carry heat between layers on either side of the surface, which are at different average temperatures. At a given point the temperature fluctuates rapidly about the constant mean temperature at that point, depending on whether a "hot" or a "cold" eddy is crossing through the point. The temperature fluctuations form a pattern with respect to both time and place, just like the fluctuations in velocity and pressure described on pages 50 to 54. The instantaneous temperature T_i at the point can be split into two parts, the constant average temperature T at the point and the fluctuating or deviating temperature T', or

$$T_i = T + T' \tag{12.44}$$

The time average of the deviating temperature T', denoted by $\overline{T}'$, is zero, and the time-average value of the total instantaneous temperature, denoted by $\overline{T}_i$, is T. The average temperature T is that measured by an ordinary thermometer. To measure T_i and so find T' requires special sensing devices that can follow rapid temperature changes.

THE EDDY DIFFUSIVITY OF HEAT. When there is no temperature gradient across the isothermal surface, all eddies have the same temperature independent of the point of origin, $dT/dy = 0$, and no net heat flow occurs. If a temperature gradient exists, an analysis equivalent to that leading to Eq. (3.17) shows that the eddies carry a net heat flux from the higher temperature to the lower, in accordance with the equation

$$\frac{q_t}{A} = -c_p\rho\overline{v'T'} \tag{12.45}$$

where v' is the deviating velocity across the surface and the overbar indicates the time average of the product $v'T'$. Although the time averages $\overline{v'}$ and $\overline{T'}$ individually are zero, the average of their product is not, because a correlation exists between these deviating quantities when $dT/dy \neq 0$, in the same way that the deviating velocities u' and v' are correlated when a velocity gradient du/dy exists.

On page 55 an eddy diffusivity for momentum transfer ε_M was defined. A corresponding eddy diffusivity for heat transfer ε_H can be defined by

$$\frac{q_t}{c_p\rho A} \equiv -\varepsilon_H\frac{dT}{dy} = -\overline{v'T'} \tag{12.46}$$

The subscript t refers to the fact that Eq. (12.46) applies to turbulent convection heat transfer. Since conduction also takes place, the total heat flux at a given point, denoted by q, is, from Eqs. (12.42) and (12.46),

$$\frac{q}{A} = \frac{q_c}{A} + \frac{q_t}{A} = -k\frac{dT}{dy} - c_p\rho\varepsilon_H\frac{dT}{dy}$$

or

$$\frac{q}{A} = -c_p\rho(\alpha + \varepsilon_H)\frac{dT}{dy} \tag{12.47}$$

where α is the thermal diffusivity, $k/c_p\rho$. The equation for the total momentum flux corresponding to Eq. (12.47) is Eq. (3.20) written as

$$\frac{\tau g_c}{\rho} = (\nu + \varepsilon_M)\frac{du}{dy} \tag{12.48}$$

where ν is the kinematic viscosity, μ/ρ.

SIGNIFICANCE OF PRANDTL NUMBER; THE EDDY DIFFUSIVITIES. The physical significance of the Prandtl number appears on noting that it is the ratio ν/α; it is therefore a measure of the magnitude of the momentum diffusivity relative to that of the thermal diffusivity. Its numerical value depends on the temperature and pressure of the fluid, and therefore it is a true property. The magnitude of the

Prandtl numbers encountered in practice covers a wide range. For liquid metals it is of the order 0.01 to 0.04. For diatomic gases it is about 0.7, and for water at 70°C it is about 2.5. For viscous liquids and concentrated solutions it may be as large as 600. Prandtl numbers for various gases and liquids are given in Appendixes 17 and 18.

The eddy diffusitives for momentum and heat, ε_M and ε_H, respectively, are not properties of the fluid but depend on the conditions of flow, especially on all factors that affect turbulence. For simple analogies, it is sometimes assumed that ε_M and ε_H are both constants and equal, but when determined by actual velocity and temperature measurements, both are found to be functions of the Reynolds number, the Prandtl number, and position in the tube cross section. Precise measurement of the eddy diffusivities is difficult, and not all reported measurements agree. Results are given in standard treatises.[6c] The ratio $\varepsilon_H/\varepsilon_M$ also varies but is more nearly constant than the individual quantities. The ratio is denoted by ψ. For ordinary liquids, where $N_{\text{Pr}} > 0.6$, ψ is close to 1 at the tube wall and in boundary layers generally and approaches 2 in turbulent wakes. For liquid metals ψ is low near the wall, passes through a maximum of about unity at $y/r_w \approx 0.2$, and decreases toward the center of the pipe.[7c]

THE REYNOLDS ANALOGY. The simplest and oldest analogy equation is that of Reynolds, which is derived for flow at high Reynolds numbers in straight round tubes. It is based, however, on several questionable assumptions:

1. The ratio of the two molecular diffusivities equals that of the two eddy diffusivities, or
$$\frac{\nu}{\alpha} \equiv N_{\text{Pr}} = \frac{\varepsilon_M}{\varepsilon_H}$$
2. The heat flux q/A varies linearly with radius r.
3. The point where the local fluid temperature equals the average bulk temperature T is at the same radial distance from the tube wall as the point where $u = \overline{V}$.

Applying these assumptions to Eqs. (12.47) and (12.48) leads to the relation

$$\frac{h}{c_p G} N_{\text{Pr}} = \frac{f}{2} \tag{12.49}$$

For the special case where $N_{\text{Pr}} = 1$ and $\varepsilon_M = \varepsilon_H$,

$$\frac{h}{c_p G} \equiv N_{\text{St}} = \frac{f}{2} \tag{12.50}$$

This is the usual form of the Reynolds analogy equation. It agrees fairly well with experimental data for most gases, which have Prandtl numbers of about unity, provided the temperature drop $T_w - T$ is not large.

THE COLBURN ANALOGY; COLBURN *j* FACTOR. Over a range of Reynolds numbers from 5000 to 200,000 the friction factor for smooth pipes is adequately given by the empirical equation

$$f = 0.046\left(\frac{DG}{\mu}\right)^{-0.2} \tag{12.51}$$

Comparison of Eq. (12.51) with Eq. (12.32) for heat transfer in turbulent flow inside long tubes shows that

$$\frac{h}{c_pG} N_{\mathrm{Pr}}^{2/3}\left(\frac{\mu_w}{\mu}\right)^{0.14} \equiv j_H = \frac{f}{2} \tag{12.52}$$

Equation (12.52) is a statement of the Colburn analogy between heat transfer and fluid friction. The factor j_H, defined as $(h/c_pG)(c_p\mu/k)^{2/3}(\mu_w/\mu)^{0.14}$, is called the *Colburn j factor*. It is used in a number of other semiempirical equations for heat transfer. While the Reynolds analogy [Eq. (12.50)] applies only to fluids for which the Prandtl number is close to unity, the Colburn analogy [Eq. (12.52)] applies over a range of Prandtl numbers from 0.6 to about 100.

Equation (12.32) can be written in *j*-factor form as follows:

$$j_H = 0.023 N_{\mathrm{Re}}^{-0.2} \tag{12.53}$$

MORE ACCURATE ANALOGY EQUATIONS. A number of more elaborate analogy equations connecting friction and heat transfer in pipes, along flat plates, and in annular spaces have been published. They cover wider ranges of Reynolds and Prandtl numbers than Eq. (12.52) and are of the general form

$$N_{\mathrm{St}} = \frac{f/2}{\Phi(N_{\mathrm{Pr}})} \tag{12.54}$$

where $\Phi(N_{\mathrm{Pr}})$ is a complicated function of the Prandtl number. One example, by Friend and Metzner,[3] applying to fully developed flow in smooth pipe is

$$N_{\mathrm{St}} = \frac{f/2}{1.20 + \sqrt{f/2}(N_{\mathrm{Pr}} - 1)(N_{\mathrm{Pr}})^{-1/3}} \tag{12.55}$$

The friction factor f used in this equation may be that given by Eq. (12.51) or, for a wider range of Reynolds numbers from 3000 to 3×10^6, by the equation

$$f = 0.00140 + \frac{0.125}{N_{\mathrm{Re}}^{0.32}} \tag{12.56}$$

Equation (12.55) is said to apply over a range of Prandtl numbers from 0.5 to 600.

All analogy equations connecting f and h have an important limitation. They apply only to wall, or skin, friction and must not be used for situations where form drag appears.

