Solution to Homework N^o 3: particulate separation systems

a.

1. In the laminar regime, separation efficiency is given by:

$$\eta = \frac{H_1}{H} = \frac{Lv_{py}}{Hv_x} \tag{1}$$

where H_1 is the vertical position (the highest inlet point of particles) in the inlet section for which particles can still be collected by gravity within a distance L inside the settling chamber whereas H is the height of the chamber. To calculate the length of the chamber assuming a chamber width $W = 2\ m$ at maximum, we can rewrite the particle collection efficiency equation as a function of W and L. Since the gas flow rate is $Q = WHv_x$, substitution in 1 gives

$$\eta = \frac{Lv_{py}W}{Hv_xW} = \frac{Lv_{py}W}{Q} \tag{2}$$

which can be solved for L imposing $W = 2\ m$ and $\eta = 1$. We get $L = Q/Wv_{yp}$ where v_{yp} is the particle settling velocity given by:

$$v_{yp} = \tau_p g = \frac{\rho_p D_p^2}{18\mu} g \tag{3}$$

2. If the sizing of the settling chamber is made considering turbulent flow conditions the particle collection efficiency becomes:

$$\eta = 1 - \exp\left(-\frac{v_{py}L}{Hv_x}\right) \tag{4}$$

$$= 1 - \exp\left(-\frac{v_{py}LW}{Q}\right) \tag{5}$$

$$\tag{6}$$

from which we can calculate L as:

$$L = -\frac{Q\ln(1-\eta)}{Wv_{py}} \tag{7}$$

b.

1. The flow is turbulent is the Reynolds number is larger than about 2000

$$Re = \frac{\rho v D_H}{\mu} \tag{8}$$

where D_H is the pipe hydraulic diameter, defined as $D_H = 4A/p$ with A area and and p wetted perimeter of the cross section of the flow. For the settling chamber we have $D_H = 4WH/2(W+H) = 2\ m$ and $v = Q/WH = 1\ m/s$. The Reynolds number is $Re = 1.5 \cdot 10^5 > 2000$ and the flow is turbulent.

2. Particle separation efficiency is the turbulent flow regime is

$$\eta(D_p) = 1 - \exp\left(-\frac{A_c u_t}{Q}\right) = 1 - \exp\left(-\frac{u_t L}{u\Delta H}\right) \tag{9}$$

To have a separation efficiency equal to 90% for particles having diameter equal to 10 μm within a given chamber length L, la distance between deposition plates ΔH should satisfy equation 9, i.e.:

$$\Delta H = -\frac{Lu_t}{u\ln(1-\eta)} = 6.51u_t = 3.94 \cdot 10^{-2} m \tag{10}$$

Therefore, we need to install $N = H/\Delta H = 50.7$ collection planes. Since this number is not an integer, we choose the larger closest integer, $N = 51$.

3. For the number of collecting plates selected, particle collection efficiency is:

$$\eta(D_p) = 1 - \exp\left(-\frac{NLWu_t}{Q}\right) = \tag{11}$$

$$= 1 - \exp\left(-\frac{Lu_t}{u\Delta H}\right) = 0.95 = 95\% \tag{12}$$

4. Overall collection efficiency depends on the fractional (or grade) collection efficiency and on the particle size distribution:

$$\eta_{tot} = \sum_i f_i(D_p)\eta(D_p) \tag{13}$$

Values of grade efficiency are shown in Table **??**, and the overall collection efficiency is $\eta_{tot} = 87.95\%$.

D_p	$1\mu m$	$5\mu m$	$10\mu m$	$15\mu m$	$20\mu m$
%	5	10	50	20	15
f_i	0.05	0.10	0.50	0.20	0.15
v_{sett}	$7.87 \cdot 10^{-5}$	0.0019	0.0078	0.0177	0.0315
$\eta(D_p)$, [%]	2.96	52.89	95.07	99.88	99.99

c.