Heat Transfer in Transition Region between Laminar and Turbulent Flow

Equation (12.32) applies only for Reynolds numbers greater than 6000 and Eq. (12.25) only for Reynolds numbers less than 2100. The range of Reynolds numbers between 2100 and 6000 is called the *transition region*, and no simple equation applies here. A graphical method therefore is used. The method is based on graphs of Eqs. (12.25) and (12.32) on a common plot of the Colburn j factor vs. N_{Re}, with lines of constant values of L/D. To obtain an equation for the laminar-flow range, it is necessary to transform Eq. (12.25) in the following manner. Substituted for the Graetz number, using Eqs. (12.13) and (12.14), is the quantity $(\pi D/4L)N_{Re}N_{Pr}$. The result is

$$N_{Nu} = 2\left(\frac{\pi D}{4L} N_{Re} N_{Pr}\right)^{1/3} \left(\frac{\mu}{\mu_w}\right)^{0.14}$$

This relation is multiplied by $(1/N_{Re})(1/N_{Pr})$ to give the j factor. The final equation can be written

$$\frac{h_i}{c_p G}\left(\frac{c_p \mu}{k}\right)^{2/3}\left(\frac{\mu_w}{\mu}\right)^{0.14} = j_H = 1.86\left(\frac{D}{L}\right)^{1/3}\left(\frac{DG}{\mu}\right)^{-2/3} \tag{12.57}$$

Equation (12.57) shows that for each value of the length-diameter ratio L/D, a logarithmic plot of the left-hand side vs. N_{Re} gives a straight line with a slope of $-\frac{2}{3}$. The straight lines on the left-hand portion of Fig. 12.3 are plots of this equation for a few values of L/D. The lines terminate at a Reynolds number of 2100.

Equation (12.57) should not be used for $L/D > 100$, since it may give coefficients smaller than the limiting values shown in Fig. 12.2.

Equation (12.32), when plotted for long tubes on the same coordinates, gives a straight line with a slope of -0.20 for Reynolds numbers above 6000. This line is drawn in the right-hand region of Fig. 12.3.

The curved lines between Reynolds numbers of 2100 and 6000 represent the transition region. The effect of L/D is pronounced at the lower Reynolds numbers in this region and fades out as a Reynolds number of 6000 is approached.

Figure 12.3 is a summary chart that can be used for the entire range of Reynolds numbers from 1000 to 30,000. Beyond its lower and upper limits, Eqs. (12.25) and (12.32) respectively, can be used.

Example 12.3. A light motor oil with the characteristics given below and in Table 12.3 is to be heated from 150 to 250°F (65.5 to 121.1°C) in a $\frac{1}{4}$-in. (6.35-mm) Schedule 40 pipe 15 ft (4.57 m) long. The pipe wall is at 350°F (176.7°C). How much oil can be heated in this pipe, in pounds per hour? What coefficient can be expected? The properties of the oil are as follows: The thermal conductivity is 0.082 Btu/ft-h-°F (0.142 W/m-°C). The specific heat is 0.48 Btu/lb-°F (2.01 J/g-°C).

Solution

Assume the flow is laminar and that the Graetz number is large enough for Eq. (12.25) to apply.

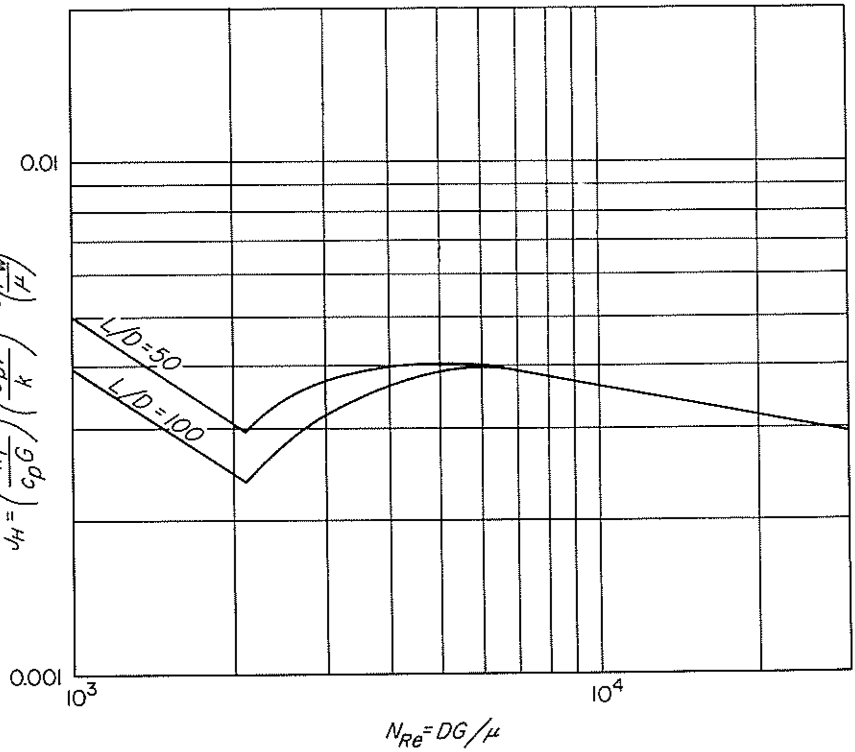

FIGURE 12.3
Heat transfer in transition range. (*By permission of author and publisher, from W. H. McAdams, Heat Transmission, 3rd ed. Copyright by author,* 1954, *McGraw-Hill Book Company.*)

Data for substitution into Eq. (12.25) are

$$\mu = \frac{6.0 + 3.3}{2} = 4.65 \text{ cP} \qquad \mu_w = 1.37 \text{ cP} \qquad D = \frac{0.364}{12} = 0.0303 \text{ ft} \qquad \text{(Appendix 5)}$$

$$\phi_v = \left(\frac{\mu}{\mu_w}\right)^{0.14} = \left(\frac{4.65}{1.37}\right)^{0.14} = 1.187 \qquad k = 0.082 \qquad c_p = 0.48$$

TABLE 12.3
Data for Example 12.3

Temperature		
°F	°C	Viscosity, cP
150	65.5	6.0
250	121.1	3.3
350	176.7	1.37

From Eq. (12.25)

$$\frac{0.0303h}{0.082} = 2 \times 1.187\left(\frac{0.48\dot{m}}{0.082 \times 15}\right)^{1/3}$$

From this, $h = 4.69\dot{m}^{1/3}$

Data for substitution into Eq. (12.18) are

$$\overline{\Delta T}_L = \frac{350 - 150 - (350 - 250)}{\ln (200/100)} = 144°\text{F}$$

$$L = 15 \qquad D = 0.0303 \qquad \overline{T}_b - T_a = 250 - 150 = 100°\text{F}$$

From Eq. (12.18)

$$h = \frac{0.48 \times 100\dot{m}}{\pi 0.0303 \times 15 \times 144} = 0.233\dot{m}$$

Then

$$4.69\dot{m}^{1/3} = 0.233\dot{m}$$

$$\dot{m} = \left(\frac{4.69}{0.233}\right)^{3/2} = 90.3 \text{ lb/h } (41.0 \text{ kg/h})$$

and

$$h = 0.233 \times 90.3 = 21.0 \text{ Btu/ft}^2\text{-h-°F } (119 \text{ W/m}^2\text{-°C})$$

$$N_{\text{Gz}} = \frac{\dot{m}c_p}{kL} = \frac{90.3 \times 0.48}{0.082 \times 15} = 35.2$$

This is large enough so that Eq. (12.25) applies. To check the assumption of laminar flow, the maximum Reynolds number, which exists at the outlet end of the pipe, is calculated:

$$N_{\text{Re}} = \frac{DG}{\mu} = \frac{D\dot{m}}{\pi(D^2/4)\mu} = \frac{4 \times 90.3}{\pi \times 0.0303 \times 3.3 \times 2.42}$$

$$= 475$$

This is well within the laminar range.

Transfer to Liquid Metals

Liquid metals are used for high-temperature heat transfer, especially in nuclear reactors. Liquid mercury, sodium, and a mixture of sodium and potassium called NaK are commonly used as carriers of sensible heat. Mercury vapor is also used as a carrier of latent heat. Temperatures of 1500°F and above are obtainable by using such metals. Molten metals have good specific heats, low viscosities, and high thermal conductivities. Their Prandtl numbers are therefore very low in comparison with those of ordinary fluids.

Equations such as (12.32), (12.34), and (12.55) do not apply at Prandtl numbers below about 0.5, because the mechanism of heat flow in a turbulent stream differs from that in fluids of ordinary Prandtl numbers. In the usual fluid, heat transfer by conduction is limited to the viscous sublayer when N_{Pr} is unity or more and occurs in the buffer zone only when the number is less than unity. In liquid metals, heat transfer by conduction is important throughout the entire turbulent core and may predominate over convection throughout the tube.

Much study has been given to liquid-metal heat transfer in recent years, primarily in connection with its use in nuclear reactors. Design equations, all based on heat-momentum analogies, are available for flow in tubes, in annuli, between plates, and outside bundles of tubes. The equations so obtained are of the form

$$N_{\mathrm{Nu}} = \alpha + \beta(\bar{\psi} N_{\mathrm{Pe}})^{\gamma} \tag{12.58}$$

where α, β, and γ are constants or functions of geometry and of whether the wall temperature or the flux is constant and $\bar{\psi}$ is the average value of $\varepsilon_H/\varepsilon_M$ across the stream. For circular pipes, $\alpha = 7.0$, $\beta = 0.025$, and $\gamma = 0.8$. For other shapes, more elaborate functions are needed. A correlation for $\bar{\psi}$ is given by the equation[1]

$$\bar{\psi} = 1 - \frac{1.82}{N_{\mathrm{Pr}}(\varepsilon_M/\nu)_m^{1.4}} \tag{12.59}$$

The quantity $(\varepsilon_M/\nu)_m$ is the maximum value of this ratio in the pipe, which is reached at a value of $y/r_w = \frac{5}{9}$. Equation (12.58) becomes, then,

$$N_{\mathrm{Nu}} = 7.0 + 0.025\left[N_{\mathrm{Pe}} - \frac{1.82 N_{\mathrm{Re}}}{(\varepsilon_M/\nu)_m^{1.4}}\right]^{0.8} \tag{12.60}$$

A correlation for $(\varepsilon_M/\nu)_m$ as a function of the Reynolds number is given in Fig. 12.4.

THE CRITICAL PECLET NUMBER. For a given Prandtl number, the Peclet number is proportional to the Reynolds number, because $N_{\mathrm{Pe}} = N_{\mathrm{Pr}} N_{\mathrm{Re}}$. At a definite value of N_{Pe} the bracketed term in Eq. (12.60) becomes zero. This situation corresponds to the point where conduction controls and the eddy diffusion no longer affects the heat transfer. Below the critical Peclet number, only the first term in Eq. (12.60) is needed, and $N_{\mathrm{Nu}} = 7.0$.

For laminar flow at uniform heat flux, by mathematical analysis $N_{\mathrm{Nu}} = \frac{48}{11} = 4.37$. This has been confirmed by experiment.

INTERPRETATION OF DIMENSIONLESS GROUPS. The relationships among several of the common dimensionless groups can be made clearer by considering them as ratios of various arbitrarily defined fluxes—that is, rates of flow per unit area.[9] The fluxes are:

A convective flux, J_{cv}
A conductive flux, J_c
A wall-transfer flux, J_w

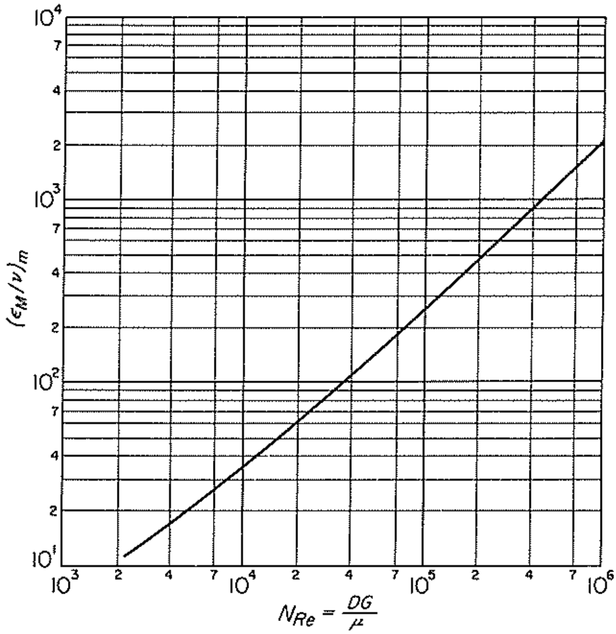

FIGURE 12.4
Values of $(\varepsilon_M/\nu)_m$ for fully developed turbulent flow of liquid metals in circular tubes.

These are defined as follows for a fluid flowing in fully developed flow in a long, straight pipe at velocity $\overline{V}$ and temperature T. The temperature of the fluid at the wall is T_w. Assume that the fluid is being cooled, so that $T > T_w$.

Convective fluxes. The mass velocity G is the mass rate of flow of the fluid per unit cross-sectional area of the pipe; it is therefore the convective flux of mass. Each kilogram or pound of fluid carries with it a certain amount of momentum, depending on the fluid velocity, so the convective flux of momentum can be approximated by the product of the mass flux and the velocity to give $G\overline{V}$ or its equivalent $\rho\overline{V}^2$. In the same way, the convective flux of heat, arbitrarily based on T_w as the reference or base temperature, is $Gc_p(T - T_w)$ or $\rho\overline{V}c_p(T - T_w)$.

Convective fluxes are vectors in the direction of the fluid flow.

Conductive fluxes. In Chap. 3, p. 47, it was pointed out that a shear stress resulting from viscous action may be considered to be a flux of momentum in the direction of the velocity gradient [see Eq. (3.4)]. The conductive flux of momentum is therefore defined here as $\mu\overline{V}/D$, where μ is the fluid viscosity and $\overline{V}/D$ is arbitrarily taken as a measure of the velocity gradient. The corresponding conductive flux of heat $(q/A)_c$ equals $k(T - T_w)/D$, where $(T - T_w)/D$ is an arbitrary measure of the temperature gradient [see Eq. (10.2)].

TABLE 12.4
Dimensionless flux ratios

Flux	For momentum transfer	For heat transfer
Convective, J_{cv}	$\rho \overline{V}^2$	$\rho \overline{V} c_p(T - T_w)$
Conductive, J_c	$\mu \overline{V}/D$	$k(T - T_w)/D$
Wall transfer, J_w	$\tau_w g_c$	$h(T - T_w)$

Conductive fluxes are in the direction normal to the direction of the fluid flow.

Wall-transfer fluxes. Also normal to the fluid-flow direction are the transfer fluxes at the pipe wall. These are the rates of transfer to or from the fluid per unit wall area. The wall flux of momentum is simply the shear stress τ_w (in fps units, $\tau_w g_c$). The wall flux of heat $(q/A)_w$ is $h(T - T_w)$, where h is the inside heat transfer coefficient.

These fluxes are summarized in Table 12.4.

Dimensionless groups.

I. For momentum transfer the ratio of the wall-transfer flux J_w to the convective flux J_{cv} is

$$\frac{J_w}{J_{cv}} = \frac{\tau_w g_c}{\rho \overline{V}^2} = \frac{f}{2} \tag{12.61}$$

For heat transfer the same ratio is

$$\frac{J_w}{J_{cv}} = \frac{h(T - T_w)}{\rho \overline{V} c_p(T - T_w)} = \frac{h}{c_p G} = N_{St} \tag{12.62}$$

II. The ratio of J_{cv} to J_c for momentum transfer is

$$\frac{J_{cv}}{J_c} = \frac{\rho \overline{V}^2}{\mu(\overline{V}/D)} = \frac{D\overline{V}\rho}{\mu} = N_{Re} \tag{12.63}$$

For heat transfer it is

$$\frac{J_{cv}}{J_c} = \frac{\rho \overline{V} c_p(T - T_w)}{k[(T - T_w)/D]} = \frac{D\overline{V}\rho c_p}{k} = N_{Pe} \tag{12.64}$$

III. Finally, the ratios of J_w to J_c are:

For momentum transfer

$$\frac{J_w}{J_c} = \frac{\tau_w g_c}{\mu(\overline{V}/D)} = \frac{D\tau_w g_c}{\mu \overline{V}} = \frac{f N_{Re}}{2} \tag{12.65}$$

For heat transfer

$$\frac{J_w}{J_c} = \frac{h(T - T_w)}{k[(T - T_w)/D]} = \frac{hD}{k} = N_{\mathrm{Nu}} \tag{12.66}$$

The dimensionless ratio in Eq. (12.65), unlike the other five, has not been given a name.