1. The particle dynamic equation is

$$m_p \frac{\mathrm{d}v_p}{\mathrm{d}t} = -(\rho_p - \rho)Vg + \tag{14}$$

$$+\frac{1}{2}\rho C_D \frac{\pi D_p^2}{4}(v - v_p) \mid v - v_p \mid \tag{15}$$

and particles are suspended in the fluid (i.e. move at the same velocity of the fluid) at starting time

($v_p(0) = v$). Heavy particles suspended in a quiescent fluid settle down by gravity. in this case, the fluid is moving upward and dragging the particles against gravity. Particle settling at the bottom of the pipe is possible if, at steady state, the particle velocity is downward directed ($v_p < 0$). At steady state, the equation becomes:

$$0 = -(\rho_p - \rho)Vg + \frac{1}{2}\rho C_D \frac{\pi D_p^2}{4}(v - v_p) \mid v - v_p \mid \quad (16)$$

$$(\rho_p - \rho)Vg = 3\pi\mu D_p(v - v_p) \quad (17)$$

assuming Stokes regime for the drag coefficient. We get:

$$(v - v_p) = \frac{(\rho_p - \rho)D_p^2}{18\mu} g \quad (18)$$

$$v_p = v - \frac{(\rho_p - \rho)D_p^2}{18\mu} g \quad (19)$$

For the particles to settle down should be

$$v_p < 0 \;\rightarrow\; v - \frac{(\rho_p - \rho)D_p^2}{18\mu} g < 0 \quad (20)$$

$$\rightarrow\; v < \frac{(\rho_p - \rho)D_p^2}{18\mu} g \quad (21)$$

i.e. the upward directed flow velocity should be smaller than the (downward directed) particle settling velocity.

2. The time evolution of the particle velocity is the solution of the differential equation

$$\rho_p \frac{\pi D_p^3}{6}\frac{\mathrm{d}v_p}{\mathrm{d}t} = -(\rho_p - \rho)\frac{\pi D_p^3}{6} g + 3\pi\mu D_p(v - v_p) \quad (22)$$

$$\frac{\mathrm{d}v_p}{\mathrm{d}t} = -\frac{(\rho_p - \rho)g}{\rho_p} + \frac{(v - v_p)}{\tau_p} \quad (23)$$

which is given by:

$$(v - v_p(t)) = \tau_p g(1 - \exp(-t/\tau_p)) \quad (24)$$

$$v_p(t) = v - \tau_p g(1 - \exp(-t/\tau_p)) \quad (25)$$

This is an exponentially decreasing function with positive initial value, which is decreasing and becomes negative at steady state. Integrating the particle velocity in time we can calculate the particle position along the vertical direction, $z_p(t)$:

$$z_p(t) = \int v_p(t)\mathrm{d}t = vt - \tau_p g(t + \tau_p \exp(-t/\tau_p)) \quad (26)$$

The upward pipe of the elutriator should be longer than the maximum height reached by the particles over time, which can calculated as the value of $z_p(t)$ at the time in which the velocity becomes zero ($z_p = max \rightarrow \mathrm{d}z_p/\mathrm{d}t = v_p = 0$). If the fluid velocity is $v = 0.8\tau_p g$ the time for motion inversion is $t = -\tau_p \ln 0.2$. Substituting this value into equation 26 we get for the maximum height reached by the particles $z_{max} = 0.52\tau_p^2 g$.

d.

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1. Using practical cyclone design equation, the separation efficiency for particles of diameter D_p is given by

$$\eta(D_p) = 1 - \exp\left[-2\left[\frac{KQ\tau}{MN_c D_c^3}\right]^{M/2}\right] \quad (27)$$

where $N_c = 450 \cdot 2 = 900$ is the number of cyclones, $M = 1/(m+1)$ depends on the flow field inside the cyclone

$$m = 1 - (1 - 0.67D_c^{0.14})\left(\frac{T}{283}\right)^{0.3} = 0.54 \quad (28)$$

For Stairmand cyclones K is 551.3 and τ s the particle characteristic time $\tau = \rho_p D_p^2/18\mu$. For $D_p = 10\mu m$, $\tau = 3.58 \cdot 10^{-4} s$ and the particle collection efficiency is 95.13%. Pressure losses are given by

$$\Delta P = \frac{N_H \rho Q^2}{2K_a^2 K_b^2 N_c^2 D_c^4} \quad (29)$$

with $N_H = 6.4$, $K_a = 0.5$ and $K_b = 0.2$ for Stairmand type cyclone. We get $\Delta P = 2.13 \cdot 10^3 Pa$.

2. To obtain the same separation efficiency using a single cyclone, the cyclone diameter should be calculated as

$$0.95 = \eta(D_p) = 1 - \exp\left[-2\left[\frac{KQ\tau}{MD_c^3}\right]^{M/2}\right] \quad (30)$$

where the unknown is D_c. We get $D_c = 2.43\ m$ and the pressure loss is $1.93 \cdot 10^5 Pa$.

3.
   - design alternative 1: multi-cyclones collector
     The total annual cost associated with this alternative is given by:

     $$TAC = k_1 N_c ab + k_2 \frac{P}{\eta} t + k_3 N_c \quad (31)$$

     with $k_1 = 7000\$/10$, $k_3 = 72/10$ and $k_2 = 0.08\$/kWh$, $t = 8000\ h$ and $\eta = 0.65$. The theoretical compressor power consumption is:

     $$P = \Delta P \cdot Q = 3.5145 \cdot 10^5 W = 351.45\ kW \quad (32)$$

   - design alternative 2: single cyclone collector
     The total annual cost is given by

     $$TAC = k_4(ab)^{0.903} \quad (33)$$

     with $k_4 = 57800\$/10$, and the theoretical compressor power consumption is:

     $$P = \Delta P \cdot Q = 3.184 \cdot 10^7 W = 31840\ kW \quad (34)$$

Cost	Alt 1	Alt 2
EC	10,417.5	3591.9
CE	346,041.2	31,349,982.4
C_{tot}	356,458.7	31,353,574.4

For both alternatives, $a = k_a D_c$ and $b = k_b D_c$ with $k_a = 0.5$ and $k_b = 0.2$. Calculated costs are: Therefore, even if equipment costs are larger, the multi-cyclones collector allows a significant abatement of operating costs.

4. The optimal number of cyclones of the multi-cyclones collector is the one minimizing the total annual cost. The total annual cost depends on the number and size of cyclonic units according to the following equation:

$$TAC = k_1 k_a k_b N_c D_c^2 + k_3 N_c + k_2 N_{ore} P = \quad (35)$$
$$= k_1 k_a k_b N_c D_c^2 + k_3 N_c + \quad (36)$$
$$+ k_2 N_{ore} Q \frac{N_H}{2} \rho \frac{Q^2}{k_a^2 k_b^2 N_c^2 D_c^4} = \quad (37)$$
$$= A N_c D_c^2 + B N_c + \frac{C}{N_c^2 D_c^4} \quad (38)$$

The number of cyclones N_c and the cyclone diameter D_c are not independent variables since to satisfy the constraint on the collection efficiency

$$\eta(D_p) = 1 - \exp(-\phi D_p^M) = \overline{\eta} \quad (39)$$
$$= 1 - \exp\left(-2 D_p^M \left[\frac{K Q \rho_p (m+1)}{18 \mu N_c D_c^3}\right]^{M/2}\right) \quad (40)$$
$$\overline{\eta} = 1 - \exp\left(-\left(\frac{E}{N_c D_c^3}\right)^{M/2}\right) \quad (41)$$

for a given $\overline{\eta}$ we get a functional relationship between N_c and D_c:

$$N_c D_c^3 = \frac{E}{-\ln(1-\overline{\eta})^{M/2}} = F \quad (42)$$

Total annual costs can be rewritten as:

$$TAC = A\frac{F}{D_c} + B\frac{F}{D_c^3} + C\frac{D_c^2}{F^2} = f(D_c) \quad (43)$$

where the cyclone diameter is now the only unknown for cost optimization. Deriving the cost with respect to D_c, we get:

$$\frac{\partial TAC}{\partial D_c} = 0 = -A\frac{F}{D_c^2} - 3B\frac{F}{D_c^4} + 2C\frac{D_c}{F^2} \quad (44)$$
$$2\frac{C}{F^2} D_c^5 - AFD_c^2 - 3BF = 0 \quad (45)$$

which can be solved in D_c to find the optimal diameter. This value, substituted into equation 42, allows to calculate the optimal number of cyclones.

e.