HEATING AND COOLING OF FLUIDS IN FORCED CONVECTION OUTSIDE TUBES

The mechanism of heat flow in forced convection outside tubes differs from that of flow inside tubes, because of differences in the fluid-flow mechanism. As has been shown on pages 59 and 106 no form drag exists inside tubes except perhaps for a short distance at the entrance end, and all friction is wall friction. Because of the lack of form friction, there is no variation in the local heat transfer at different points in a given circumference, and a close analogy exists between friction and heat transfer. An increase in heat transfer is obtainable at the expense of added friction simply by increasing the fluid velocity. Also, a sharp distinction exists between laminar and turbulent flow, which calls for different treatment of heat-transfer relations for the two flow regimes.

On the other hand, as shown on pages 143 to 151, in the flow of fluids across a cylindrical shape boundary-layer separation occurs, and a wake develops that causes form friction. No sharp distinction is found between laminar and turbulent flow, and a common correlation can be used for both low and high Reynolds numbers. Also, the local value of the heat-transfer coefficient varies from point to point around a circumference. In Fig. 12.5 the local value of the Nusselt number is plotted radially for all points around the circumference of the tube. At low Reynolds numbers, $N_{\mathrm{Nu},\theta}$ is a maximum at the front and back of the tube and a minimum at the sides. In practice, the variations in the local coefficient h_θ are often of no importance, and average values based on the entire circumference are used.

Radiation may be important in heat transfer to outside tube surfaces. Inside tubes, the surface cannot see surfaces other than the inside wall of the same tube, and heat flow by radiation does not occur. Outside tube surfaces, however, are necessarily in sight of external surfaces, if not nearby, at least at a distance, and the surrounding surfaces may be appreciably hotter or cooler than the tube wall. Heat flow by radiation, especially when the fluid is a gas, is appreciable in comparison with heat flow by conduction and convection. The total heat flow is then a sum of two independent flows, one by radiation and the other by conduction and convection. The relations given in the remainder of this section have to do with conduction and convection only. Radiation, as such and in combination with conduction and convection, is discussed in Chap. 14.

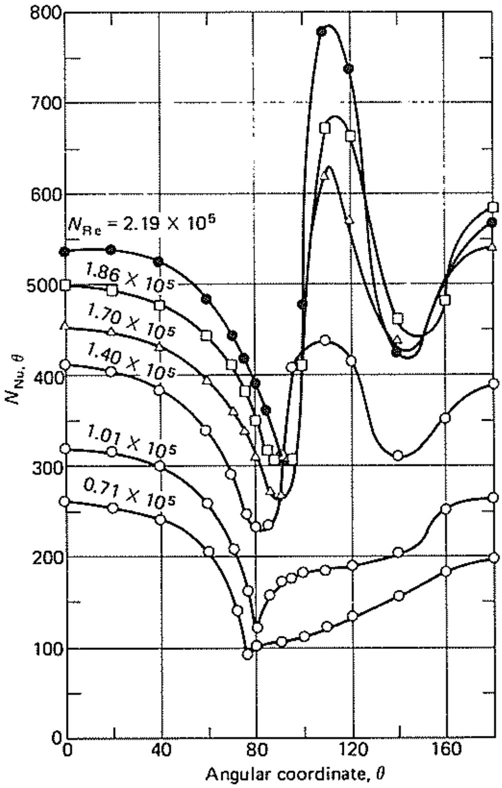

FIGURE 12.5
Local Nusselt number for airflow normal to a circular cylinder. (*Adapted with permission from W. H. Giedt, Trans. ASME*, 71:375, 1949.)

FLUIDS FLOWING NORMALLY TO A SINGLE TUBE. The variables affecting the coefficient of heat transfer to a fluid in forced convection outside a tube are D_o, the outside diameter of the tube; c_p, μ, and k, the specific heat at constant pressure, the viscosity, and the thermal conductivity, respectively, of the fluid; and G, the mass velocity of the fluid approaching the tube. Dimensional analysis gives, then, an equation of the type of Eq. (12.27):

$$\frac{h_o D_o}{k} = \psi_0\left(\frac{D_o G}{\mu}, \frac{c_p \mu}{k}\right) \tag{12.67}$$

Here, however, ends the similarity between the two types of process—the flow of heat to fluids inside tubes and the flow of heat to fluids outside tubes—and the functional relationships in the two cases differ.

For simple gases, for which the Prandtl number is nearly independent of temperature, the Nusselt number is a function only of the Reynolds number. Experimental data for air are plotted in this way in Fig. 12.6. The effect of radiation is not included in this curve, and radiation must be calculated separately.

The subscript f on the terms k_f and μ_f indicates that in using Fig. 12.6 these terms must be evaluated at the average film temperature T_f midway between the

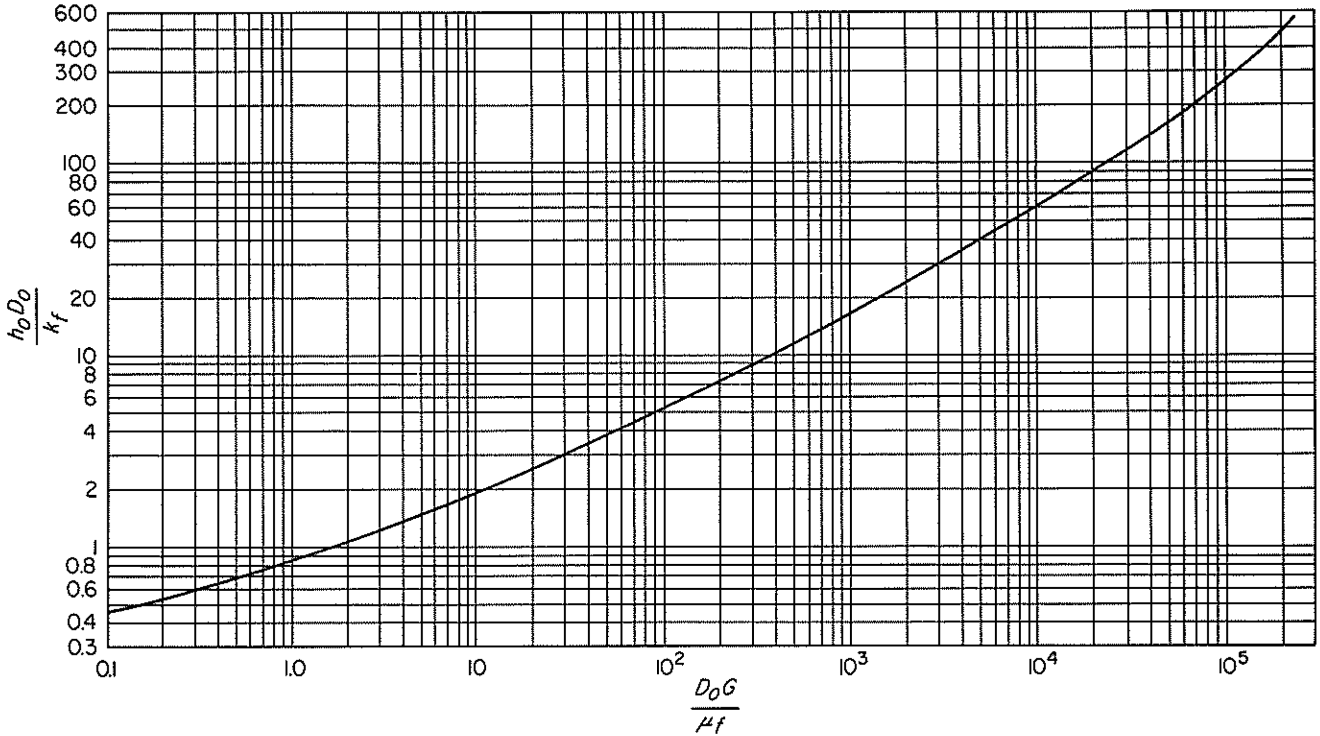

FIGURE 12.6
Heat transfer to air flowing normally to a single tube. (*By permission of author and publisher, from W. H. McAdams, Heat Transmission,* 3rd *ed. Copyright by author,* 1954, *McGraw-Hill Book Company.*)

wall temperature and the mean bulk temperature of the fluid $\overline{T}$. Therefore, T_f is given by the equation

$$T_f = \tfrac{1}{2}(T_w + \overline{T}) \tag{12.68}$$

Figure 12.6 can be used for both heating and cooling.