1. The particle collection efficiency for a plate-plate electrostatic precipitator is given by:

$$\eta(D_p) = 1 - \exp\left(-\frac{A_c u_e}{Q}\right) = \quad (46)$$
$$= 1 - \exp\left(-\frac{u_e L H}{u W H}\right) \quad (47)$$

where W is the distance between plates, H and L are width and length of the ESP. Equation 46 can be rewritten as a function of the volume V of the ESP:

$$\eta(D_p) = 1 - \exp\left(-\frac{u_e L H W}{u W^2 H}\right) = \quad (48)$$
$$= 1 - \exp\left(-\frac{u_e V}{Q W}\right) \quad (49)$$

from which we get:

$$W = -\frac{u_e V}{Q \ln(1-\eta)} \quad (50)$$

Considering that

$$u_e = \frac{q_p E}{3\pi\mu D_p} = \frac{q_p \Delta V}{3\pi\mu D_p W} \quad (51)$$

we get

$$W = \sqrt{-\frac{q_p \Delta V V}{3\pi\mu D_p Q \ln(1-\eta)}} = 1.28\ m \quad (52)$$

2. The overall collection efficiency depends on the fractional collection efficiency and the particle size distribution:

$$\eta_{tot} = \sum_i f_i(D_p)\eta(D_p) \quad (53)$$

Fractional collection efficiency calculated for each particle diameter are shown in Table **??**; the overall collection efficiency for the given particle size distribution is $\eta_{tot} = 97.49\%$.

D_p, $[\mu m]$	f %	q_p	v_{ele}, $[m/s]$	η	$\eta \cdot f$
0.5	0.2	$1.60 \cdot 10^{-15}$	0.736	0.899	0.179
1.0	0.5	$6.40 \cdot 10^{-15}$	1.473	0.989	0.494
2.0	0.2	$2.56 \cdot 10^{-14}$	2.946	0.999	0.199
5.0	0.1	$1.60 \cdot 10^{-13}$	7.366	0.999	0.1

f.

1. Being the maximum power available to handle the flow fixed, the maximum pressure drop for the flow is fixed. This pressure drop is mainly given by the flow through the cyclone, since pressure drops are usually reduced inside ESPs. The pressure drop for the flow through the cyclone is given by:

$$\Delta p = N_H \frac{1}{2} \rho v_E^2 \tag{54}$$

with $N_H = 9.24$ for Swift type cyclone. The maximum pressure drop is $\Delta p = P/Q = 1500\ Pa$. The gas density at actual temperature and pressure is:

$$\rho = \frac{Mp}{RT} = 1.17\ kg/m^3 \tag{55}$$

and using this value we obtain $v_E = 16.66\ m/s$. The inlet velocity depends on the size of cyclone

$$v_E = \frac{Q}{k_a k_b D_c^2} \tag{56}$$

with $k_a = 0.44$, $k_b = 0.21$ for Swift type cyclone, from which we can calculate the cyclone diameter, $D_c = 2.55\ m$. For any larger diameter, the pressure drop through the cyclone will be lower than the fixed maximum.

2. The overall collection efficiency for the cyclone is given by

$$\eta_{tot} = \sum_i f_i(D_p)\eta(D_p) \tag{57}$$

where $\eta(D_p)$ are the grade efficiency values calculated for each particle size in the particle size distribution. We can calculate m as

$$m = 1 - (1 - 0.67 D_c^{0.14}) \left(\frac{T}{283}\right)^{0.3} = 0.76 \tag{58}$$

from which $M = 1/(m+1) = 0.568$. Cyclone efficiency is given by:

$$\eta(D_p) = 1 - \exp(-\psi D_p^M) \tag{59}$$

with

$$\psi = 2\left[\frac{KQ\rho_p(m+1)}{18\mu D_c^3}\right]^{M/2} = 554.26 \tag{60}$$

and $K = 699.2$. Grade efficiency values are shown in the Table and the overall collection efficiency is $\eta_{tot} = 37.78\%$

3. Particles having $D_p = 1\ \mu m$ can be collected with 19.48% separation efficiency in the first stage of separation (cyclone). The overall collection efficiency for the two-stages separation system is given by:

$$1 - \eta_{tot} = (1 - \eta_{cicl})(1 - \eta_{ESP}) = 1 - 0.999 = 0.001 \tag{61}$$

D_p, [μm]	η	peso	f	$f\eta$
0.5	0.1360	25	0.0694	0.0094
1	0.1948	125	0.3472	0.0676
5	0.4175	100	0.2778	0.1160
10	0.5512	75	0.2083	0.1148
20	0.6951	30	0.0833	0.0579
50	0.8645	5	0.0139	0.0120

From this we can calculate $\eta_{ESP}(1\ \mu m) = 99.875\%$. The fractional collection efficiency of the ESP depends on its geometrical characteristics:

$$\frac{A_c}{W} = -\ln(1 - \eta_{esp}(D_p))\frac{3\pi\mu D_p Q}{\sigma\pi D_p^2 \Delta V} = 190\ m^2/m \tag{62}$$

The required ratio of collecting area to plate distance can be obtained choosing $W = 0.2\ m$ and fixing a collecting area equal to 38 m^2, for instance obtained using 4 charged plates and 5 grounded plates (corresponding to 8 collection areas) having $H = 1.5\ m$ and $L = 3.167\ m$. The overall ESP width in this case would be $W_T = 8 \cdot 0.2 = 1.6\ m$.

4. Similarly as the cyclonic collector, the overall collection efficiency for the ESP is given by

$$\eta_{tot} = \sum_i f_i(D_p)\eta(D_p) \tag{63}$$

Values of grade efficiency are calculated for each particle size represented in the particle size distribution as shown in the Table and the overall collection efficiency for the ESP is equal to $\eta_{tot} = 99.71\%$.

D_p, [μm]	η_{ESP}	f %	$\eta \cdot f$
0.5	0.9646	0.0694	0.06698
1.0	0.9987	0.3472	0.34678
5	1	0.2777	0.27777
10	1	0.2083	0.20833
20	1	0.0833	0.08333
50	1	0.0138	0.01388

g.

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1. The air flow rate through the cyclone is

$$Q = \frac{\dot{m}}{\rho} = 46.29\ m^3/s \tag{64}$$

From practical cyclone design equation, we can calculate the (Stairmand type) cyclone diameter which allows to separate the particles with the desired efficiency. We have $M = 1/(m+1) = 0.57$ and

$$\eta(D_p = 100\ \mu m) = 0.99 = 1 - \exp(-\psi D_p^M) \tag{65}$$

from which

$$\psi = -\frac{\ln(1-\eta)}{D_p^M} = 877.50 \tag{66}$$

Since ψ is defined as

$$\psi = 2\left[\frac{KQ\rho_p(m+1)}{18\mu D_c^3}\right]^{M/2} \tag{67}$$

we can write D_c as

$$\frac{1}{D_c^3} = \left[\frac{\psi}{2}\right]^{2/M} \frac{18\mu}{KQ\rho_p(m+1)} \tag{68}$$

where $K = 551.3$ for Stairmand type cyclone. From calculations we get $D_c = 2.8\ m$.

2. The compressor power consumption is given by $P = \Delta p \cdot Q$ where

$$\Delta p = \frac{1}{2} N_H \rho v_E^2 \tag{69}$$

For a Stairmand type cyclone $k_a = 0.5$, $k_b = 0.2$, $N_H = 6.4$ and we get $v_E = Q/(k_a k_b D_c^2) = 58.39\ m/s$ and $\Delta p = 9819.59\ Pa$, from which $P = 454.61\ kW$.

3. Overall collection efficiency can be computed as

$$\eta_{tot} = \sum_i f_i(D_p)\eta(D_p) \tag{70}$$

where values of grade efficiency for each particle size are calculated using equation 65 and summarized in Table. The overall collection efficiency is $\eta_{tot} = 84.64\%$.

D_p, $[\mu m]$	η_{ESP}	f %	$\eta \cdot f$
1	0.2821	0.05	0.0141
5	0.5645	0.1	0.0564
10	0.7092	0.2	0.1418
50	0.9549	0.3	0.2864
100	0.99	0.2	0.198
150	0.9969	0.15	0.1495

h.