For heating and cooling liquids flowing normally to single cylinders the following equation is used[4a]:

$$\frac{h_o D_o}{k_f}\left(\frac{c_p \mu_f}{k_f}\right)^{-0.3} = 0.35 + 0.56\left(\frac{D_o G}{\mu_f}\right)^{0.52} \tag{12.69}$$

This equation can also be used for gases from $N_{Re} = 1$ to $N_{Re} = 10^4$, but it gives lower values of the Nusselt number than Fig. 12.6 at higher Reynolds numbers. Equation (12.69) is plotted in j-factor form in Fig. 21.4, in the section of Chap. 21 dealing with the analogies between heat and mass transfer.

Heat-transfer data for flow normal to cylinders of noncircular cross section are given in the literature.[4b] Banks of tubes across which the fluid flows are common in industrial exchangers. Problems of heat flow in tube banks are discussed in Chap. 15.

FLOW PAST SINGLE SPHERES. For heat transfer between a flowing fluid and the surface of a single sphere the following equation is recommended:

$$\frac{h_o D_p}{k_f} = 2.0 + 0.60\left(\frac{D_p G}{\mu_f}\right)^{0.50}\left(\frac{c_p \mu_f}{k_f}\right)^{1/3} \tag{12.70}$$

where D_p is the diameter of the sphere. In a completely stagnant stream the Nusselt number, $h_o D_p / k_f$, is equal to 2.0. A plot of Eq. (12.70) is given in Fig. 21.5.

HEAT TRANSFER IN PACKED BEDS. Data for heat transfer between fluids and beds of various kinds of particles can be obtained from Fig. 21.5 or from Eq. (21.62) by replacing N_{Sc} with N_{Pr} and N_{Sh} with N_{Nu}.

NATURAL CONVECTION

As an example of natural convection, consider a hot, vertical plate in contact with the air in a room. The temperature of the air in contact with the plate will be that of the surface of the plate, and a temperature gradient will exist from the plate out into the room. At the bottom of the plate, the temperature gradient is steep, as shown by the full line marked "$Z = 10$ mm" in Fig. 12.7. At distances above the bottom of the plate, the gradient becomes less steep, as shown by the full curve marked "$Z = 240$ mm" of Fig. 12.7. At a height of about 600 mm from the bottom of the plate, the temperature-distance curves approach an asymptotic condition and do not change with further increase in height.

The density of the heated air immediately adjacent to the plate is less than that of the unheated air at a distance from the plate, and the buoyancy of the hot

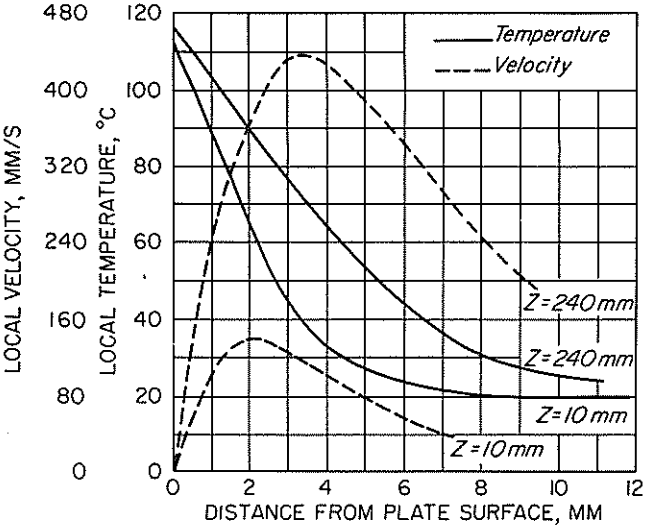

FIGURE 12.7
Velocity and temperature gradients, natural convection from heated vertical plate. (*By permission of author and publisher, from W. H. McAdams, Heat Transmission, 3rd ed. Copyright by author,* 1954, *McGraw-Hill Book Company.*)

air causes an unbalance between the vertical layers of air of differing density. The unbalanced forces generate a circulation by which hot air near the plate rises and cold air flows toward the plate from the room to replenish the rising airstream. A velocity gradient near the plate is formed. Since the velocities of the air in contact with the plate and that out in the room are both zero, the velocity is a maximum at a definite distance from the wall. The velocity reaches its maximum a few millimeters from the surface of the plate. The dashed curves in Fig. 12.7 show the velocity gradients for heights of 10 and 240 mm above the bottom of the plate. For tall plates, an asymptotic condition is approached.

The temperature difference between the surface of the plate and the air in the room at a distance from the plate causes a transfer of heat by conduction into the current of gas next to the wall, and the stream carries the heat away by convection in a direction parallel to the plate.

The natural convection currents surrounding a hot, horizontal pipe are more complicated than those adjacent to a vertical heated plate, but the mechanism of the process is similar. The layers of air immediately next to the bottom and sides of the pipe are heated and tend to rise. The rising layers of hot air, one on each side of the pipe, separate from the pipe at points short of the top center of the pipe and form two independent rising currents with a zone of relatively stagnant and unheated air between them.

Natural convection in liquids follows the same pattern, because liquids are also less dense hot than cold. The buoyancy of heated liquid layers near a hot surface generate convection currents just as in gases.

On the assumption that h depends upon pipe diameter, specific heat, thermal conductivity, viscosity, coefficient of thermal expansion, the acceleration of gravity, and temperature difference, dimensional analysis gives

$$\frac{hD_o}{k} = \Phi\left(\frac{c_p\mu}{k}, \frac{D_o^3\rho^2 g}{\mu^2}, \beta\,\Delta T\right) \tag{12.71}$$

Since the effect of β is through buoyancy in a gravitational field, the product $g\beta\,\Delta T$ acts as a single factor, and the last two groups fuse into a dimensionless group called the *Grashof number* N_{Gr}.

For single horizontal cylinders, the heat-transfer coefficient can be correlated by an equation containing three dimensionless groups, the Nusselt number, the Prandtl number, and the Grashof number, or specifically,

$$\frac{hD_o}{k_f} = \Phi\left(\frac{c_p\mu_f}{k_f}, \frac{D_o^3\rho_f^2\beta g\,\Delta T_o}{\mu_f^2}\right) \tag{12.72}$$

where h = average heat-transfer coefficient, based on entire pipe surface
D_o = outside pipe diameter
k_f = thermal conductivity of fluid
c_p = specific heat of fluid at constant pressure
ρ_f = density of fluid
β = coefficient of thermal expansion of fluid
g = acceleration of gravity
ΔT_o = average difference in temperature between outside of pipe and fluid distant from wall
μ_f = viscosity of fluid

The fluid properties μ_f, ρ_f, and k_f are evaluated at the mean film temperature [Eq. (12.68)]. Radiation is not accounted for in this equation.

The coefficient of thermal expansion β is a property of the fluid, defined as the fractional increase in volume at constant pressure of the fluid per degree of temperature change, or mathematically,

$$\beta = \frac{(\partial v/\partial T)_p}{v} \tag{12.73}$$

where v = specific volume of fluid
$(\partial v/\partial T)_p$ = rate of change of specific volume with temperature at constant pressure

For liquids, β can be considered constant over a definite temperature range and Eq. (12.73) written as

$$\beta = \frac{\Delta v/\Delta T}{\bar{v}} \tag{12.74}$$

where $\bar{v}$ is the average specific volume. In terms of density.

$$\beta = \frac{1/\rho_2 - 1/\rho_1}{(T_2 - T_1)(1/\rho_1 + 1/\rho_2)/2} = \frac{\rho_1 - \rho_2}{\bar{\rho}_a(T_2 - T_1)} \tag{12.75}$$

where $\bar{\rho}_a = (\rho_1 + \rho_2)/2$
ρ_1 = density of fluid at temperature T_1
ρ_2 = density of fluid at temperature T_2

For an ideal gas, since $v = RT/p$,

$$\left(\frac{\partial v}{\partial T}\right)_p = \frac{R}{p}$$

and using Eq. (12.73),

$$\beta = \frac{R/p}{RT/p} = \frac{1}{T} \tag{12.76}$$

The coefficient of thermal expansion of an ideal gas equals the reciprocal of the absolute temperature.

In Fig. 12.8 is shown a relationship, based on Eq. (12.72), which satisfactorily correlates experimental data for heat transfer from a single horizontal cylinder to

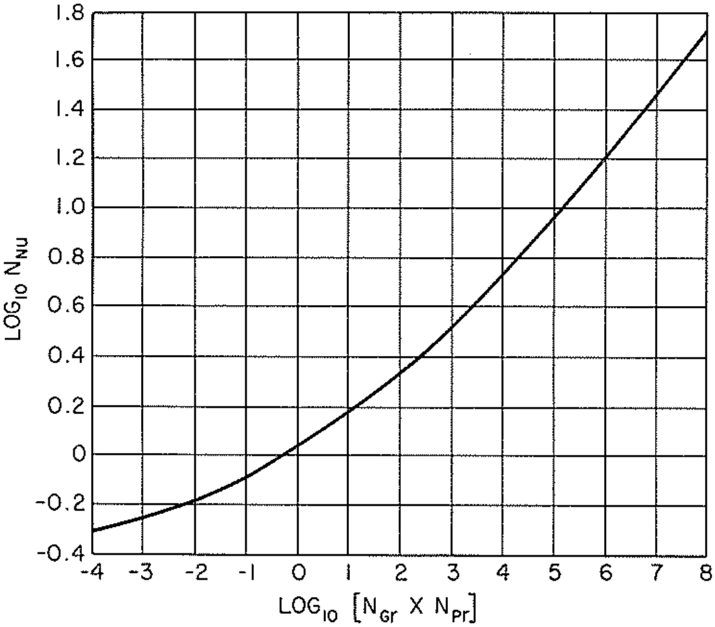

FIGURE 12.8
Heat transfer between single horizontal cylinders and fluids in natural convection.

liquids or gases. The range of variables covered by the single line of Fig. 12.8 is very great.

For magnitudes of $\log_{10} N_{Gr}N_{Pr}$ of 4 or more, the line of Fig. 12.8 follows closely the empirical equation[7b]

$$N_{Nu} = 0.53(N_{Gr}N_{Pr})_f^{0.25} \tag{12.77}$$

NATURAL CONVECTION TO AIR FROM VERTICAL SHAPES AND HORIZONTAL PLANES. Equations for heat transfer in natural convection between fluids and solids of definite geometric shape are of the form[7a]

$$\frac{hL}{k_f} = b\left[\frac{L^3\rho_f^2 g\beta_f\,\Delta T}{\mu_f^2}\left(\frac{c_p\mu}{k}\right)_f\right]^n \tag{12.78}$$

where b, n = const

L = height of vertical surface or length of horizontal square surface

Properties are taken at the mean film temperature. Equation (12.78) can be written

$$N_{Nu,f} = b(N_{Gr}N_{Pr})_f^n \tag{12.79}$$

Values of the constants b and n for various conditions are given in Table 12.5.

EFFECTS OF NATURAL CONVECTION IN LAMINAR-FLOW HEAT TRANSFER. In laminar flow at low velocities, in large pipes, and at large temperature drops, natural convection may occur to such an extent that the usual equations for laminar-flow heat transfer must be modified. The effect of natural convection in tubes is found almost entirely in laminar flow, as the higher velocities characteristic of flow in the transition and turbulent regimes overcome the relatively gentle currents of natural convection.

The effect of natural convection on the coefficient of heat transfer to fluids in laminar flow through horizontal tubes can be accounted for by multiplying the

TABLE 12.5
Values of constants in Eq. (12.79)†

System	Range of $N_{Gr}N_{Pr}$	b	n
Vertical plates, vertical cylinders	10^4–10^9	0.59	0.25
	10^9–10^{12}	0.13	0.333
Horizontal plates:			
Heated, facing upward or cooled, facing down	10^5–2×10^7	0.54	0.25
	2×10^7–3×10^{10}	0.14	0.333
Cooled, facing upward or heated, facing down	3×10^5–3×10^{10}	0.27	0.25

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coefficient h_i, computed from Eq. (12.57) or Fig. 12.3, by the factor[5]

$$\phi_n = \frac{2.25(1 + 0.010N_{\mathrm{Gr}}^{1/3})}{\log_{10} N_{\mathrm{Re}}} \tag{12.80}$$

Natural convection also occurs in vertical tubes, increasing the rate of heat flow, when the fluid flow is upward, to above that found in laminar flow only. The effect is marked at values of N_{Gr} between 10 and 10,000 and depends[7e] on the magnitude of the quantity $N_{\mathrm{Gr}}N_{\mathrm{Pr}}D/L$.

Example 12.4. Air at 1 atm pressure is passed through a horizontal 2-in (51-mm) Schedule 40 steam-jacketed steel pipe at a velocity of 1.5 ft/s (0.457 m/s) and an inlet temperature of 68°F (20°C). The pipe-wall temperature is 220°F (104.4°C). If the outlet air temperature is to be 188°F (86.7°C), how long must the heated section be?

Solution

To establish the flow regime, the Reynolds number based on the average temperature is calculated. The quantities needed are

$$\overline{T} = \frac{68 + 188}{2} = 128°\mathrm{F} \qquad D = \frac{2.067}{12} = 0.1723 \text{ ft} \qquad \text{(Appendix 5)}$$

$$\mu\,(\text{at } 128°\mathrm{F}) = 0.019 \text{ cP} \qquad \text{(Appendix 8)}$$

$$\rho\,(\text{at } 68°\mathrm{F}) = \frac{29}{359}\,\frac{492}{68 + 460} = 0.0753 \text{ lb/ft}^3$$

$$\overline{V}\rho = G = 1.5 \times 0.0753 \times 3600 = 406.4 \text{ lb/ft}^2\text{-h}$$

$$N_{\mathrm{Re}} = \frac{DG}{\mu} = \frac{0.1723 \times 406.4}{0.019 \times 2.42} = 1522$$

Hence flow is laminar, and Eq. (12.25) applies. The results may later require correction for the effect of natural convection using Eq. (12.80). To use Eq. (12.25), the following quantities are needed:

$$c_p\,(\text{at } 128°\mathrm{F}) = 0.25 \text{ Btu/lb-°F} \qquad \text{(Appendix 15)}$$

$$k\,(\text{at } 128°\mathrm{F}) = 0.0163 \text{ Btu/ft-h-°F} \qquad \text{(Appendix 12)}$$

(By linear interpolation)

$$\mu_w\,(\text{at } 220°\mathrm{F}) = 0.021 \text{ cP} \qquad \text{(Appendix 8)}$$

The internal cross-sectional area of pipe is

$$S = 0.02330 \text{ ft}^2 \qquad \text{(Appendix 5)}$$

The mass flow rate is

$$\dot{m} = GS = 406.4 \times 0.02330 = 9.47 \text{ lb/h}$$

The heat load is

$$q = \dot{m}c_p(\overline{T}_b - T_a) = 9.47 \times 0.25(188 - 68) = 284.1 \text{ Btu/h}$$

The logarithmic mean temperature difference is

$$\Delta T_1 = 220 - 188 = 32°\text{F} \qquad \Delta T_2 = 220 - 68 = 152°\text{F}$$

$$\overline{\Delta T_L} = \frac{152 - 32}{\ln(152/32)} = 77.0°\text{F}$$

The heat-transfer coefficient $h = q/A\,\overline{\Delta T_L}$. From Appendix 5, for 2-in. Schedule 40 pipe, $A = 0.541L$. Hence

$$h = \frac{284.1}{0.541L \times 77} = \frac{6.820}{L}$$

Also, from Eq. (12.25), the heat-transfer coefficient is

$$h = \frac{2k}{D}\left(\frac{\dot{m}c_p}{kL}\right)^{1/3}\left(\frac{\mu}{\mu_w}\right)^{0.14}$$

$$= \frac{2 \times 0.0163}{0.1723}\left(\frac{9.47 \times 0.25}{0.0163L}\right)^{1/3}\left(\frac{0.019}{0.021}\right)^{0.14} = \frac{0.9813}{L^{1/3}}$$

Equating the two relationships for h gives

$$\frac{0.9813}{L^{1/3}} = \frac{6.820}{L}$$

from which $L = 18.32$ ft (5.58 m).