1. For a Lapple type cyclone is $k_a = 0.5$, $k_b = 0.25$, $K = 402.9$ and $N_H = 8$. From practical cyclone design equations we get

$$\eta(D_p) = 1 - \exp(-\psi D_p^M) \rightarrow \psi = \frac{-\ln(1-\eta)}{D_p^M} \tag{71}$$

and assuming $m = 0.38$ as first trial value we get $M = 1/(m+1) = 0.725$. To achieve the target collection efficiency $\eta = 0.5$ for $D_{p,1} = 10\ \mu m$ and $D_{p,2} = 2.5\ \mu m$ it should be $\psi_1 = 2910.81$ and $\psi_2 = 7948.57$. The value of ψ depends on the cyclone size

$$\psi = 2\left[\frac{KQ\rho_p(m+1)}{18\mu D_c^3}\right]^{M/2} \tag{72}$$

and isolating D_c we get:

$$\frac{1}{D_c^3} = \left[\frac{\psi}{2}\right]^{2/M} \frac{18\mu}{KQ\rho_p(m+1)} \tag{73}$$

from which

$$D_c = \left(\frac{2^{2/M} KQ\rho_p(m+1)}{18\mu\psi^{2/M}}\right)^{1/3} \tag{74}$$

Using the numerical values $D_{c,1} = 11.78\ cm$ and $D_{c,2} = 4.67\ cm$. The cyclone size should be smaller to collect smaller size particles. Checking the values of m used as first trial we get

$$m = 1 - \left(1 - 0.67 \cdot D_c^{0.14}\right)\left(\frac{T}{285}\right)^{0.3} \tag{75}$$

$m_1 = 0.47$ for the first cyclone, and $M = 0.68$, $\psi = 1746.57$ and $D_c = 12.82\ cm$ as corrected values; for the second cyclone we have $m_2 = 0.41$, and $M = 0.71$, $\psi = 6515.05$ and $D_c = 4.80\ cm$ as corrected values.

2. To separate $PM10$ and $PM2.5$ from the sample particles the larger particles should be collected first to collect only the smaller particles from the second cyclone. If the second cyclone $D_{c,2}$ would be the first in the series, separated particles would include 50% of the $D_p = 2.5\ \mu m$ particles together with a significant fraction (more than 50%) of all the particles larger in size.

3. The overall pressure drop across the two-stage separation system is given by the sum of the pressure drop across each cyclone:

$$\Delta p_{tot} = \Delta p_1 + \Delta p_2 \tag{76}$$

where

$$\Delta p_i = \frac{1}{2}\rho K_H v_{E,i}^2 = \frac{1}{2}\rho K_H \frac{Q^2}{k_a^2 k_b^2 D_{c,i}^4} \tag{77}$$

with $D_{c,i}$ cyclone diameter, $i = 1, 2$. The gas density is given by ideal gas law

$$\rho = \frac{pMM}{RT} = 1.04\ kg/m^3 \tag{78}$$

Using the numerical values we get $\Delta p_{tot} = 0.405 + 20.47 = 20.875\ Pa$.

i.

1. Oil droplets and sand particles separate by gravity. For a given particle size, the separation is easier for the phase for which the density difference with water is larger. The density difference is $\Delta\rho = 1000\ kg/m^3$ for sand particles and only $200\ kg/m^3$ for oil droplets. Therefore, for a given particle diameter, oil separation is harder than sand separation. We need to identify process conditions which allow to achieve the target collection efficiency. The collection efficiency is given by:

$$\eta(D_p) = 1 - \exp\left(-\frac{A_c v_t}{Q}\right) \quad (79)$$

where $A_c = LW$ is the collection area and $v_t = \tau_p g \Delta\rho/\rho_p$ is the particle settling velocity. Being the geometrical data known, we can calculate the flow rate which, flowing through one tank, allows to separate the oil droplets with the target efficiency

$$Q = \frac{A_c u_t}{-\ln(1-\eta)} \quad (80)$$

Since we have $v_t = 1.09\ mm/s$ and $A_c = 3\ m^2$, we get $Q = Q_{max} = 1.42 \cdot 10^{-3} m^3/s = 1.42\ l/s$. Any smaller flow rate can be conveniently processed by one tank. Since the flow o be processed is larger, we need to split it into many tanks working in parallel

$$N_{vasche} = \frac{Q}{Q_{max}} = 2.11 \to 3 \quad (81)$$

We need at least 3 tanks.

2. To calculate the sand and oil overall collection efficiency we use the equation

$$\eta_{tot} = \sum_i f_i(D_p)\eta(D_p) \quad (82)$$

where the values of grade efficiency calculated for each particle diameter are given by equation 79 where $Q' = Q/N_{vasche}$ and the settling velocity is that of the oil droplet or sand particle of the given size. Results are summarized in Tables. The overall collection efficiency is $\eta_{tot} = 99.45\%$ for sand particles and $\eta_{tot} = 84.78\%$ for oil droplets.

Sand

D_p, [μm]	m_p	f, [%]	v_t	η	$f \cdot \eta$
0.00005	325	0.325	0.0013	0.98	0.319
0.0001	450	0.45	0.0054	0.99	0.449
0.00015	225	0.225	0.0122	1	0.225

Oil

D_p, [μm]	m_p	f, [%]	v_t	η	$f \cdot \eta$
0.00005	0.5	0.294	0.0002	0.5584	0.1642
0.0001	1	0.588	0.001	0.9619	0.5658
0.0002	0.2	0.117	0.004	0.9999	0.1176

j.

1. In the first configuration, the overall collection efficiency is obtained from the two collecting devices working in series:

$$(1-\eta_{tot}) = (1-\eta_{grav})(1-\eta_{cicl}) \quad (83)$$

Since the geometrical characteristics of the the settling chamber are given, we can calculate η_{grav} and derive η_{cicl} from the equation above. For the settling chamber:

$$\eta_{grav} = 1 - \exp\left(-\frac{A_c u_t}{Q}\right) \quad (84)$$

with $A_c = LW = 10\ m^2$ collecting area and $u_t = \Delta\rho g D_p^2/18\mu$ settling velocity. The gas density can be calculated using the ideal gas law

$$\rho = \frac{pM}{RT} = 0.934\ kg/m^3 \quad (85)$$

and we get $\eta_{grav} = 0.5309 = 53.09\%$. From 83 we get $\eta_{cicl} = 0.9573 = 95.73\%$.

2. The size of the single cyclone (first design alternative) can be calculated from practical cyclone design equations. For a Swift type cyclone, $k_a = 0.44$, $k_b = 0.21$, $N_h = 9.24$ and $K = 699.2$. Using $m = 0.7$ we get $M = 1/(m+1) = 0.588$ and from the collection efficiency:

$$\eta_{cicl}(D_p) = 1 - \exp(-\psi D_p^M) \to \psi = \frac{-\ln(1-\eta)}{D_p^M} \quad (86)$$

$\psi = 711.13$. Using

$$\psi = 2\left[\frac{KQ\rho_p(m+1)}{18\mu D_c^3}\right]^{M/2} = 711.13 \quad (87)$$

and deriving D_c we get:

$$\frac{1}{D_c^3} = \left[\frac{\psi}{2}\right]^{2/M} \frac{18\mu}{KQ\rho_p(m+1)} \quad (88)$$

From calculation we get $D_c = 1.98\ m$. Pressure drops are given by

$$\Delta p = \frac{1}{2}\rho K_H v_E^2 = \frac{1}{2}\rho K_H \frac{Q^2}{k_a^2 k_b^2 D_{c,i}^4} \quad (89)$$

which for problem data give is $\Delta p = 131.088\ Pa$.

3. The size of a cyclone of the multi-cyclone collector system can be calculated using the same equations as before imposing a collection efficiency equal to

the overall target collection efficiency. Considering $\eta_{cicl} = 0.98$, and assuming $m = 0.59$ we get $M = 0.6289$ and $\psi = 1282.698$. The size of each cyclone is $D_c = 0.587\ m$ and the pressure drop is $\Delta p = 42.349\ Pa$.

4. The best design alternative depends on design constraints (e.g. space available for the installation of the particle collection system), on equipment costs and on operating costs. The pressure drop is three times larger for the first design alternative. The fingerprint on the ground of each design alternative is similar. In these conditions, the multi-cyclone collector could be more profitable (lower operating costs).

k.

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1. Particle collection efficiency for an ESP with 5 plates (4 channels) is given by:

$$\eta(D_p) = 1 - \exp\left(-\frac{A_c v_t}{Q}\right) \tag{90}$$

where $A_c = 4HL$ and v_t is the electric migration velocity given by

$$v_t = \frac{qE}{3\pi\mu D_p} = \frac{q\Delta V}{3\pi\mu D_p W'} \tag{91}$$

where $W' = W/N_c$ is the plate to plate distance. For $D_p = 20\ \mu m$ is $v_t = 0.1768\ m/s$. The volumetric gas flow rate through the ESP is

$$Q = \frac{\dot{m}}{\rho} = Q_{cn}\frac{\rho_{cn}}{\rho} = Q_{cn}\frac{T}{T_{cn}} = 4.78\ m^3/s \tag{92}$$

ESP collection efficiency is $\eta = 30.91\%$.