This result is now corrected for the effect of natural convection using Eq. (12.80). This requires calculation of the Grashof number, for which the additional quantities needed are

$$\beta\,(\text{at } 128°\text{F}) = \frac{1}{460 + 128} = 0.0017°\text{R}^{-1}$$

$$\Delta T = 220 - 128 = 92°\text{F} \qquad \rho\,(\text{at } 128°\text{F}) = 0.0676\ \text{lb/ft}^3$$

The Grashof number is therefore

$$N_{\text{Gr}} = \frac{D^3\rho^2 g\beta\,\Delta T}{\mu^2}$$

$$= \frac{0.1723^3 \times 0.0676^2 \times 32.174 \times 0.0017 \times 92}{(0.019 \times 6.72 \times 10^{-4})^2} = 0.7192 \times 10^6$$

Hence, from Eq. (12.80),

$$\phi_n = \frac{2.25[1 + 0.01(0.7192 \times 10^6)^{1/3}]}{\log_{10} 1522} = 1.34$$

This factor is used to correct the value of L. Hence $L = 18.32/1.34 = 13.7$ ft(4.17 m).

SYMBOLS

A	Area, m^2 or ft^2
b	Constant in Eq. (12.78)
C	Constant
c_p	Specific heat at constant pressure, J/g-°C or Btu/lb-°F
D	Diameter, m or ft; D_e, equivalent diameter, $4r_H$; D_i, inside diameter; D_{ij}, inside diameter of jacket; D_{it}, inside diameter of inner tube; D_o, outside diameter; D_{ot}, outside diameter of inner tube; D_p, of spherical particle; $\overline{D_L}$, logarithmic mean
f	Fanning friction factor, dimensionless
G	Mass velocity, kg/m^2-s or lb/ft^2-s
g	Gravitational acceleration, m/s^2 or ft/s^2
g_c	Newton's-law proportionality factor, 32.174 ft-lb/lb$_f$-s^2
h	Individual heat-transfer coefficient, W/m^2-°C or Btu/ft^2-h-°F; h_a, based on arithmetic mean temperature drop; h_i, average over inside of tube; h_o, for outside of tube or particle; h_x, local value; h_{x1}, at trailing edge of plate; h_∞, for fully developed flow in long pipes; h_θ, local value outside tube
J	Flux, rate per unit area; J_c, conductive flux; J_{cv}, convective flux; J_w, wall transfer flux
j_H	Colburn j factor, $N_{St}(N_{Pr})^{2/3}\phi_v$, dimensionless
K'	Flow consistency index of non-newtonian fluid
k	Thermal conductivity, W/m-°C or Btu/ft-h-°F; k_f, at mean film temperature; k_m, of tube wall
L	Length, m or ft
m	Parameter in Eq. (12.26), $K'8^{n'-1}$; m_w, value at T_w
$\dot{m}$	Mass flow rate, kg/h or lb/h
N_{Fo}	Fourier number, $4kL/c_p\rho D^2\overline{V}$, dimensionless
N_{Gr}	Grashof number, $D^3\rho^2 g\beta\,\Delta T/\mu^2$, dimensionless
N_{Gz}	Graetz number, $\dot{m}c_p/kL$, dimensionless
N_{Nu}	Nusselt number, hD/k, dimensionless; $N_{Nu,f}$, at mean film temperature; $N_{Nu,x}$, local value on flat plate; $N_{Nu,\theta}$, local value on outside of tube
N_{Pe}	Peclet number, $\rho\overline{V}c_pD/k$, dimensionless
N_{Pr}	Prandtl number, $c_p\mu/k$, dimensionless
N_{Re}	Reynolds number, DG/μ, dimensionless; $N_{Re,x}$, local value on flat plate, $u_0x\rho/\mu$; $N_{Re,x1}$, at trailing edge of plate
N_{St}	Stanton number, h/c_pG, dimensionless
n	Constant in Eq. (12.78)
n'	Flow behavior index of non-newtonian fluid, dimensionless
p	Pressure, N/m^2 or lb$_f$/ft^2
q	Heat flow rate, W or Btu/h; q_c, by conduction; q_t, by turbulent convection
R	Gas-law constant

r	Radius, m or ft; r_H, hydraulic radius of channel; r_m, of tube; r_w, radius of pipe
S	Cross-sectional area of tube, m^2 or ft^2
T	Temperature, °C or °F; T_a, at inlet; T_b, at outlet; T_f, mean film temperature; T_i, instantaneous value; T_w, at wall or plate; T_∞, of approaching fluid; $\overline{T}$, average fluid temperature in tube; $\overline{T}_b$, bulk average fluid temperature at outlet; $\overline{T}_i$, time average of instantaneous values; T', fluctuating component; $\overline{T}'$, time average of fluctuating component
t_T	Total time of heating or cooling, s or h
U	Overall heat-transfer coefficient, W/m²-°C or Btu/ft²-h-°F; U_o, based on outside area; U_1, U_2, at ends of exchanger
u	Fluid velocity, m/s or ft/s; u_0, of approaching fluid; u', fluctuating component
$\overline{V}$	Volumetric average fluid velocity, m/s or ft/s
v	Specific volume, m³/kg or ft³/lb for liquids, m³/kg mol or ft³/lb mol for gases; $\overline{v}$, average value
v'	Fluctuating component of velocity in y direction; $\overline{v}'$, time-average value
x	Distance from leading edge of plate or from tube entrance, m or ft; x_w, wall thickness; x_0, at start of heated section; x_1, length of plate
y	Radial distance from wall, m or ft; also, boundary-layer thickness
Z	Height, m or ft

Greek letters

α	Thermal diffusivity, $k/\rho c_p$, m²/h or ft²/h; also constant in Eq. (12.58)
β	Coefficient of volumetric expansion, 1/°R or 1/K; also constant in Eq. (12.58); β_f, at mean film temperature
γ	Constant in Eq. (12.58)
ΔT	Temperature drop, °C or °F; ΔT_i, from inner wall of pipe to fluid; ΔT_o, from outside surface to fluid distant from wall; $\overline{\Delta T}_a$, arithmetic mean temperature drop; $\overline{\Delta T}_L$, logarithmic mean temperature drop
δ	Parameter in Eq. (12.26), $(3n' + 1)/4n'$
ε	Turbulent diffusivity, m²/h or ft²/h; ε_H, of heat; ε_M, of momentum
θ	Angular position on outside of tube
μ	Absolute viscosity, kg/m-s or lb/ft-s; μ_f, average value of liquid film; μ_w, value at wall temperature
ν	Kinematic viscosity, m²/h or ft²/h
ρ	Density, kg/m³ or lb/ft³; ρ_f, of liquid film; $\overline{\rho}_a$, arithmetic average value
τ	Shear stress, N/m² or lb_f/ft^2; τ_w, shear stress at pipe wall; τ_0, yield stress of plastic fluid
Φ, Φ_1	Function
ϕ_n	Natural-convection factor [Eq. (12.80)]
ϕ_v	Viscosity-correction factor, $(\mu/\mu_w)^{0.14}$

ψ Function in Eq. (12.35); also ratio of turbulent diffusivities, $\varepsilon_H/\varepsilon_M$; $\bar{\psi}$, average value

ψ_0 Function in Eq. (12.67)

PROBLEMS

12.1. Glycerin is flowing at the rate of 700 kg/h through a 30-mm-ID pipe. It enters a heated section 2.5 m long, the walls of which are at a uniform temperature of 115°C. The temperature of the glycerin at the entrance is 15°C. (*a*) If the velocity profile is parabolic, what would be the temperature of the glycerin at the outlet of the heated section? (*b*) What would the outlet temperature be if flow were rodlike? (*c*) How long would the heated section have to be to heat the glycerin essentially to 115°C?

12.2. Oil at 50°F is heated in a horizontal 2-in Schedule 40 steel pipe 60 ft long having a surface temperature of 120°F. The oil flow rate is 150 gal/h at inlet temperature. What will be the oil temperature as it leaves the pipe and after mixing? What is the average heat-transfer coefficient? Properties of the oil are given in Table 12.6.