2. The overall target collection efficiency can be calculated as

$$\eta_{tot} = \frac{\dot{m}_{in} - \dot{m}_{out}}{\dot{m}_{in}} = 1 - \frac{C_2}{C_1} = 95\% \tag{93}$$

and will be obtained by the two devices (ESP and filter) in series:

$$(1 - \eta_{tot}) = (1 - \eta_{esp})(1 - \eta_{filter}) \tag{94}$$

We get

$$(1-\eta_{filter}) = \frac{(1 - \eta_{tot})}{(1 - \eta_{esp})} \rightarrow \eta_{filter} 1 - \frac{(1 - \eta_{tot})}{(1 - \eta_{esp})} = 92.76\% \tag{95}$$

l.

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1. The gas temperature at filter inlet ($T = 300^oC = 523.15\ K$) drives the selection of the filtering material which should by thermally resistant. From Table 10.3 of Benitez (p. 430), fiberglass could be a suitable material (fiberglass, $T_{lim} = 530\ K$). From Table 10.1 of Benitez (p.427) we can select a proper value for the gas to cloth ratio, i.e. for the filtration velocity, which depends on the material to be filtered and on the cleaning strategy: for fly ashes and reverse air cleaning we get $V = 1.02\ cm/s$. To achieve this filtration velocity, the filtration area should be $A = Q/V$, where Q is the volumetric gas flow rate at actual working conditions. The volumetric gas flow rate is given by

$$Q = \frac{\dot{m}}{\rho} = Q_{cn}\frac{\rho_{cn}}{\rho} = Q_{cn}\frac{T}{T_{cn}} = 10.49\ m^3/s \tag{96}$$

from which we calculate $A = 1028.4\ m^2$. From Table 10.2 of Benitez we get that, to use a reverse air or shaker cleaning strategy, the filtering area should be multiplied by a factor larger than 1 to account for the fraction of filtering area which is by-passed by the flow during the cleaning operations. From problem data (net filtering area $A = 1028\ m^2$) the multiplication factor is 1.5 and the overall filtering area is $A' = 1.5A = 1542.6\ m^2$.

2. The pressure drop across the filter increases in time according to

$$\Delta p(t) = S_e V + K_2 C_i V^2 t \tag{97}$$

where V is the filtration velocity, S_e is the pressure loss through the clean filter, and C_i is the particle concentration in the incoming stream (kg/m^3). From problem data we can calculate:

$$C_i = \frac{Q_{cn} * C}{Q} = 238.32\ mg/m^3 = 0.238\dot{1}0^{-3} kg/m^3 \tag{98}$$

and imposing $\Delta p(t_{clean} = p_{max}$ we can calculate the time after which the cleaning cycle should be performed to get the pressure drop below the maximum allowed:

$$t_{clean} = \frac{p_{max} - S_e V}{K_2 C_i V^2} \tag{99}$$

we get $t_{clean} = 1.019 \cdot 10^6 s \simeq 11.7\ days$.

3. 40% of the mass of particles has $D_p < 25\ \mu m$. This fraction of particles together with 5% of the particles larger than $25\mu m$ which are not collected by the pre-separator are the new particle load to the filter. The overall mass of particles is

$$\dot{m} = C \cdot Q_{cn} = 2500\ mg/s \tag{100}$$

of which $0.4\dot{m} = 1000\ mg/s$ is fines and $0.6\dot{m} \cdot 0.05 = 75\ mg/s$ is the unseparated fraction of particles larger than 25 μm. Overall mass downstream the cyclonic pre-separator is $\dot{m}_{tot} = 1075\ mg/s$. The concentration of particles loading the filter in this condition is $C_i = \dot{m}_{tot}/Q = 102.48\ mg/m^3 = 0.102 \cdot 10^{-3} kg/m^3$ (five times less than before) and using this value in 99 we get $t_{clean} \simeq 58 days$.