TABLE 12.6
Data for Prob. 12.2

	60°F	120°F
Specific gravity, 60°F/60°F	0.79	0.74
Thermal conductivity, Btu/ft-h-°F	0.072	0.074
Viscosity, cP	18	8
Specific heat, Btu/lb-°F	0.75	0.75

12.3. Oil is flowing through a 75-mm-ID iron pipe at 1 m/s. It is being heated by steam outside the pipe, and the steam-film coefficient may be taken as 11 kW/m^2-°C. At a particular point along the pipe the oil is at 50°C, its density is 880 kg/m^3, its viscosity is 2.1 cP, its thermal conductivity is 0.135 W/m-°C, and its specific heat is 2.17 J/g-°C. What is the overall heat-transfer coefficient at this point based on the inside area of the pipe? If the steam temperature is 120°C, what is the heat flux at this point based on the outside area of the pipe?

12.4. Kerosene is heated by hot water in a shell-and-tube heater. The kerosene is inside the tubes, and the water is outside. The flow is countercurrent. The average temperature of the kerosene is 110°F, and the average linear velocity is 8 ft/s. The properties of the kerosene at 110°F are specific gravity 0.805, viscosity 1.5 cP, specific heat 0.583 Btu/lb-°F, and thermal conductivity 0.0875 Btu/ft-h-°F. The tubes are low-carbon steel $\frac{3}{4}$ in. OD by BWG 16. The heat-transfer coefficient on the shell side is 300 Btu/ft^2-h-°F. Calculate the overall coefficient based on the outside area of the tube.

12.5. Assume that the kerosene of Prob. 12.4 is replaced with water at 110°F and flowing at a velocity of 8 ft/s. What percentage increase in overall coefficient may be expected if the tube surfaces remain clean?

12.6. Both surfaces of the tube of Prob. 12.5 become fouled with deposits from the water. The fouling factors are 330 on the inside and 200 on the outside surfaces, both in

Btu/ft^2-h-°F. What percentage decrease in overall coefficient is caused by the fouling of the tube?

12.7. From the Colburn analogy, how much would the heat-transfer coefficient inside a 1-in. Schedule 40 steel pipe differ from that inside a 1-in. BWG 16 copper tube if the same fluid were flowing in each and the Reynolds number in both cases was 4×10^4?

12.8. Water must be heated from 15 to 50°C in a simple double-pipe heat exchanger at a rate of 3500 kg/h. The water is flowing inside the inner tube with steam condensing at 110°C on the outside. The tube wall is so thin that the wall resistance may be neglected. Assume that the steam-film coefficient h_o is 11 kW/m^2-°C. What is the length of the shortest heat exchanger that will heat the water to the desired temperature? Average properties of water are as follows:

$$\rho = 993 \text{ kg/m}^3 \qquad k = 0.61 \text{ W/m-°C} \qquad \mu = 0.78 \text{ cP} \qquad c_p = 4.19 \text{ J/g-°C}$$

Hint: Find the optimum diameter for the tube.

12.9. Since the Prandtl number and the heat capacity of air are nearly independent of temperature, Eq. (12.32) seems to indicate that h_i for air increases with $\mu^{0.2}$. (*a*) Explain this anomaly and determine the approximate dependence of h_i on temperature, using $h_i \propto T^n$. (*b*) How does h_i for air vary with temperature if the linear velocity, rather than the mass velocity, is kept constant?

12.10. Air is flowing through a steam-heated tubular heater under such conditions that the steam and wall resistances are negligible in comparison with the air-side resistance. Assuming that each of the following factors is changed in turn but that all other original factors remain constant, calculate the percentage variation in $q/\overline{\Delta t}_L$ that accompanies each change. (*a*) Double the pressure on the gas but keep fixed the mass flow rate of the air. (*b*) Double the mass flow rate of the air. (*c*) Double the number of tubes in the heater. (*d*) Halve the diameter of the tubes.

12.11. A sodium-potassium alloy (78 percent K) is to be circulated through $\frac{1}{2}$-in.-ID tubes in a reactor core for cooling. The liquid-metal inlet temperature and velocity are to be 580°F and 32 ft/s. If the tubes are 3 ft long and have an inside surface temperature of 720°F, find the coolant temperature rise and the energy gain per pound of liquid metal. Properties of NaK (78 percent K) are as follows:

$$\rho = 45 \text{ lb/ft}^3 \qquad k = 179 \text{ Btu/ft-h-°F} \qquad \mu = 0.16 \text{ cP} \qquad c_p = 0.21 \text{ Btu/lb-°F}$$

12.12. In a catalytic cracking regenerator, catalyst particles at 600°C are injected into air at 700°C in a fluidized bed. Neglecting the chemical reaction, how long would it take for a 50-μm particle to be heated to within 5°C of the air temperature? Assume the heat-transfer coefficient is the same as for a spherical particle falling at its terminal velocity.

12.13. In a pilot plant, a viscous oil is being cooled from 200 to 110°C in a 1.0-in. jacketed pipe with water flowing in the jacket at an average temperature of 30°C. To get greater cooling of the oil, it has been suggested that the exchanger be replaced with one having a greater inside diameter (1.5-in.) but the same length. (*a*) If the oil is in laminar flow in the 1.0-in. pipe, what change in exit temperature might result from using the larger exchanger? (*b*) Repeat assuming the oil is in turbulent flow.

12.14. In the manufacture of nitric acid, air containing 10 percent ammonia is passed through a pack of fine-mesh wire screens of Pt/Rh alloy. (*a*) Calculate the heat-transfer coefficient for air at 500°C flowing at a superficial velocity of 20 ft/s past wires 0.5 mm in diameter. (*b*) If the surface area of the wire screen is 3.7 cm^2/cm^2 of cross section, what is the temperature change in air, initially at 500°C, flowing through one screen if the surface of the wires is at 900°C?

12.15. Water at 15°C is flowing at right angles across a heated 25-mm-OD cylinder, the surface temperature of which is 120°C. The approach velocity of the water is 1 m/s. (*a*) What is the heat flux, in kilowatts per square meter, from the surface of the cylinder to the water? (*b*) What would be the flux if the cylinder were replaced by a 25-mm-OD sphere, also with a surface temperature of 120°C?

12.16. Water is heated from 15 to 65°C in a steam-heated horizontal 50-mm-ID tube. The steam temperature is 120°C. The average Reynolds number of the water is 450. The individual coefficient of the water is controlling. By what percentage would natural convection increase the total rate of heat transfer over that predicted for purely laminar flow? Compare your answer with the increase indicated in Example 12.4.

12.17. A large tank of water is heated by natural convection from submerged horizontal steam pipes. The pipes are 3-in Schedule 40 steel. When the steam pressure is atmospheric and the water temperature is 80°F, what is the rate of heat transfer to the water in Btu per hour per foot of pipe length?

12.18. Calculate (*a*) the overall coefficient U for heat transfer through a vertical glass window from a room at 70°F to still air at 0°F. Assume that a single pane of glass is $\frac{1}{8}$ in. thick and 4 ft high. (*b*) Calculate U for a thermopane window with a $\frac{1}{2}$-in. air space between the two panes. For this glass $k = 0.4$ Btu/h-ft-°F.

12.19. How does U for a thermopane window depend on the spacing between the panes?

REFERENCES

1. Dwyer, O. E.: *AIChE J.*, **9**:261 (1963).
2. Eckert, E. R. G., and J. F. Gross: *Introduction to Heat and Mass Transfer*, McGraw-Hill, New York, 1963 pp. 110–114.
3. Friend, W. L., and A. B. Metzer: *AIChE J.*, **4**:393 (1958).
4. Gebhart, B.: *Heat Transfer*, 2nd ed., McGraw-Hill, New York, 1971; (*a*) p. 272, (*b*) p. 274, (*c*) p. 283.
5. Kern, D. Q., and D. F. Othmer: *Trans. AIChE*, **39**:517 (1943).
6. Knudsen, J. G., and D. L. Katz: *Fluid Dynamics and Heat Transfer*, McGraw-Hill, New York, 1958; (*a*) pp. 361–390, (*b*) pp. 400–403, (*c*) p. 439.
7. McAdams, W. H.: *Heat Transmission*, 3rd ed., McGraw-Hill, New York, 1954; (*a*) pp. 172, 180, (*b*) p. 177, (*c*) p. 215, (*d*) p. 230, (*e*) p. 234.
8. Sieder, E. N., and G. E. Tate: *Ind. Eng. Chem.*, **28**:1429 (1936).
9. Smith, J. C.: *Can. J. Chem. Eng.*, **39**:106 (1961).
10. Wilkinson, W. L.: *Non-Newtonian Fluids*, Pergamon, London, 1960, p. 104.